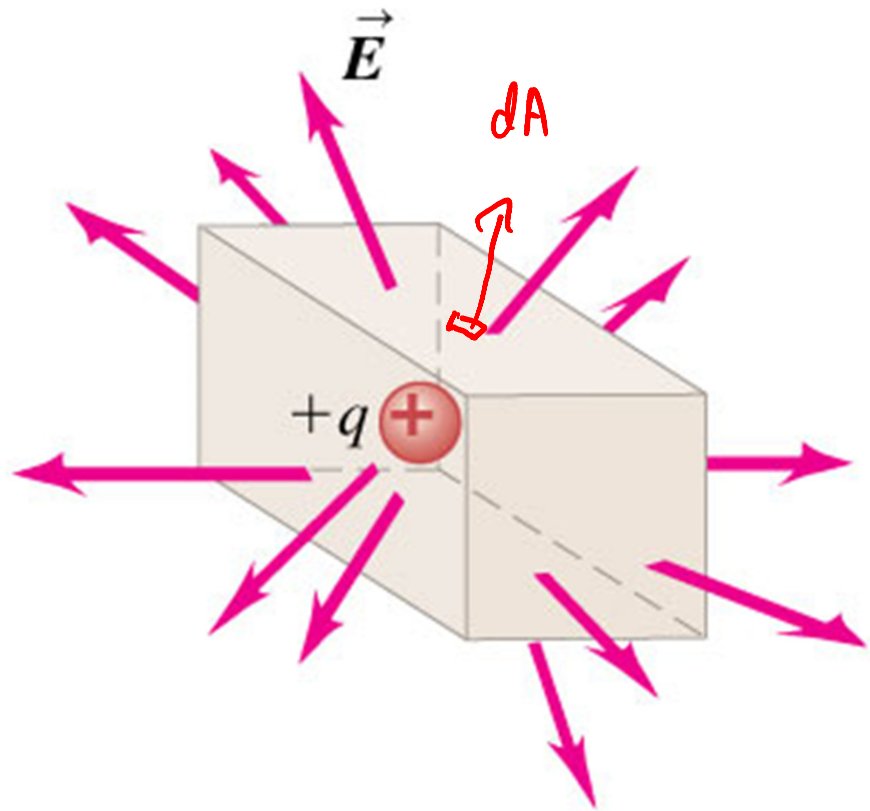
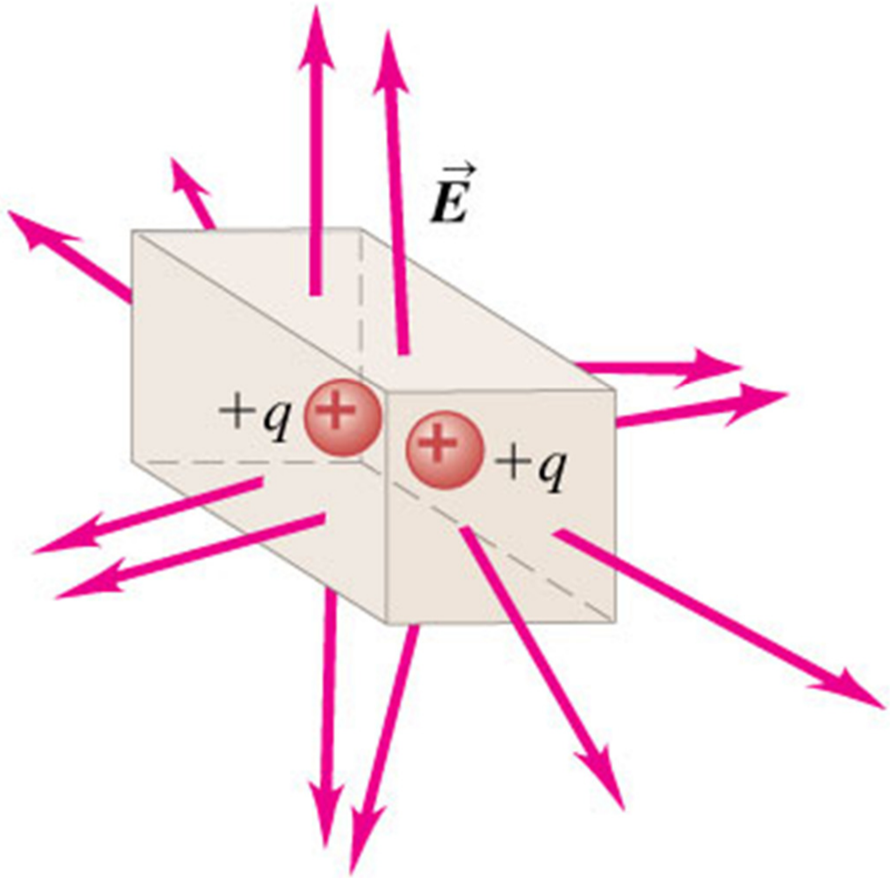
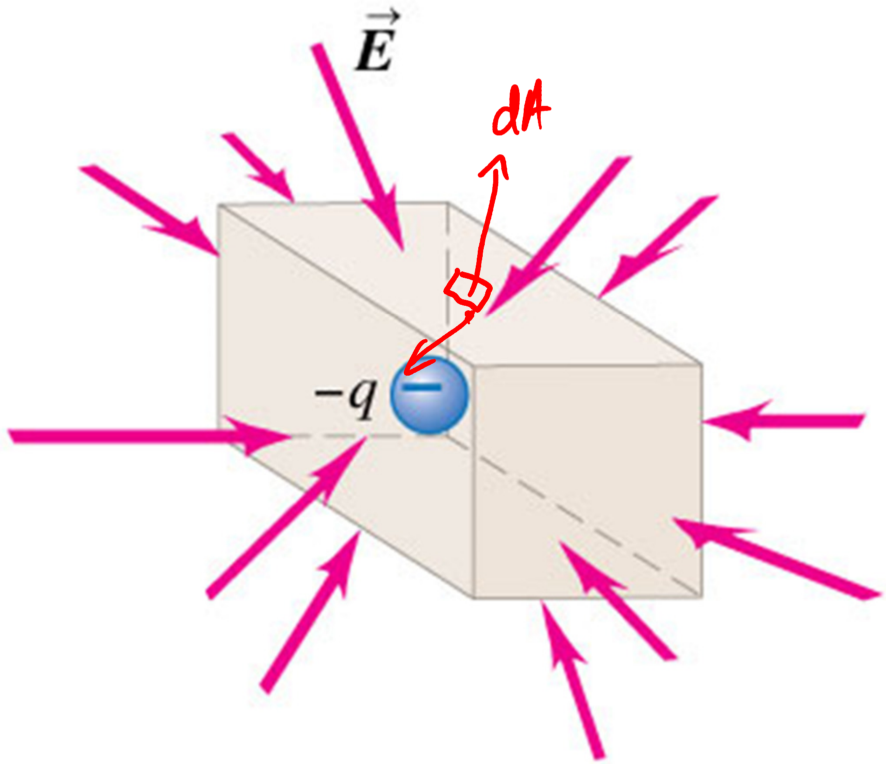
(a) Positive charge inside box, outward flux



