Recipe for Applying Gauss' Law

1. Make a sketch of the charge distribution.

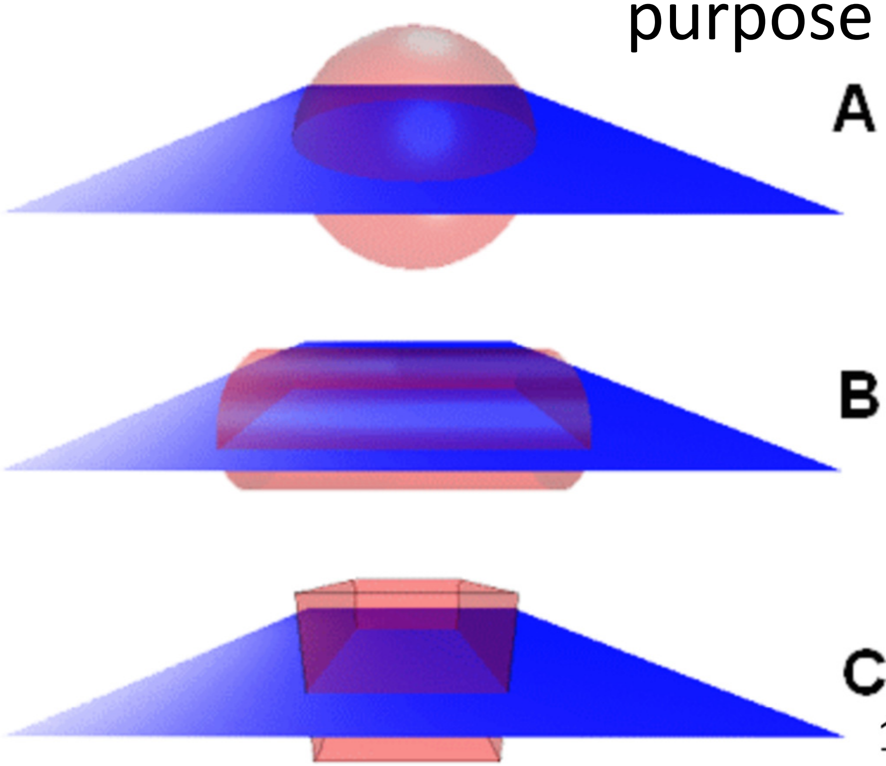
2. Identify the symmetry of the distribution and its effect on the electric field.

3. Gauss' law is true for **any** closed surface. Choose one that makes the calculation of the flux Φ as easy as possible.

4. Use Gauss' law to determine the electric field vector:

$$\Phi = \int \vec{E} \cdot d\vec{A} \qquad \Phi = \frac{q_{\text{enc}}}{\varepsilon_0}$$

You are told to use Gauss' law to calculate the electric field near an infinite sheet of charge. Which of the following Gaussian surfaces is best suited for this purpose?



A.

B.

✔ C.

D. None of them

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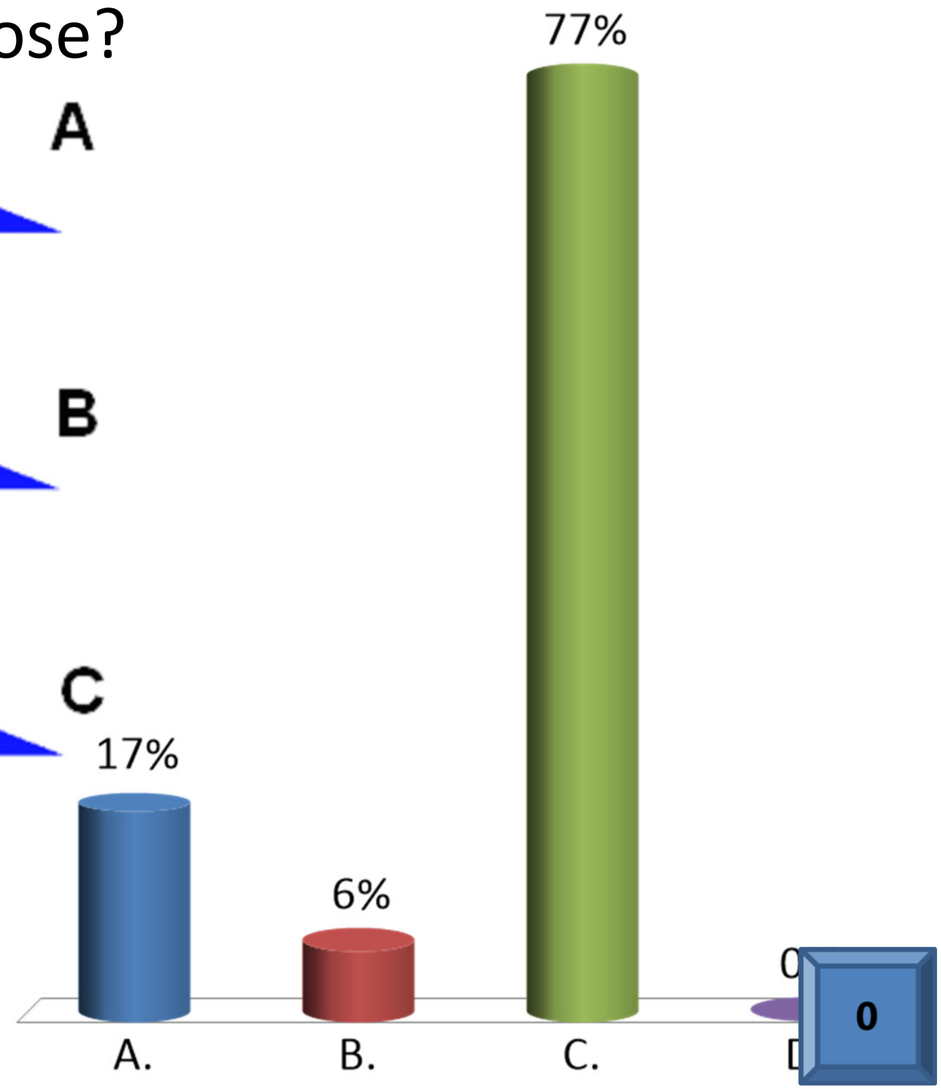


Now you try

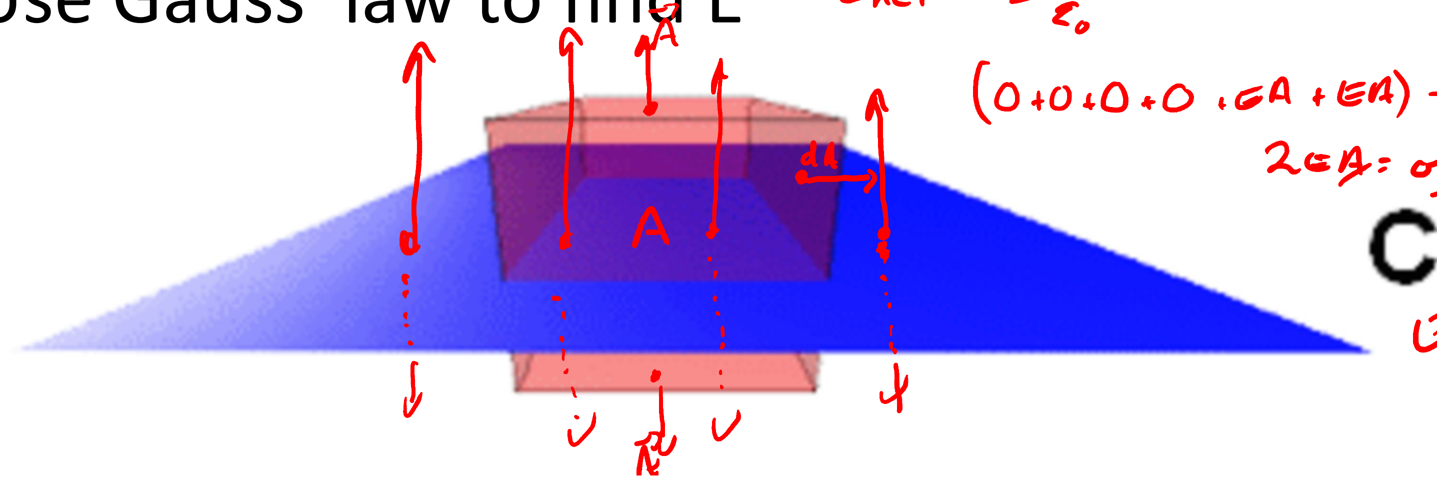
- In terms of σ (in C/m²), how much charge is enclosed in the box? $q_{enc} = \sigma A$
- What is the flux through the fours side walls of the box? $\oint \vec{E} \cdot d\vec{A} = \Phi$ $\quad \vec{E} \cdot d\vec{A} = 0$ for all sides so $\Phi_{sides} = 0$
- What is the flux through the top and bottom of the box? $\Phi = \int \vec{E} \cdot d\vec{A} = \int |E|\,|dA| \cos 0 = E \int dA = EA$
- Use Gauss' law to find E