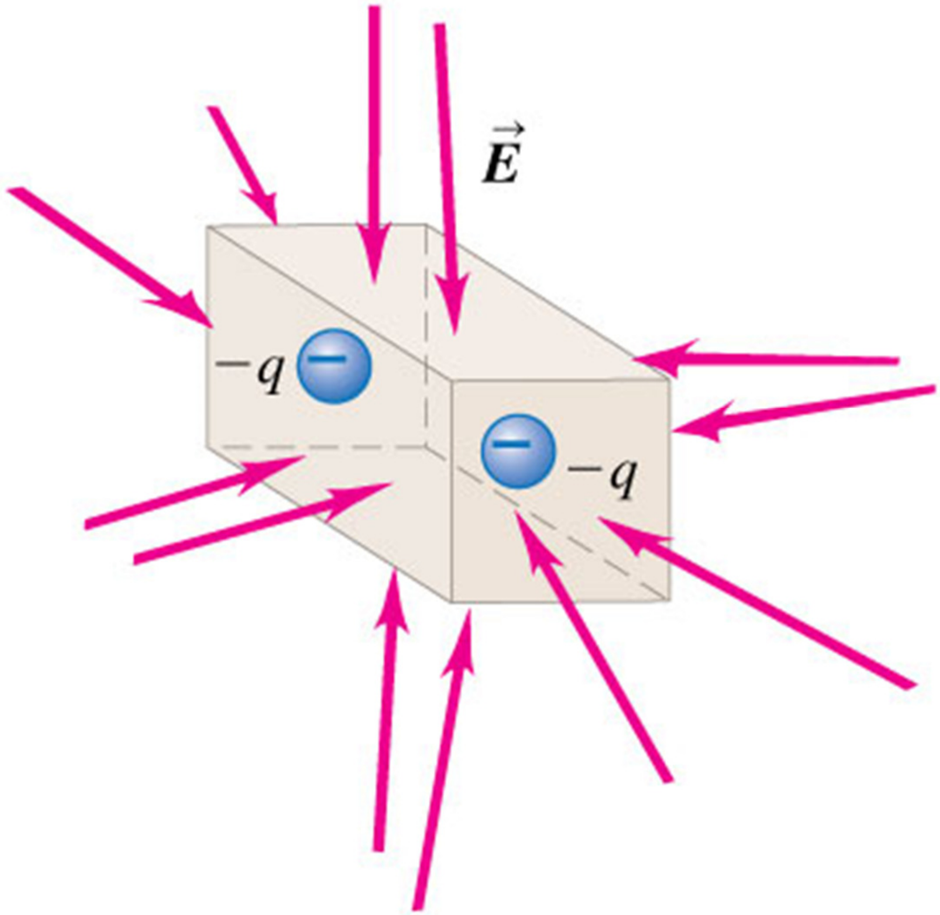
(b) Positive charges inside box, outward flux



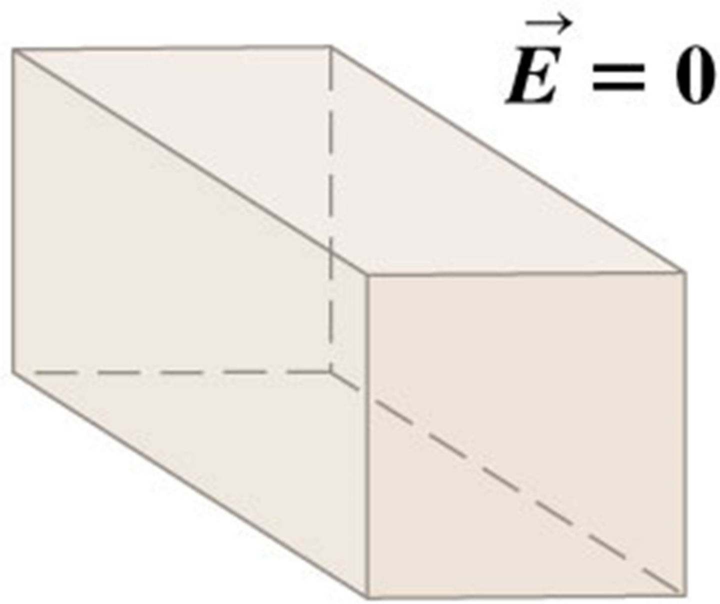
(c) Negative charge inside box, inward flux



(d) Negative charges inside box, inward flux

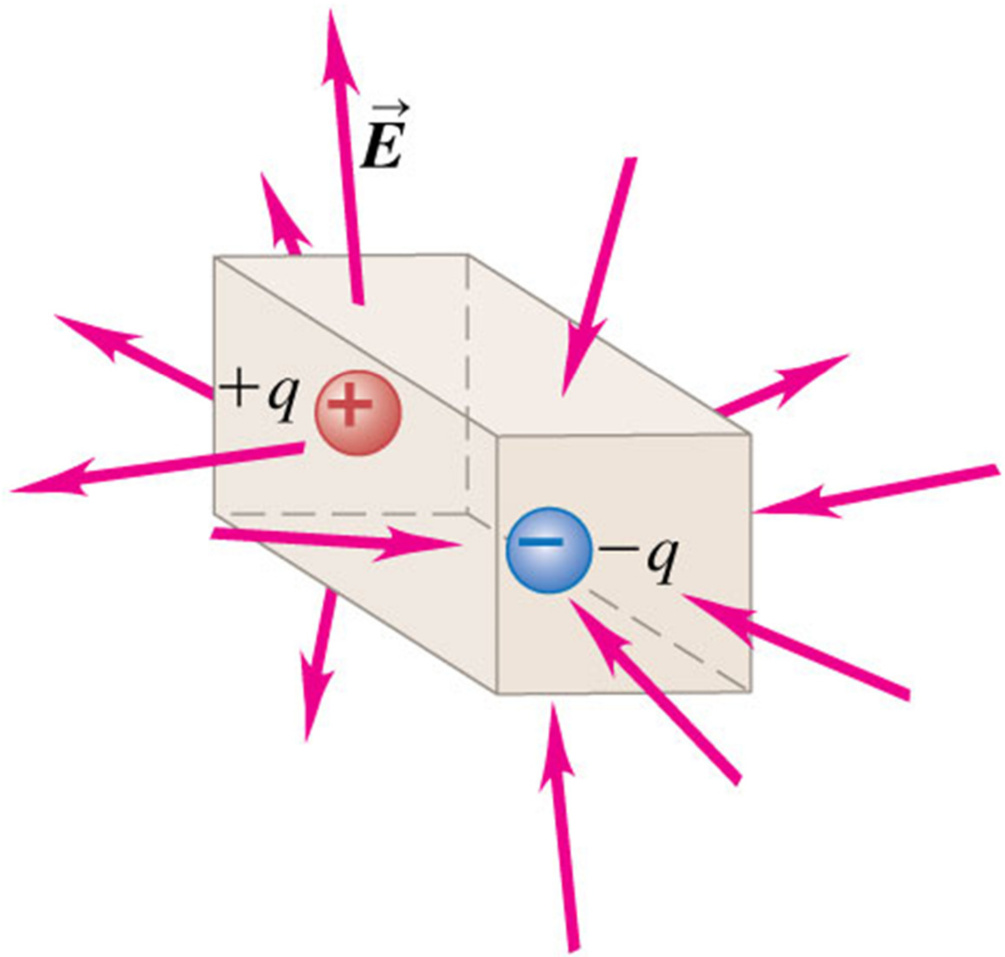


(a) No charge inside box, zero flux

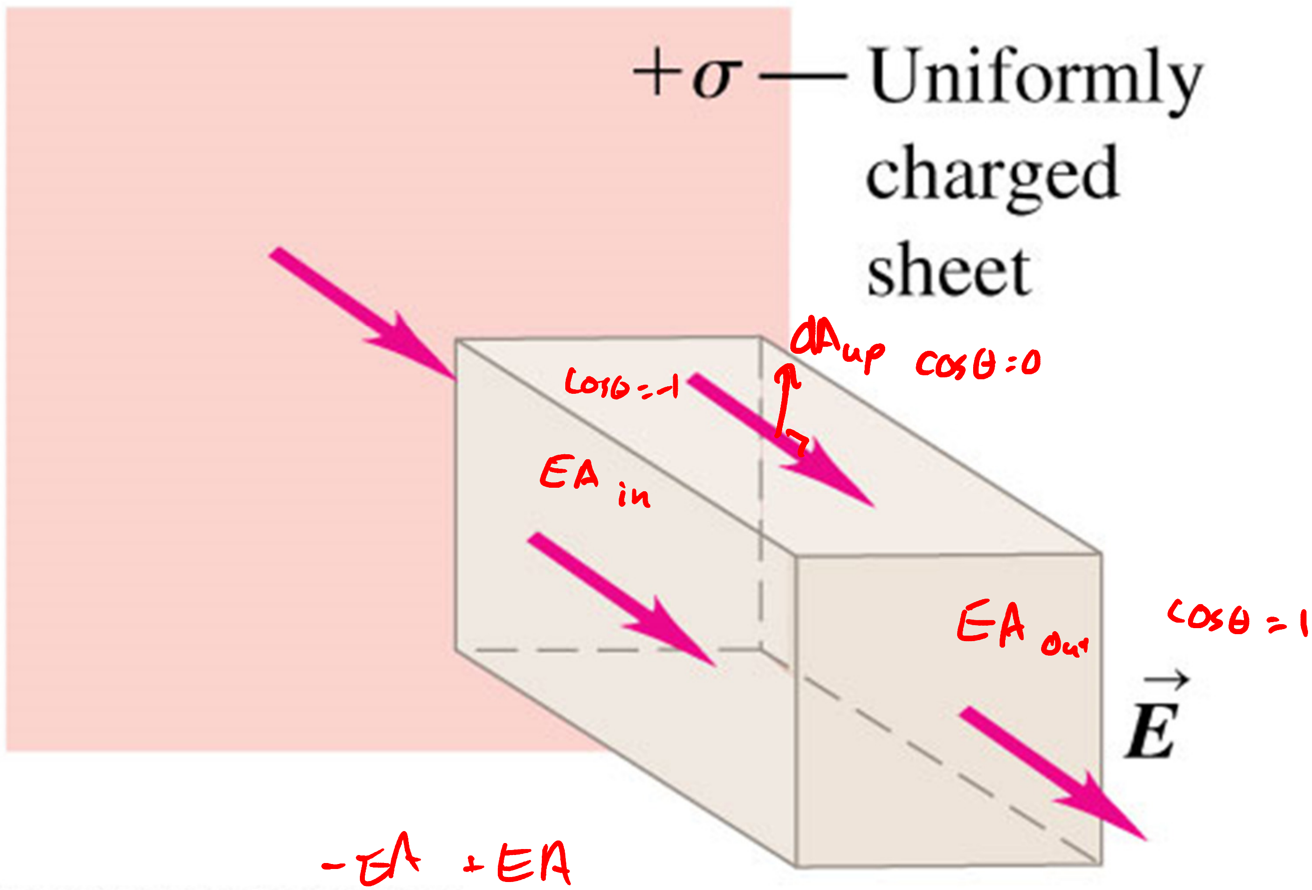


(b) Zero *net* charge inside box, inward flux cancels outward flux.

net $\Phi_E = 0$



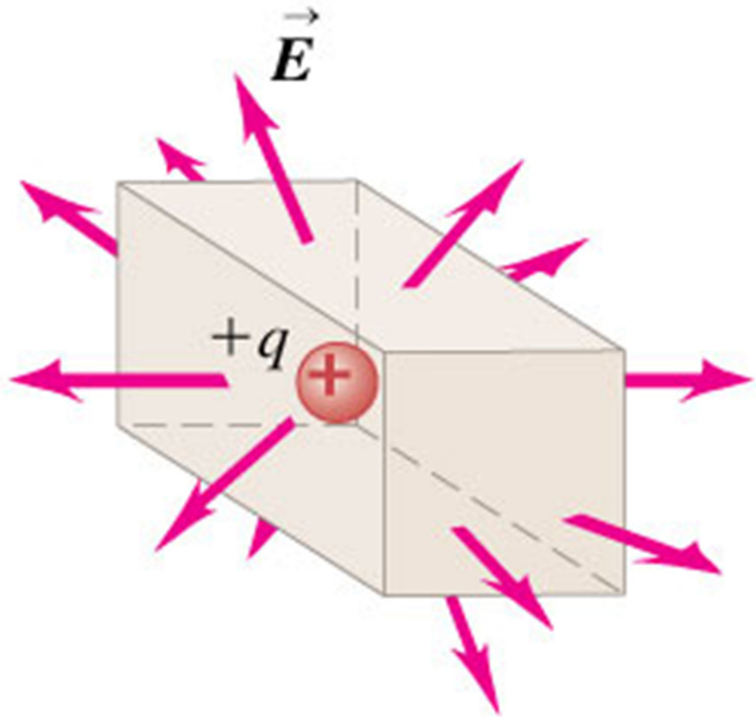
(c) No charge inside box,
inward flux cancels outward flux.