$\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}$

$(0+0+0+0+EA+EA) = \frac{\sigma A}{\varepsilon_0}$

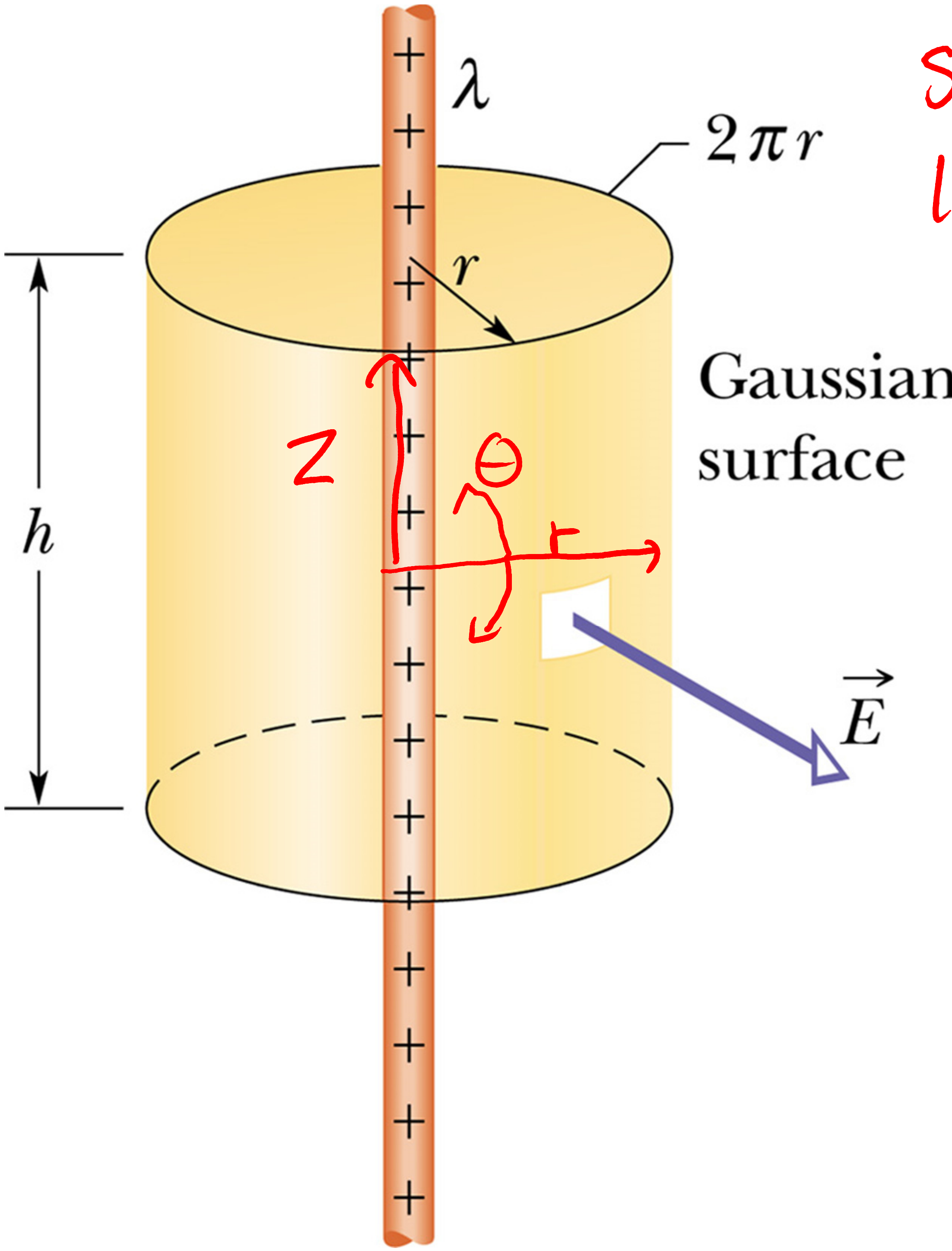
$2EA = \frac{\sigma A}{\varepsilon_0}$

$E = \frac{\sigma}{2\varepsilon_0}$



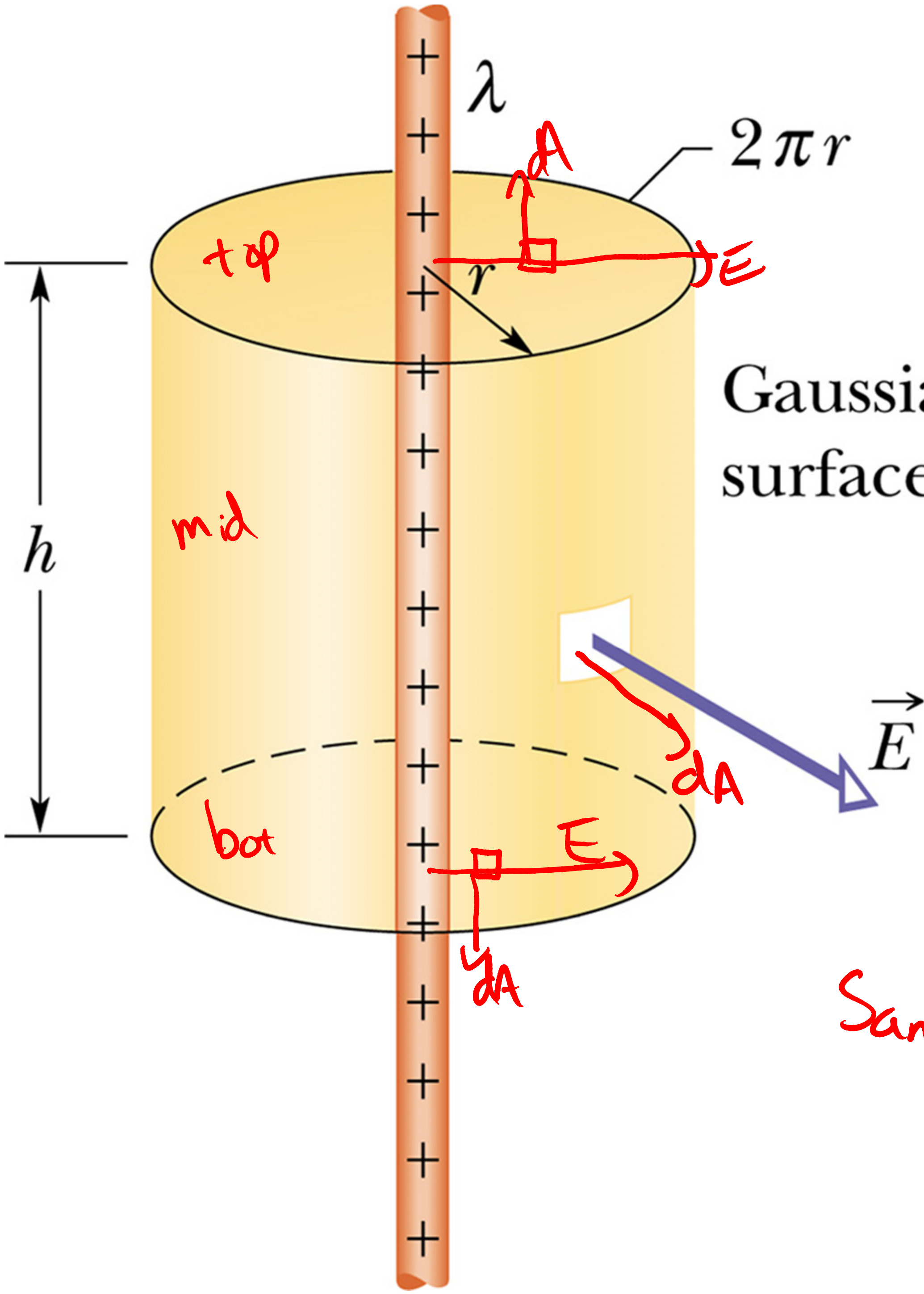
One more example…

- How bout a long line of charge?
- … you did this in Discussion, but here’s my take on it for reference in the posted notes



Symettric about The long rod.

draw Z-axis There for a polar coordinate System.