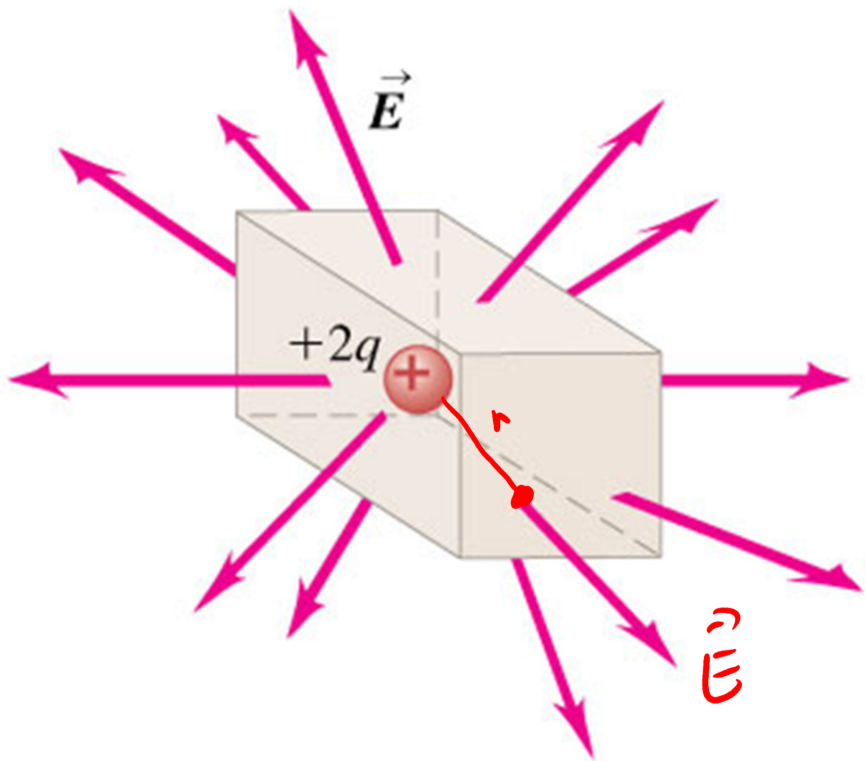



$$\Phi_{net} = \Phi_B + \Phi_F + \Phi_{Top} + \Phi_{Bot} + \Phi_L + \Phi_R = 0$$

(a) A box containing a charge



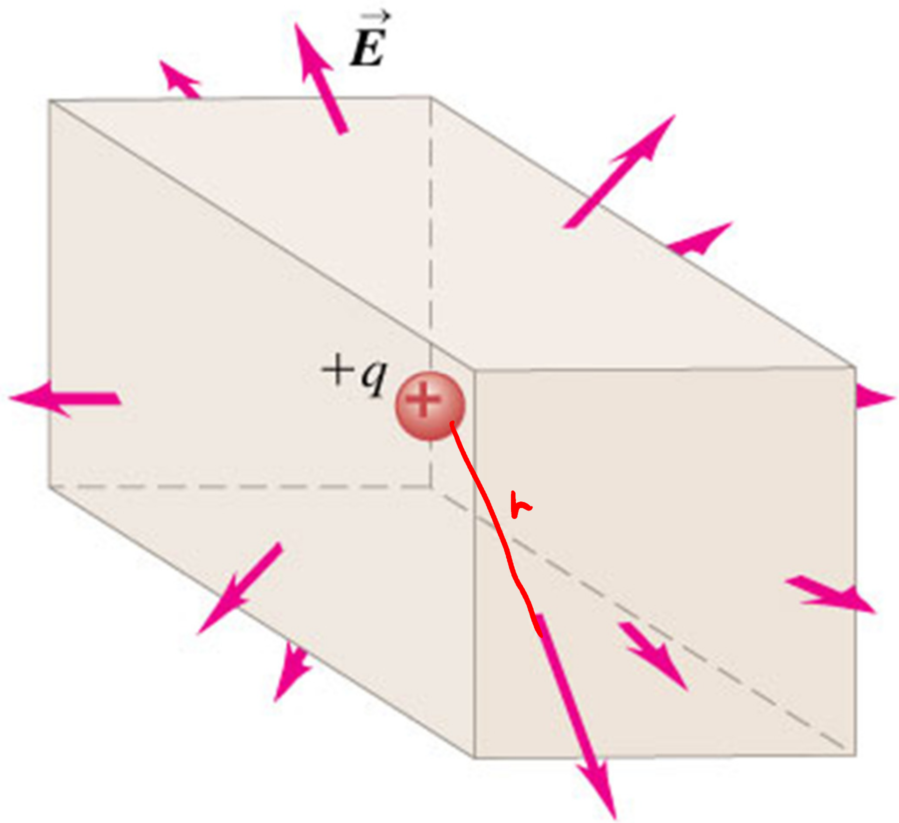
(b) Doubling the enclosed charge doubles the flux.



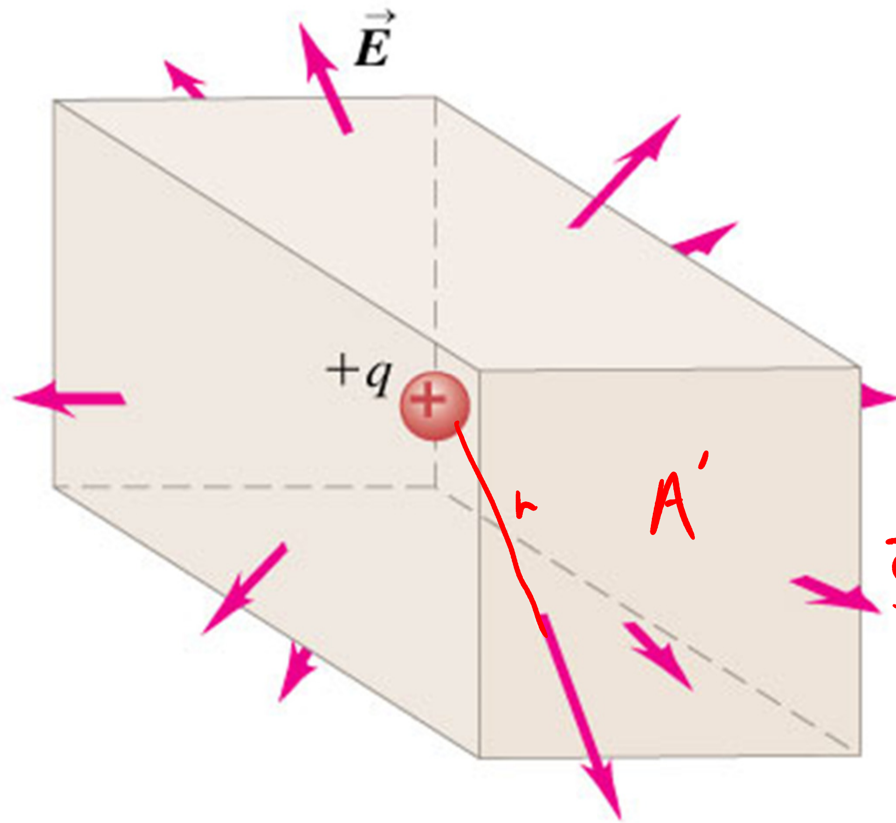



$$\vec{E} = \frac{k\,2q}{r^2}\hat{r}$$

What if we double the size of the box: what happens to the net flux?



**(c)** Doubling the box dimensions *does not change* the flux.






$A' = 4A$

$r' = 2r$

$E' = \frac{kq}{r'^2}$

$= \frac{1}{4}$ old $E$

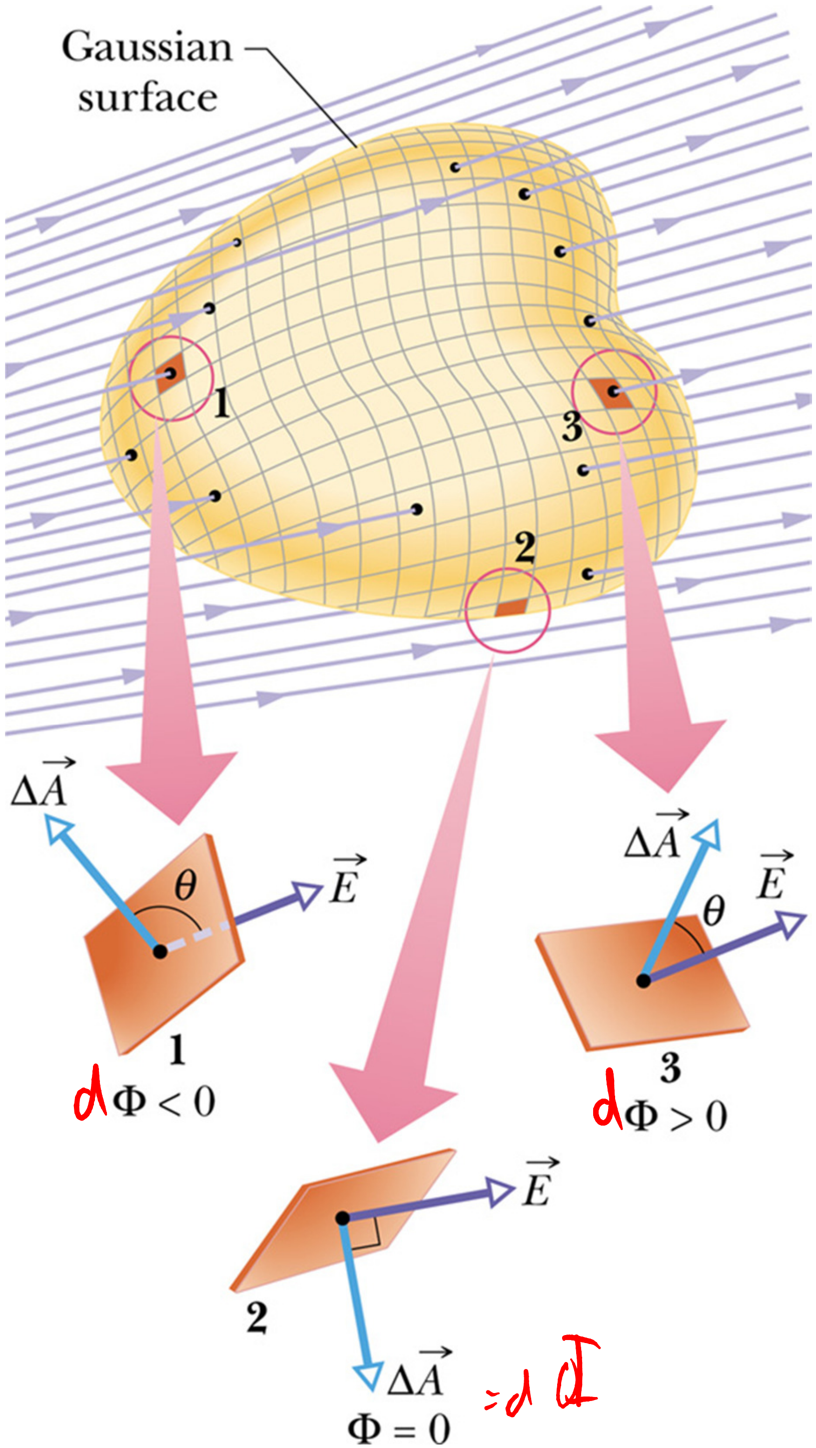
$\Phi_E = E' \cdot A' =$ same

# Gauss' Law


$$\Phi_{net} = q_{enc}/\varepsilon_0$$

where $\varepsilon_0 = 8.854\text{x}10^{-12}$ $\frac{C^2}{N\,m^2}$
(measures how well electric field lines can poke through empty space)