Choose to invent a Can-shaped surface centered on z-axis.

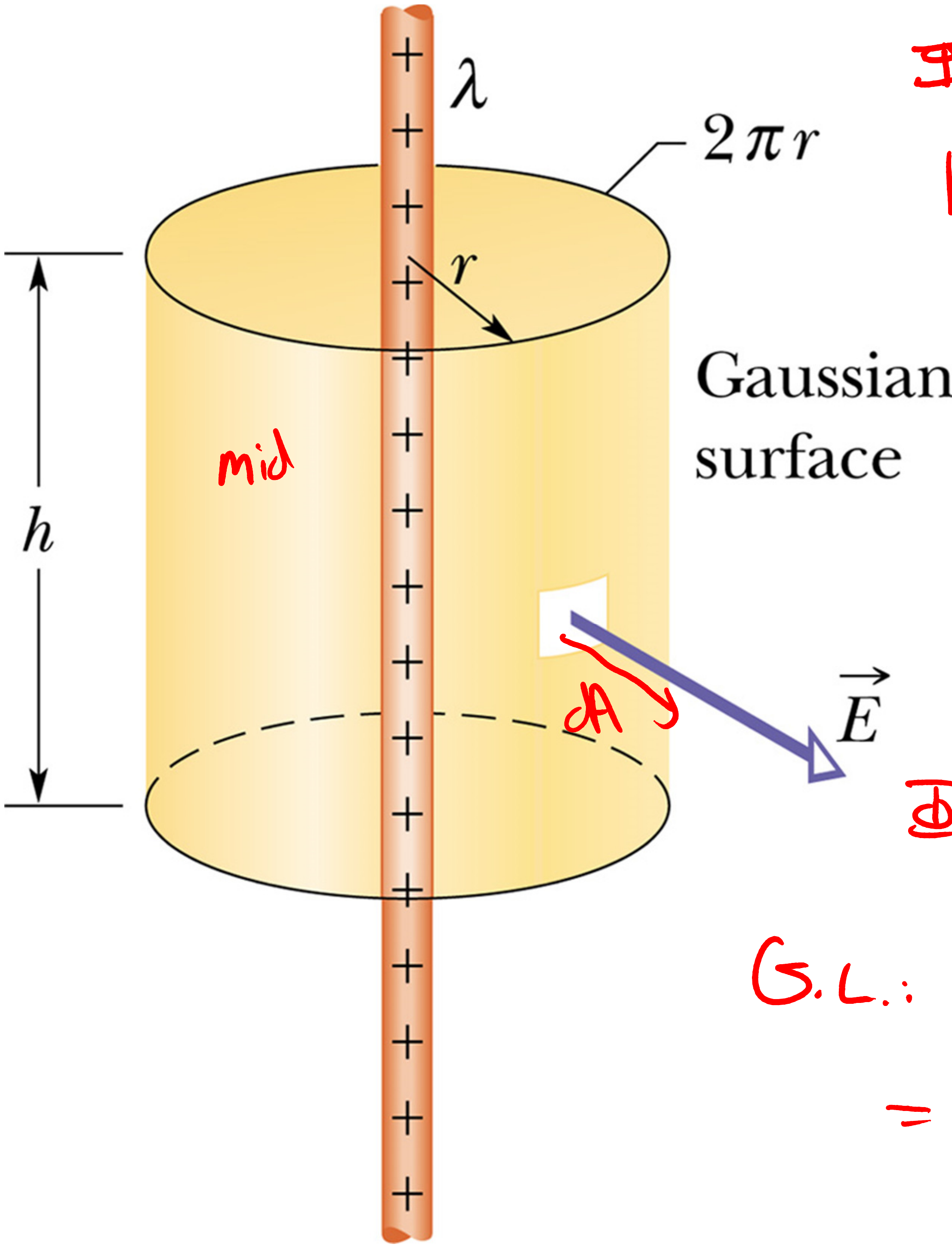
$\Phi_{net} = \Phi_{top} + \Phi_{mid} + \Phi_{bot}$

Φ_{top}: $\vec{E}$, $d\vec{A}$ are $\perp$

so $\int \vec{E} \cdot d\vec{A} = \int |E||dA| \cos 90^\circ$

$\Phi_{top} = 0$.

Same for Φ_{bot}.



$\Phi_{mid} = \int \vec{E}\cdot d\vec{A} = \int |E||dA|\cos\theta$

here, all $d\vec{A}$ || to $\vec{E}$ so $\theta = 0$

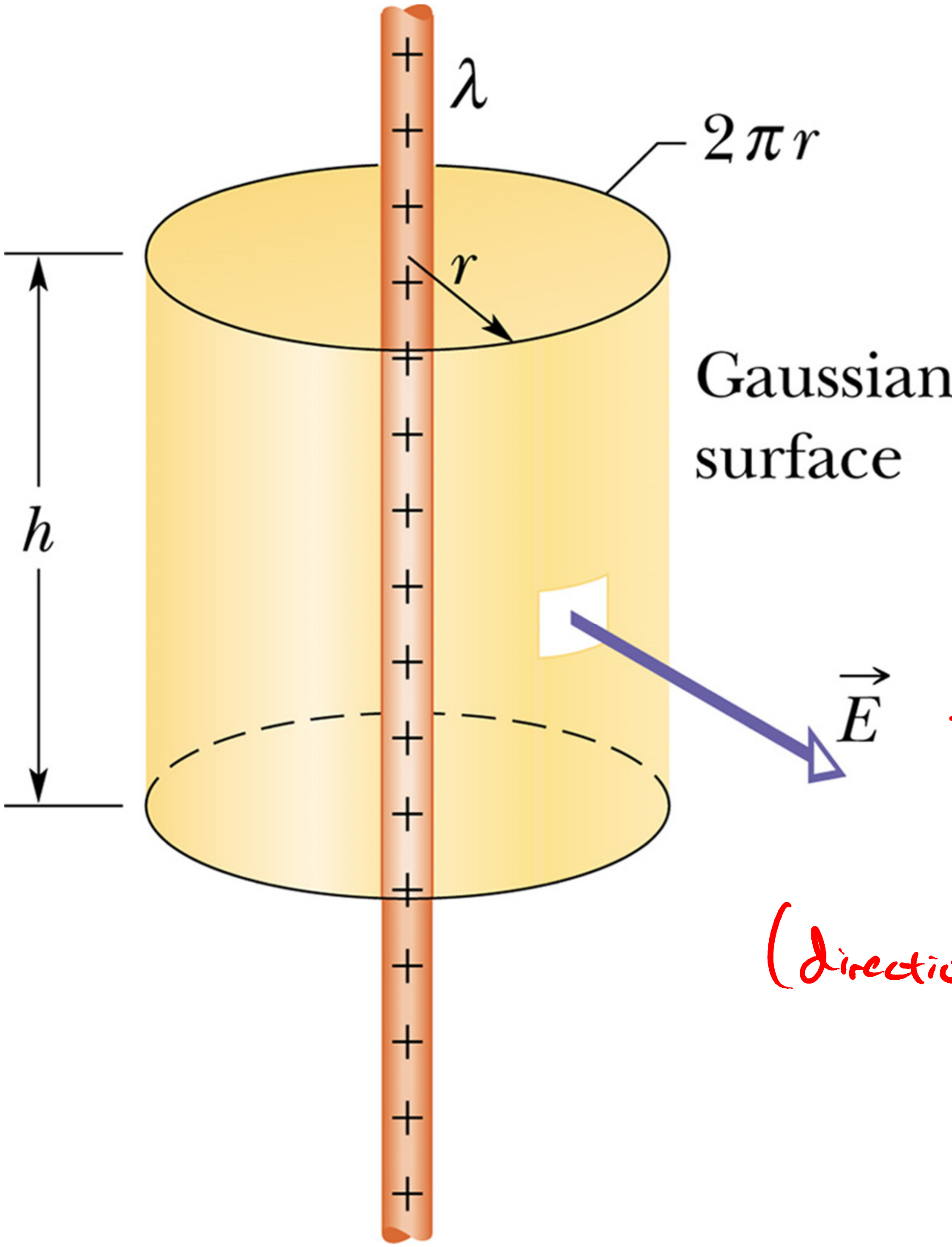
$\int |E||dA|\cos 0$ (cos 0 = 1)