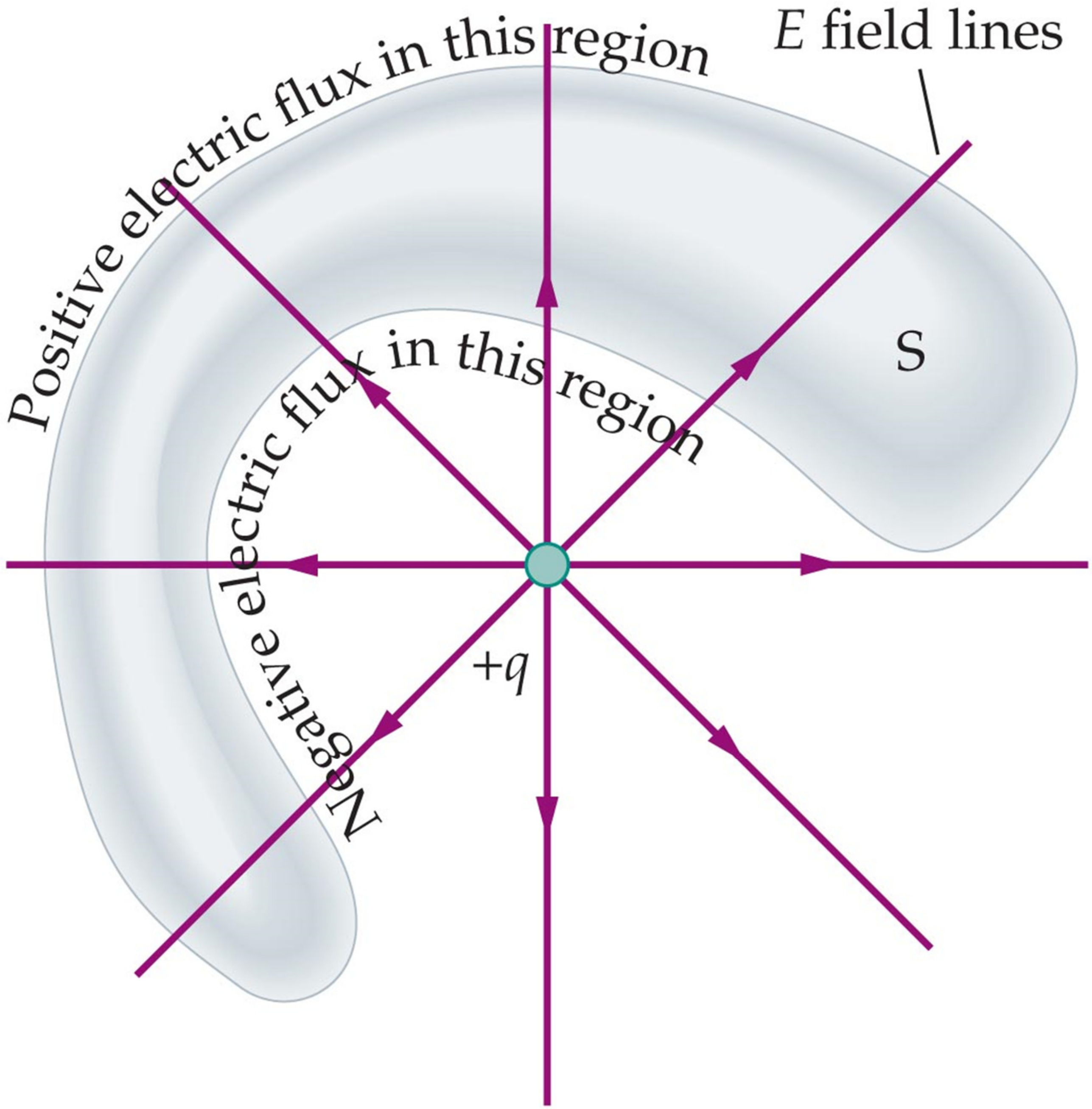
Try page w/ cylinders




$(\Delta\vec{A} = d\vec{A})$

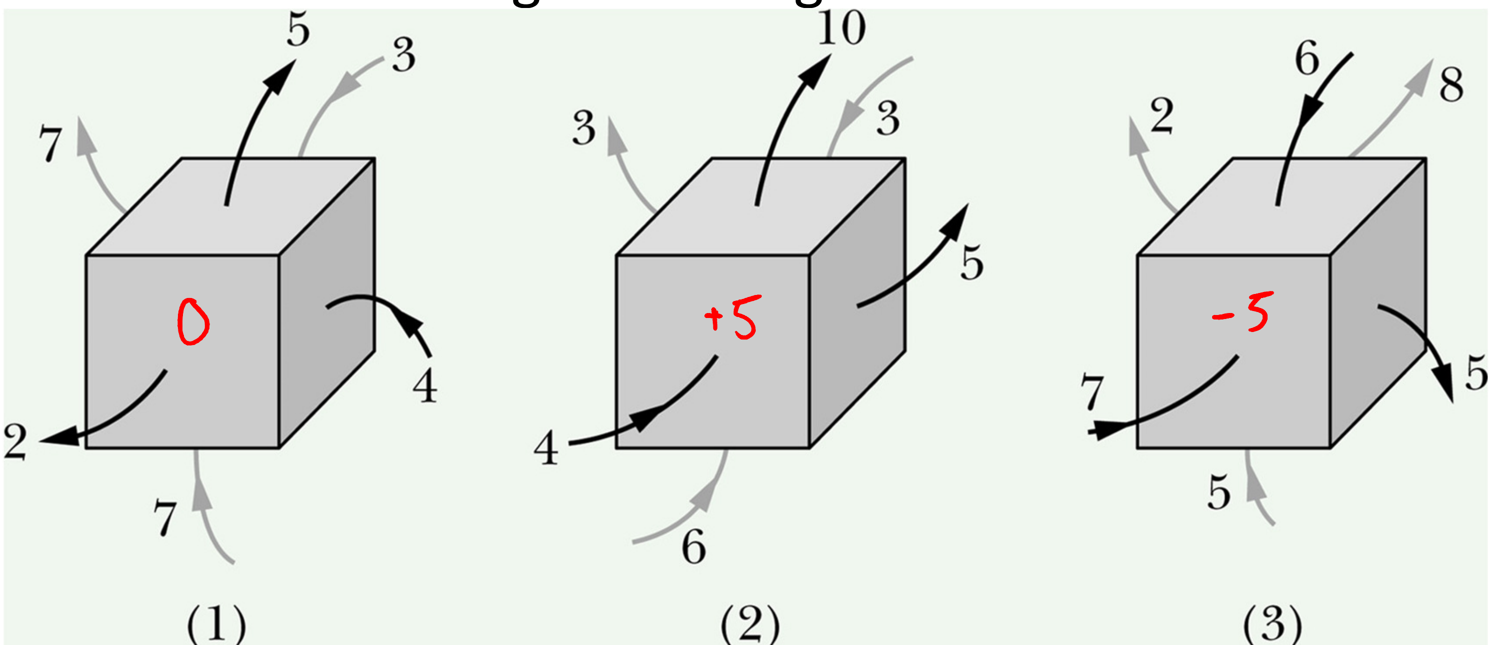
if no $Q$ inside this surface,

$\underline{\sum d\vec{A}\cdot\vec{E}} = 0$

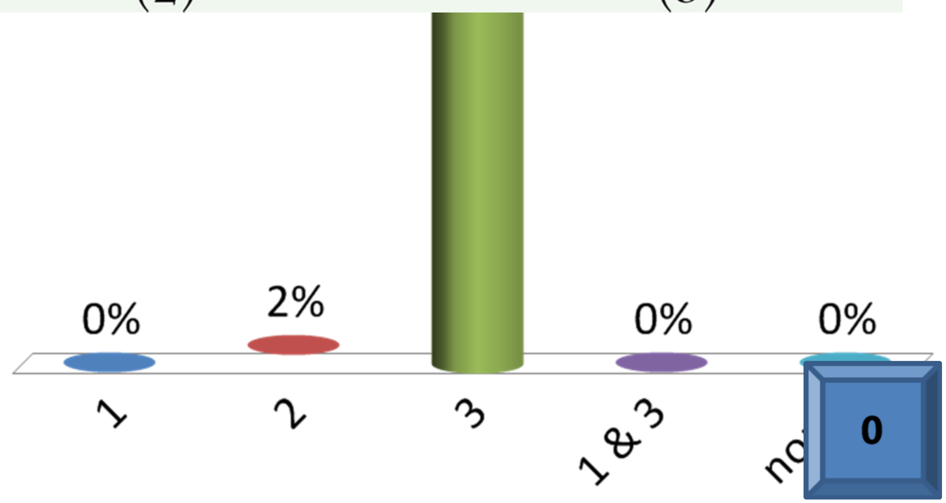

E field lines
Positive electric flux in this region
Negative electric flux in this region
S
+q

The numbers and arrows indicate the amount of electric flux and direction of the field lines for each face of a Gaussian cube. Which cube contains a net negative charge?

1. 1
2. 2
3. 3
4. 1 & 3
5. none