$\Phi_{mid} = E\int dA$ since every dA same distance from all charges

$\Phi_{mid} = EA = E(2\pi r)h$

G.L.: $\Phi_{tot} = 0 + E(2\pi r)h + 0$

$= 2\pi r E h = \frac{q_{enc}}{\varepsilon_0}$



How much q is enclosed?

h long rod, λ C/m,

so $q_{enc} = \lambda h$

So: $\varepsilon_0 \Phi_{net} = q_{enc}$

$\varepsilon_0 (2\pi r \not{h} E) = \lambda \not{h}$

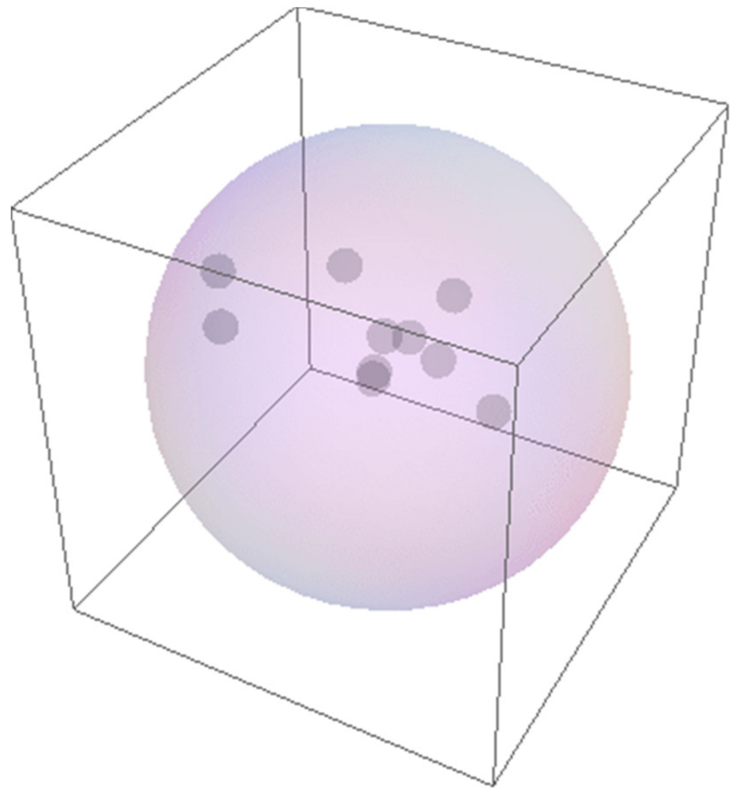
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

(direction from drawing)

In a conductor

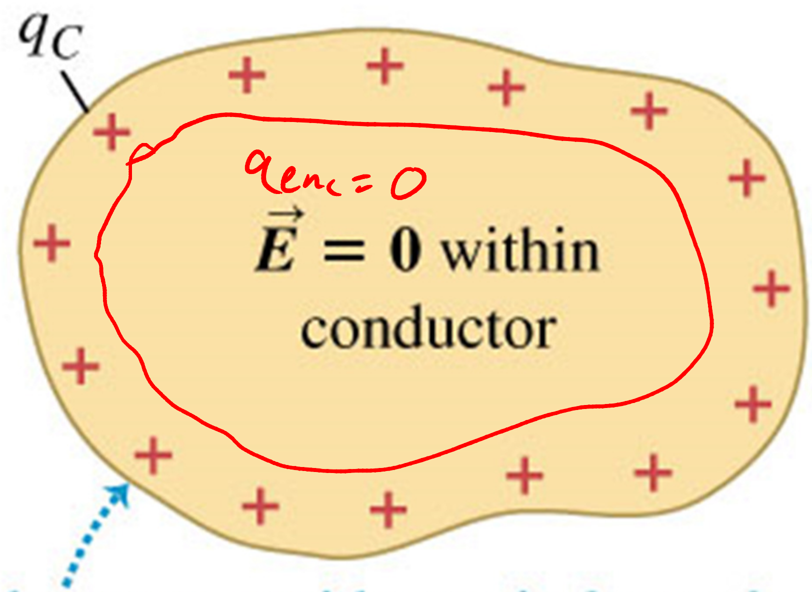
- Charge can move freely
- If, at some moment, there's an Electric Field, it will make a force on some charge, so thus move it $\vec{F} = \vec{E} \cdot q$
- Wait a bit, charges move to where they're going to go
- After a while, charges end up where they'll stay put… so no F, no E remains
  - If "in equilibrium" (*ie, wait a while*)

A simulation…



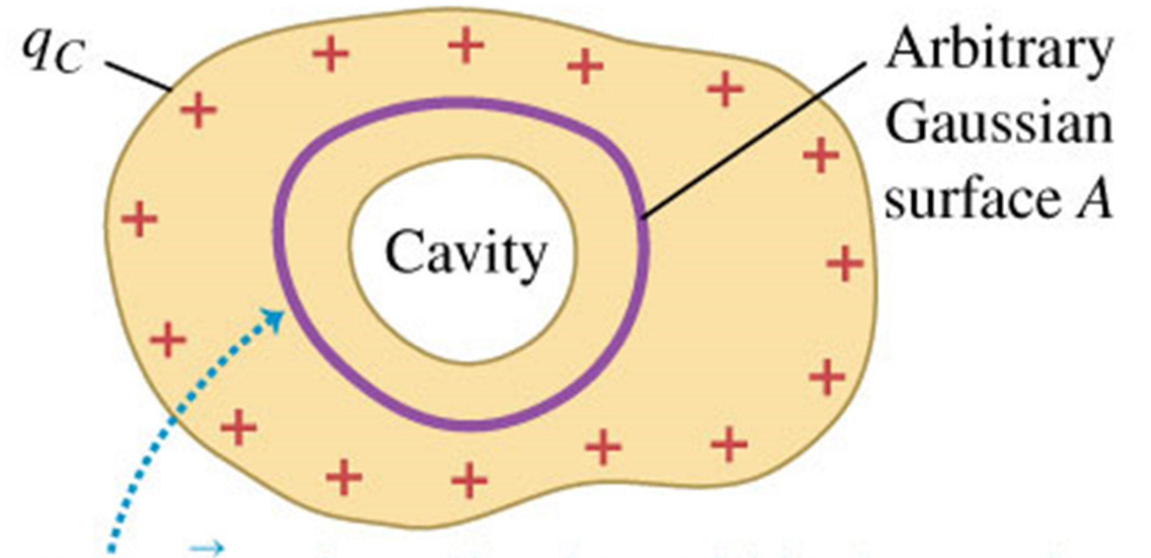
The perspective is a bit weird, if you could spin this around, you'd see all the charges on the edges at the end.

(a) Solid conductor with charge q_C



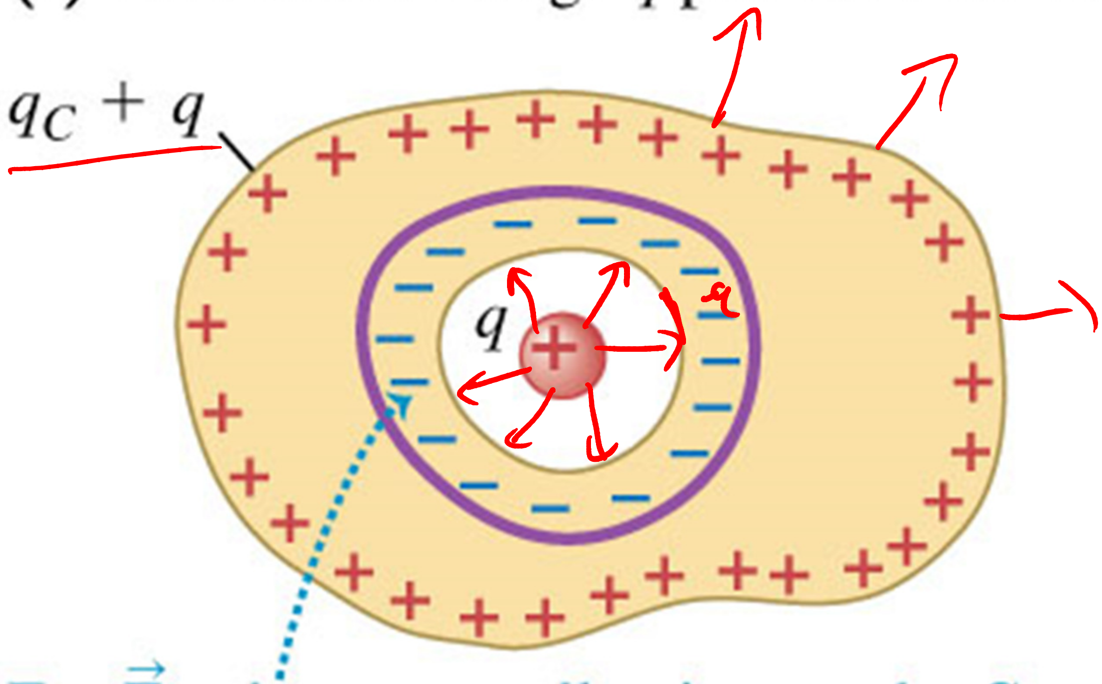
The charge q_C resides entirely on the surface of the conductor. The situation is electrostatic, so $\vec{E} = 0$ within the conductor.

(b) The same conductor with an internal cavity



Because $\vec{E} = 0$ at all points within the conductor, the electric field at all points on the Gaussian surface must be zero.

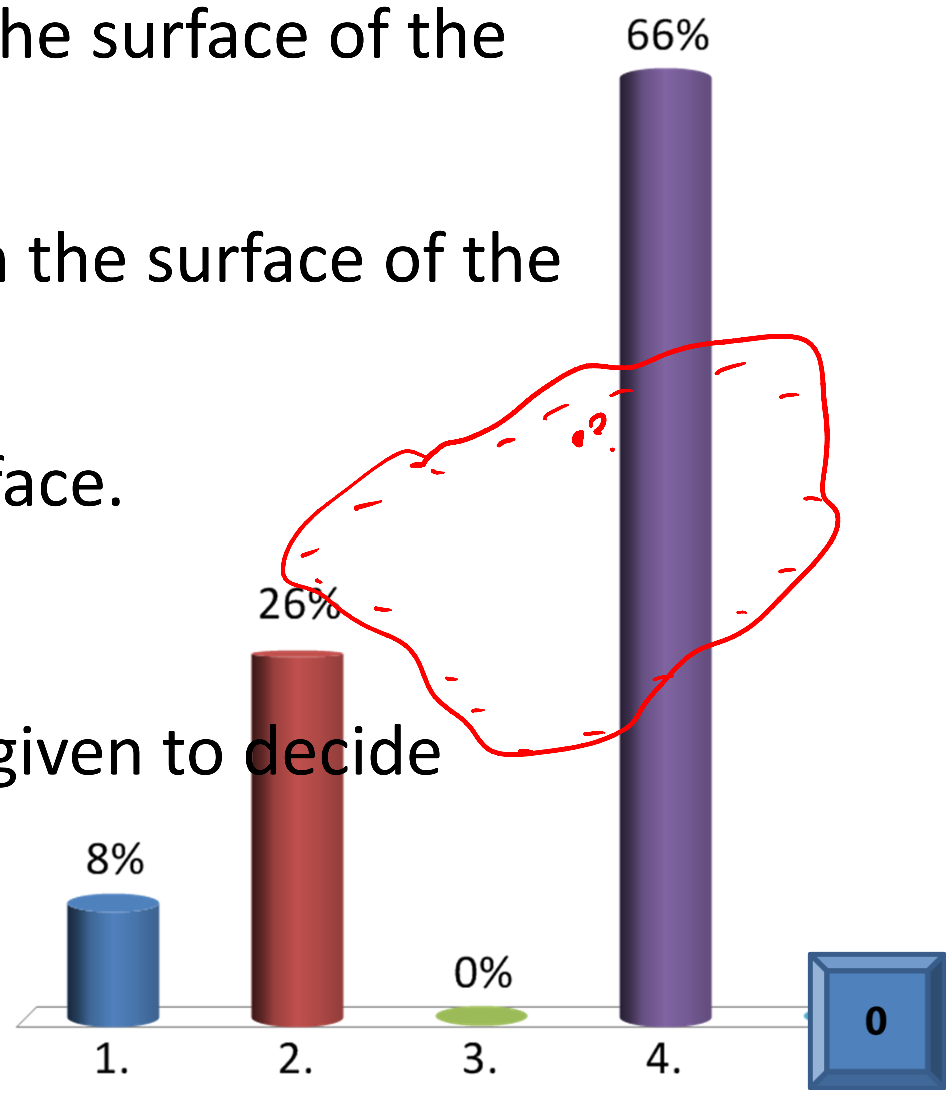
(c) An isolated charge q placed in the cavity



For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

There is a negative surface charge density in a certain region on the surface of a solid conductor.
Just beneath the surface of this region, the electric field

1. points outward, toward the surface of the conductor.
2. points inward, away from the surface of the conductor.
3. points parallel to the surface.
4. is zero.
5. not enough information given to decide



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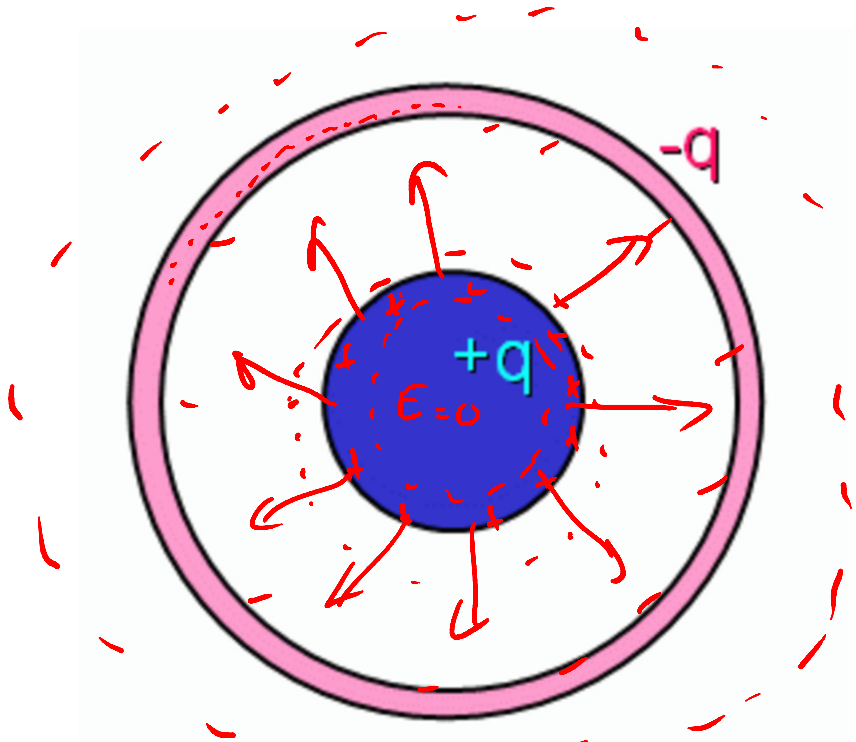
A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.