$\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}$

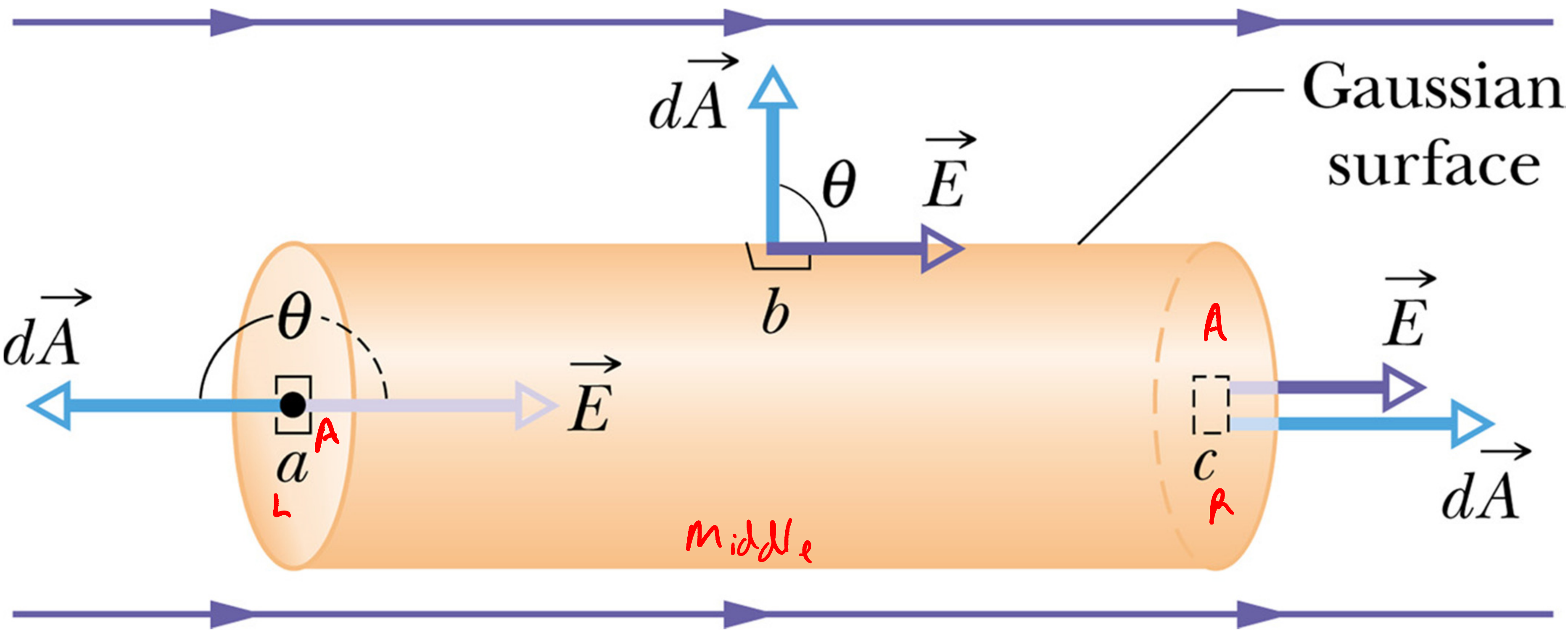


Let's be careful with this shape, in a uniform field, and see if net flux really is 0

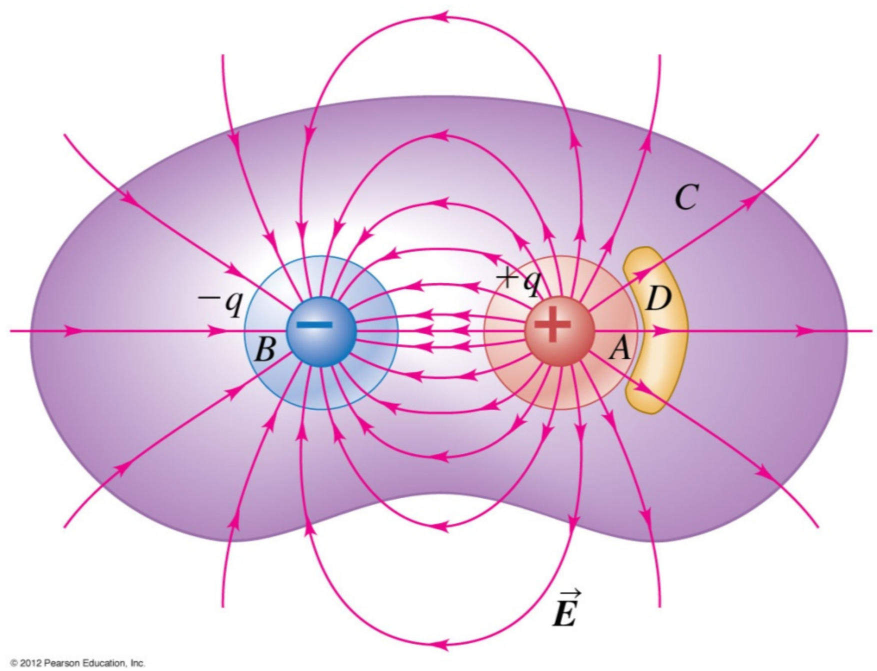
Does this charge-free pipe really have no net flux?

$$\Phi = \vec{E} \cdot \vec{A}$$
$$= |E||A| \cos\theta$$

$$\Phi_{net} = \Phi_L + \Phi_m + \Phi_R$$
$$= -EA + 0 + EA$$
$$= 0$$



Two point charges, +*q* (in red) and –*q* (in blue), are arranged as shown. Through which closed surface(s) is the net electric flux equal to zero?