$\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}$

$E = 0$ (in conductor)



Draw electric field lines for all areas! And, draw how the charges would be distributed, in this setup.

Once your table agrees on a picture, have a representative draw it on the board.

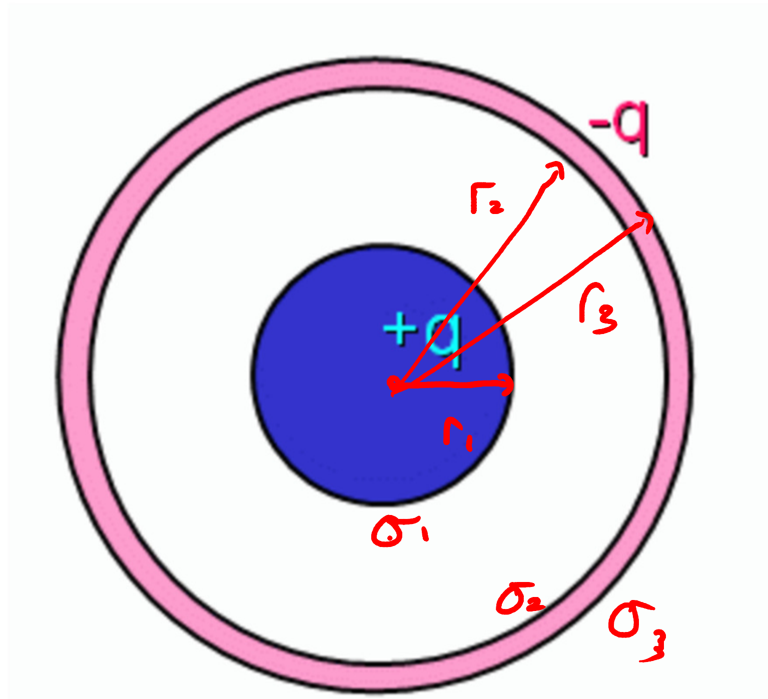
A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.

+q is on r_1

so $\sigma_1 = \frac{+q}{4\pi r_1^2}$

$\sigma_2 = \frac{-q}{4\pi r_2^2}$

$\sigma_3 = 0$

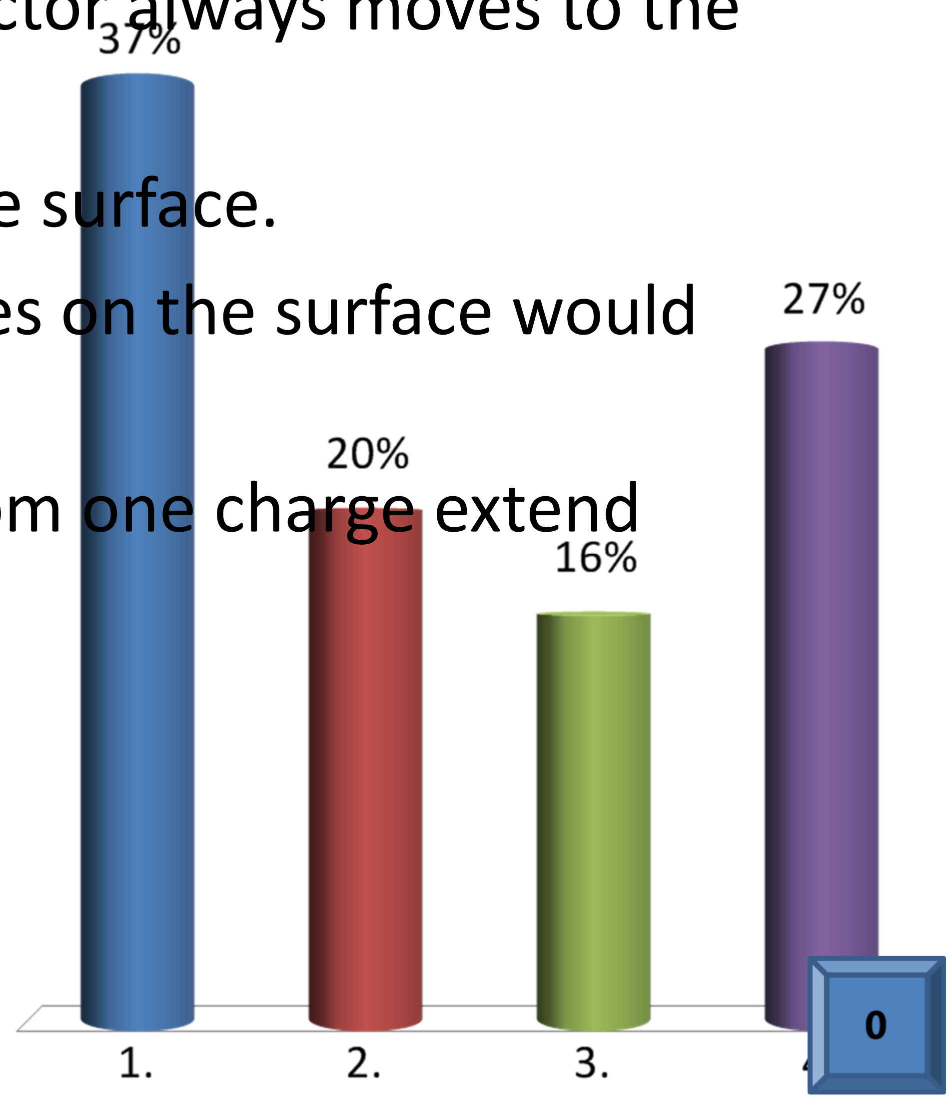


Draw electric field lines for all areas! And, draw how the charges would be distributed, in this setup.

What's the surface charge density σ on each surface in the picture?

Why must the electric field at the surface of a conductor be perpendicular to the surface?

1. Excess charge in a conductor always moves to the surface of the conductor.
2. Flux is always perp. to the surface.
3. ✓ If it was not perp., charges on the surface would be moving.
4. The electric field lines from one charge extend radially.



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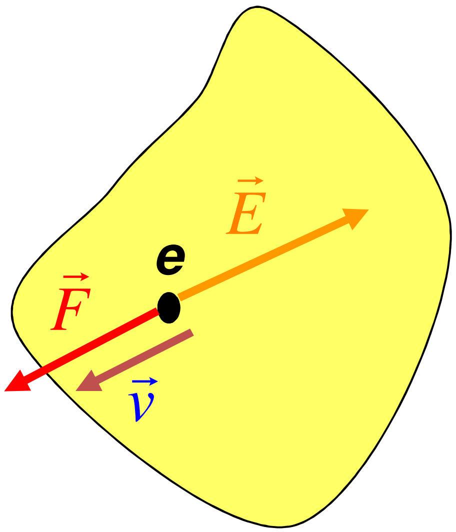
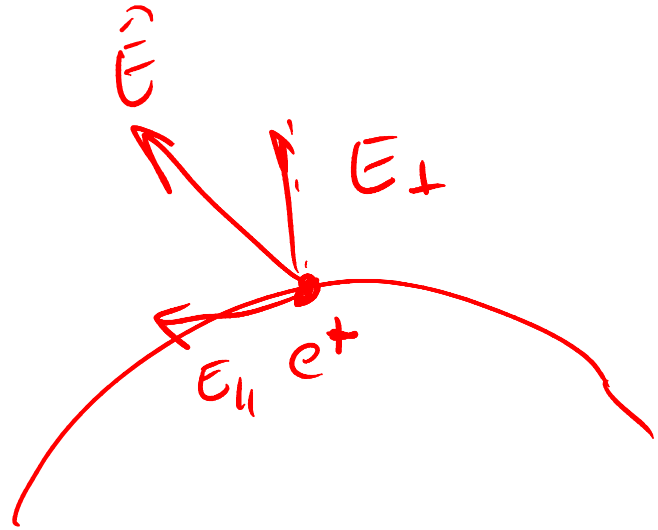
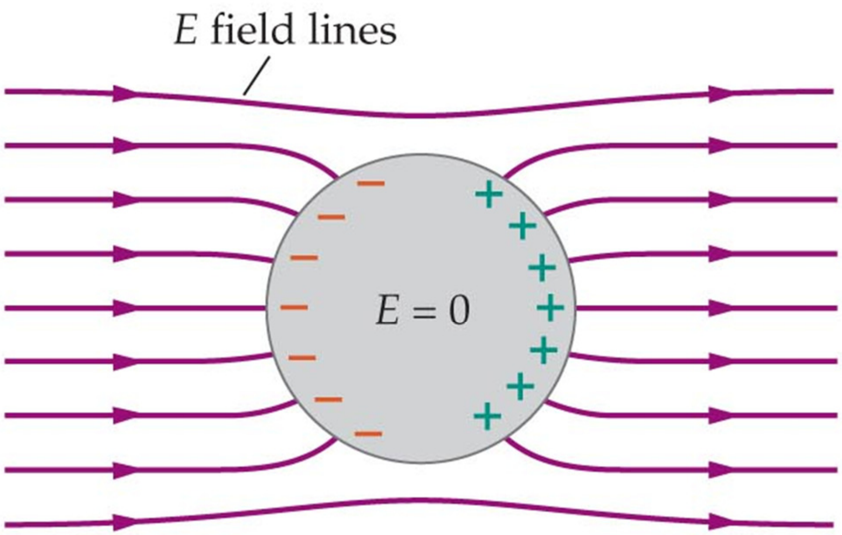