1. Surface A
2. Surface B
3. Surface C
4. Surface D
5. both C and D

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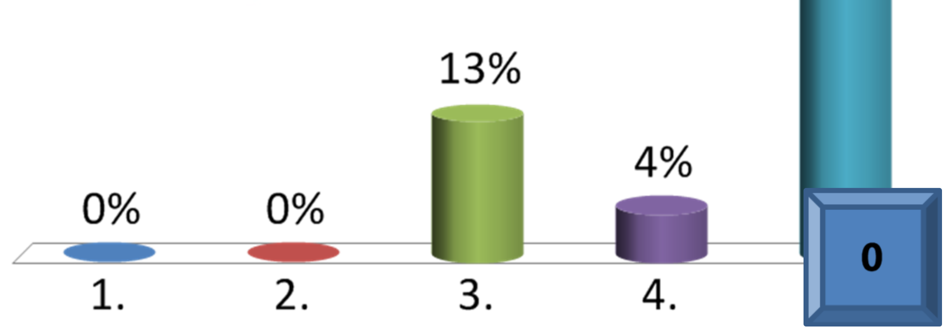
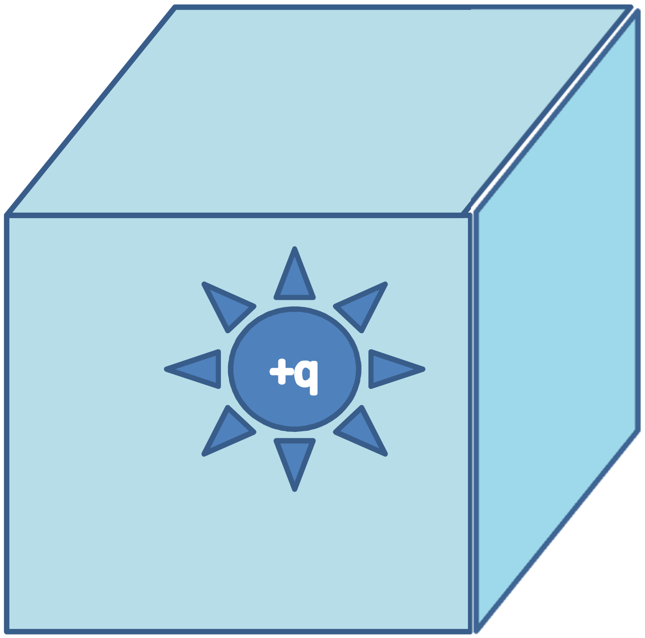


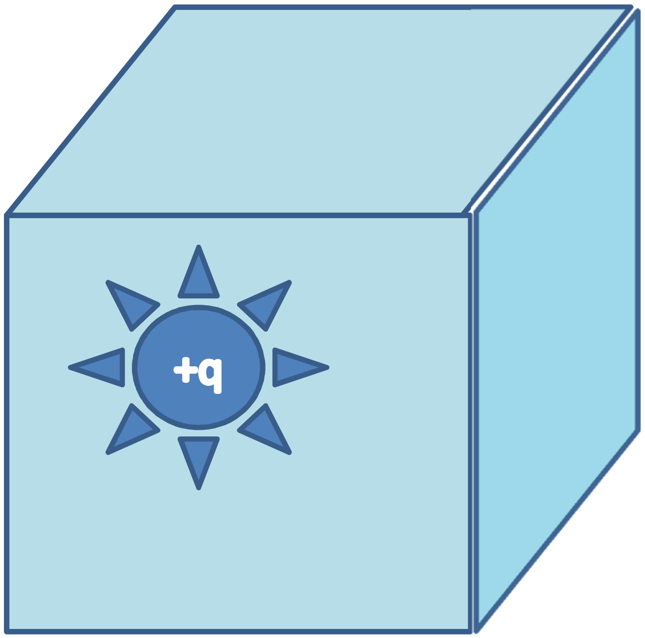
What is electric flux through <u>one</u> side of this cube, if +q is right in the middle?



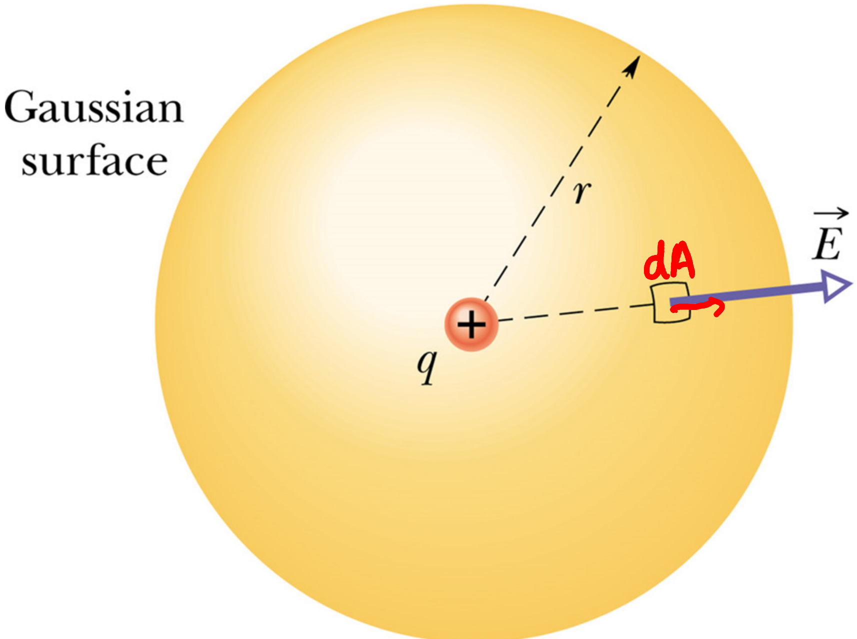

$$\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}$$

$$\Phi_{side} = \frac{+q}{6\,\varepsilon_0}$$

What if the q is moved a little to the left?



Turn Problem around.
Can we use Gauss' Law to figure out what the E from some charge is?




$$\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}$$

$$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}$$

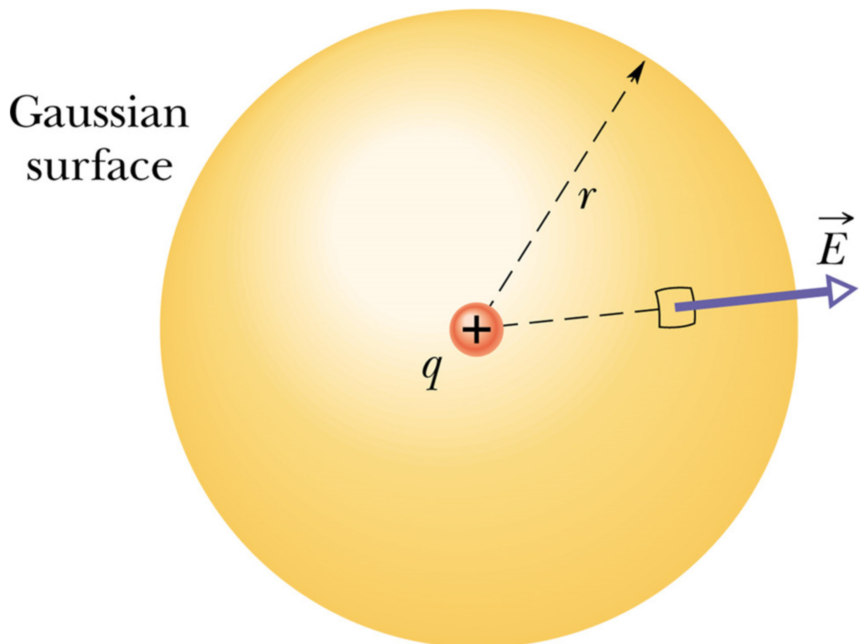
$$\frac{q_{enc}}{\varepsilon_0} = \oint |E||dA| \overbrace{\cancel{\cos\theta}}^{1}$$

$$\frac{q_{enc}}{\varepsilon_0} = E\cdot 1 \oint dA = EA$$

$$\frac{q_{enc}}{\varepsilon_0} = E(4\pi r^2)$$

$$E = \frac{q}{\varepsilon_0 4\pi r^2}$$

$$\left(k = \frac{1}{4\pi\varepsilon_0} = 8.99\times 10^9\right)$$


Gaussian surface
r
$\vec{E}$
q

# Recipe for Applying Gauss' Law

**1.** Make a sketch of the charge distribution.

**2.** Identify the symmetry of the distribution and its effect on the electric field.

**3.** Gauss' law is true for **any** closed surface. Choose one that makes the calculation of the flux $\Phi$ as easy as possible.

**4.** Use Gauss' law to determine the electric field vector:

$\Phi = \int \vec{E} \cdot d\vec{A}$

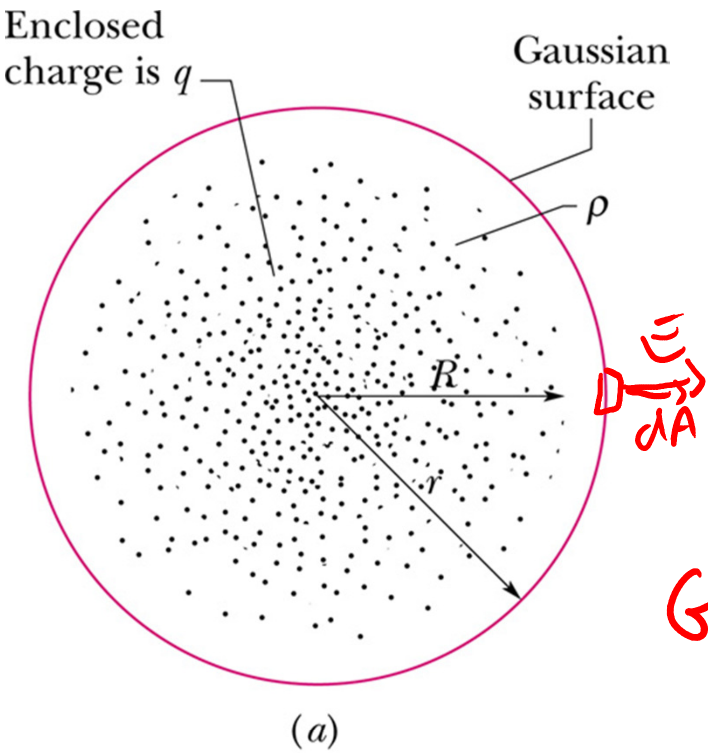
$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0}$$

# How ‘bout that charge $q_{enc}$?

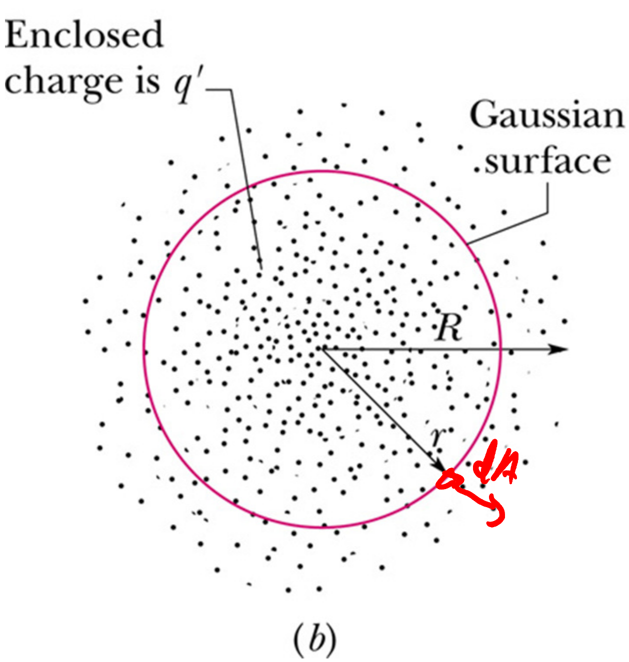
- Point charge?  It’s “q” Coulombs
- Line of charge?  It’s “$\lambda$” C/m
- Sheet of charge?  It’s “$\sigma$” $C/m^2$
- Volume of charge?  It’s “$\rho$” $C/m^3$

- SO: You might be multiplying times a length, an area, or a volume to get back to Coulombs for figuring out the total “$q_{enc}$”

You try the worksheet with the spherical <u>distribution</u> of charge




(a)




(b)

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

E outside:

$$\Phi = \int \vec{E}\cdot d\vec{A}$$

$\vec{E}$, $dA$ parallel so

$$= \int |E||dA| \cancel{\cos\theta}^{\,1} = E\int dA = EA$$

Every $dA$ has same $\vec{E}$, so

G.L: $\varepsilon_0 \Phi = q_{enc}$

$$\varepsilon_0 EA = Q = \varepsilon_0 E(4\pi r^2)$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

direction? out $\hat{r}$

---

E inside. Same, only $q_{enclosed} = \rho\,\text{Vol}$

$$= \frac{Q}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi r^3$$

$$= \frac{Qr^3}{R^3}$$

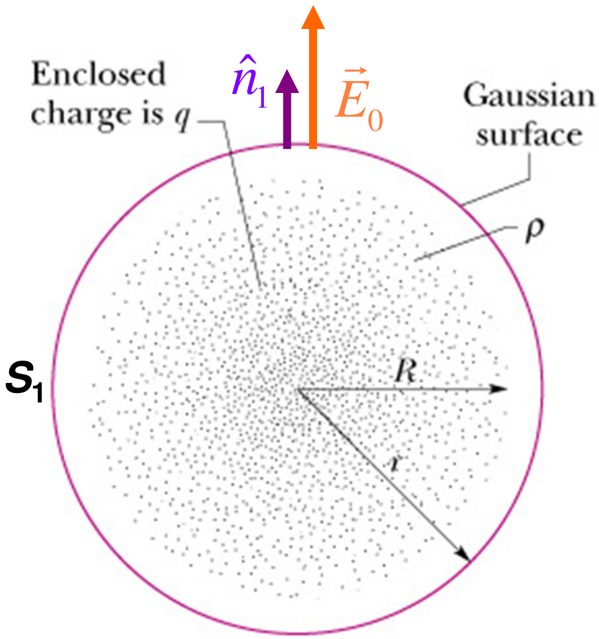
So: $\varepsilon_0 EA = \frac{Qr^3}{\varepsilon_0 R^3}$

$$\varepsilon_0 E(4\pi r^2) = \frac{Qr^3}{\varepsilon_0 R^3}$$

$$E = \frac{Q\,r}{4\pi\varepsilon_0 R^3}$$

# Electric Field Generated by a Uniformly Charged Sphere of Radius $R$ and Charge $q$

**Summary :**

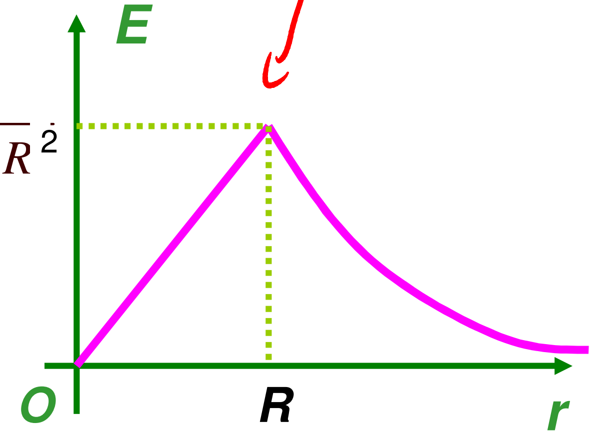


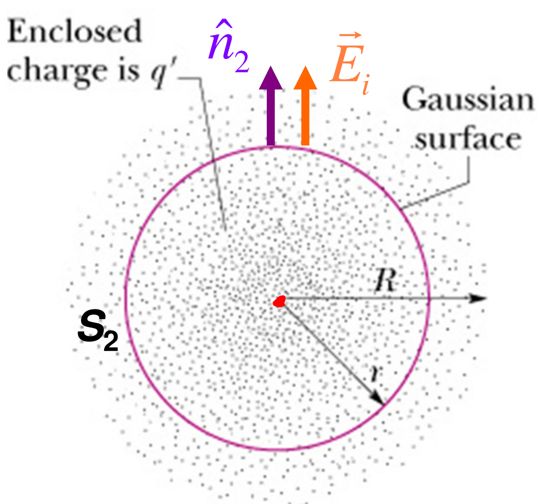
(a)

$E(r)$

$$\boxed{E_0 = \frac{q}{4\pi\varepsilon_0 r^2}}$$

$$E(R) = \frac{q}{4\pi\varepsilon_0 R^2}$$



(b)

$$\boxed{E_i = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r}$$