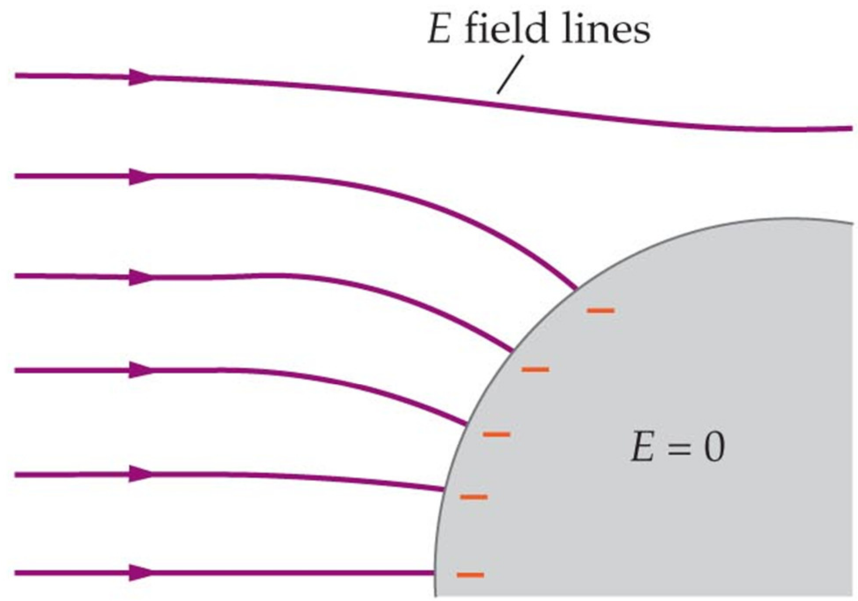
$\vec{E}$
e
$\vec{F}$
$\vec{v}$



$\hat{E}$
$E_\perp$
$E_\parallel$
e^+

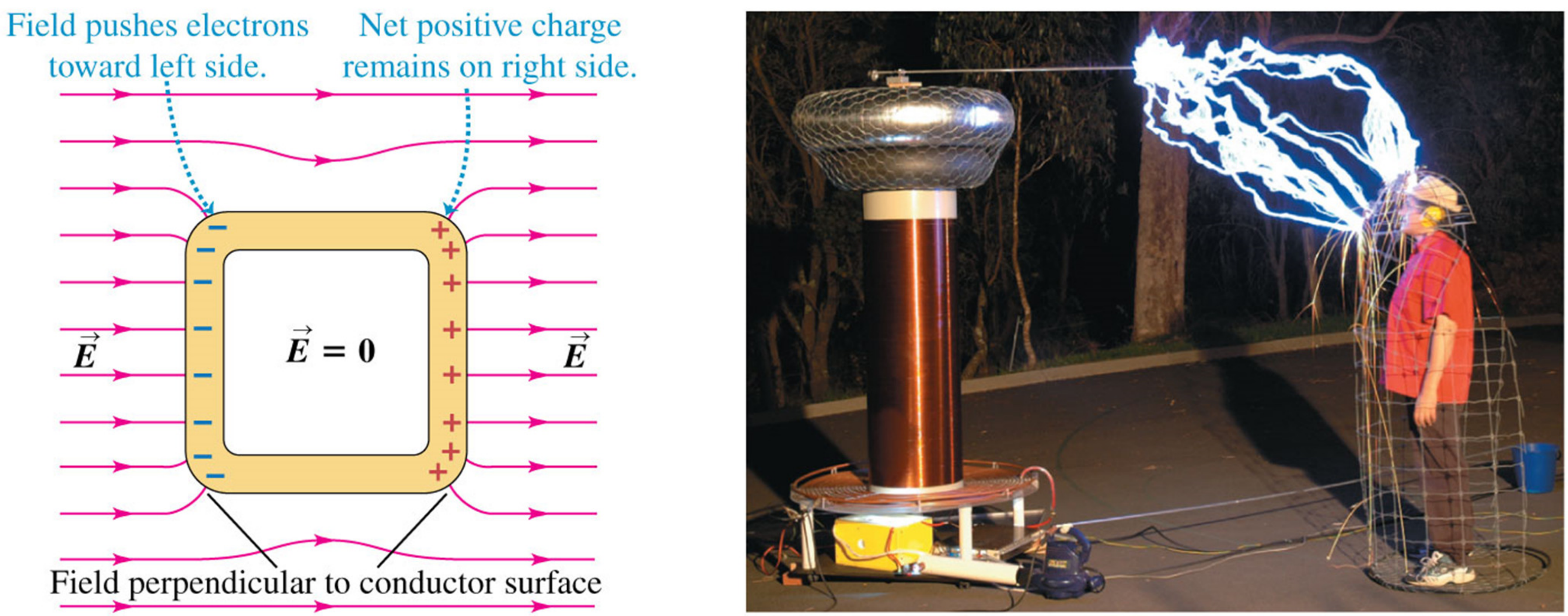


(a) The electric field E vanishes inside a conductor



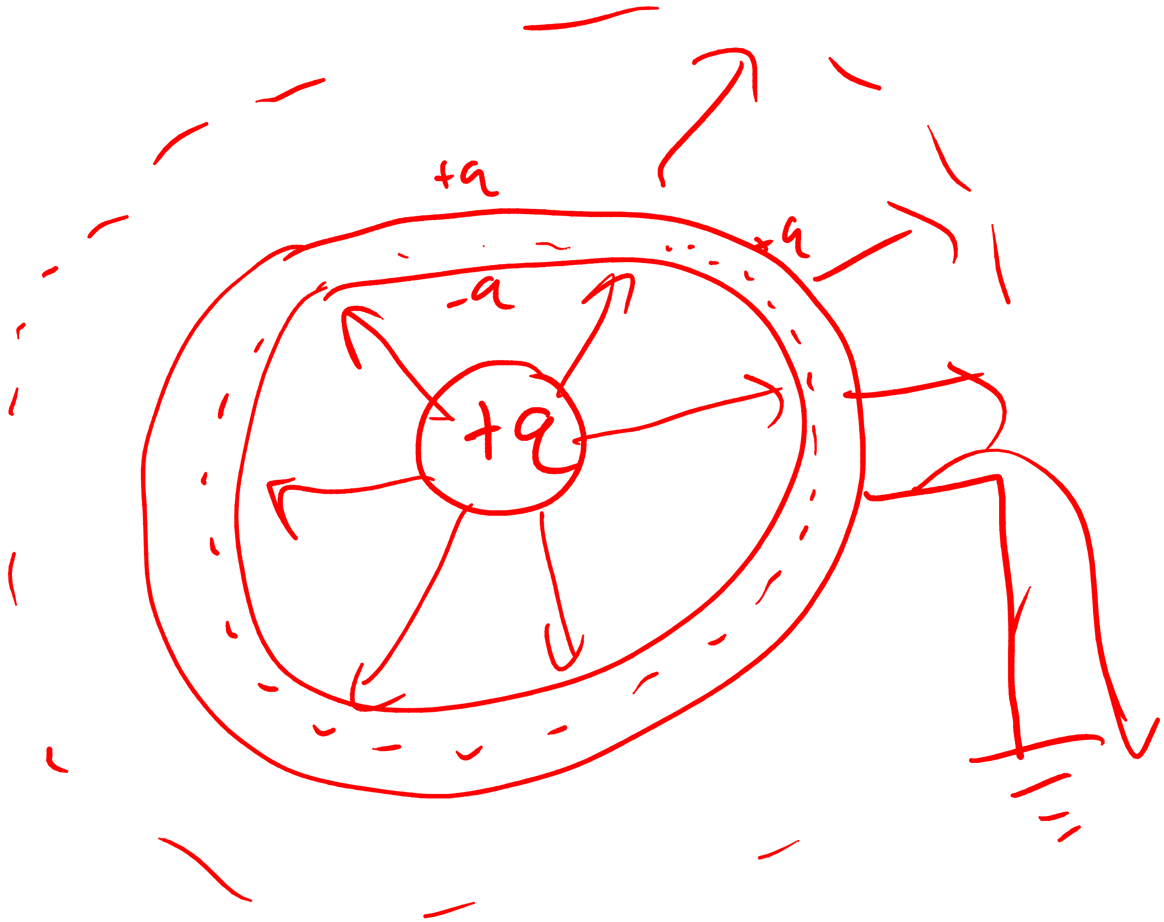
(b) E field lines meet a conducting surface at right angles

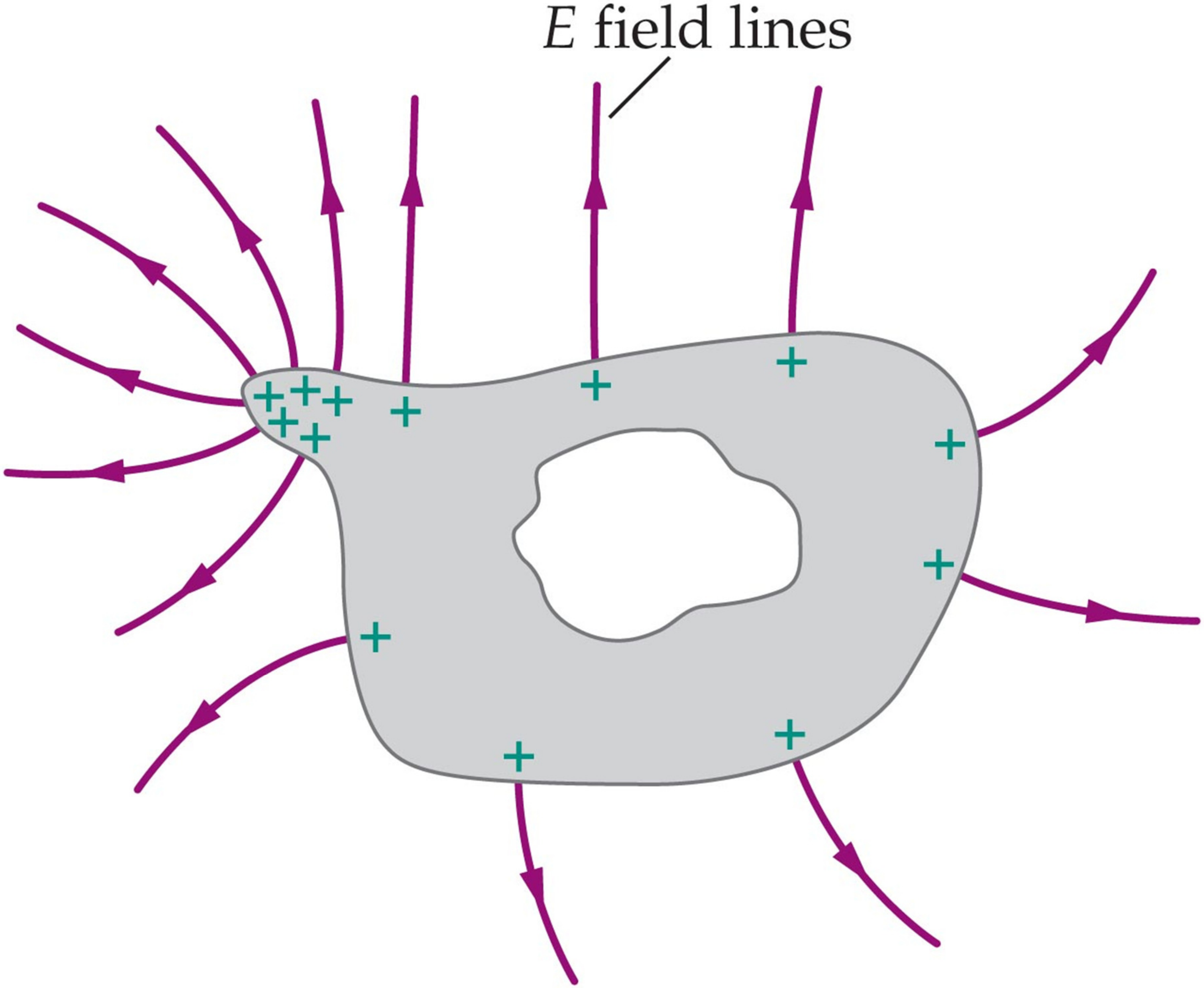
(a) (b)

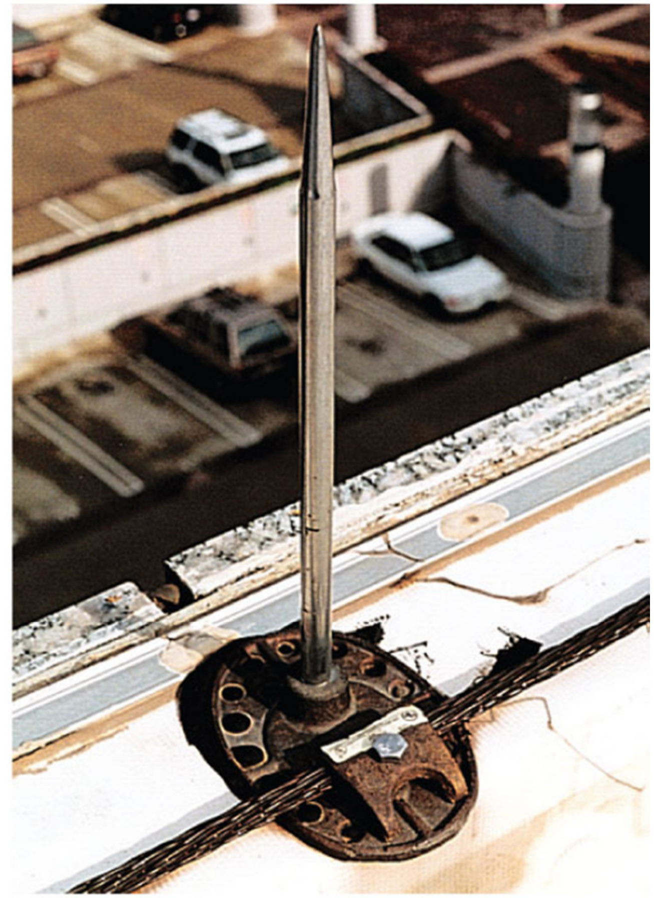


Field pushes electrons toward left side.
Net positive charge remains on right side.
$\vec{E}$
$\vec{E} = 0$
$\vec{E}$
Field perpendicular to conductor surface

What about the reverse: how would you shield the world from a ball of charge?




E field lines



Test Wednesday, 9/26

- Covers Ch. 23-26
- Some problems, some multiple choice
  - See assignments page for an example test plus solutions
- Bring a regular letter-sized sheet of paper with whatever numbers, formulae, etc you think you might need
- Note that phones can’t be used as calculators: bring a real calculator