Notes Sheet Suggestions

- Yes, you can use both sides of the page. Yes, you can put whatever you want on there.
- That said, suggestions for using it most efficiently:
  - Start with something in the style your Physics I prof might have included for those exams: the formulas and the values of the constants you've used so far
  - Other things you'd like not to have to remember
    - Which component goes with which sin or cos? What are the areas and volumes of simple shapes? Units?

Can I include examples?

- Sure, if you want to
- Beware: it’s very easy to crowd so much stuff into this one page that it becomes useless to you as a reference.
- Beware part#2: Don’t fall into the trap of “oh, I’ve got everything written down, this will be easy”

Could I put an Integral Table there?

- If you really want to. But, all I'd ask for actual integrating are polynomials (x^n, where n might be negative), sines and cosines, or exponentials. So, it would be the world's simplest integral table

You have two point charges, q1 and q2.
Each is +5nC, and they are 8cm apart on the x-axis.

a) What's the electric potential midway between them?
b) What's the electric field at this same point?
c) What's the electric potential 6cm above the leftmost charge?

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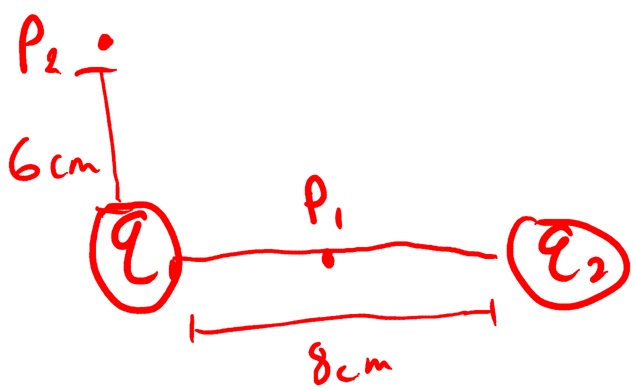

a) $V(P_1)? = V_1 + V_2 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$

here, $q_1 = q_2 = 5nC$, $r_1 = r_2 = 4cm$

So. $V(P_1) = \frac{2kq}{r} = 2\frac{(8.99\times10^9 \frac{N m^2}{C^2})(5\times10^{-9} C)}{0.04 m}$

$V = 2250V$

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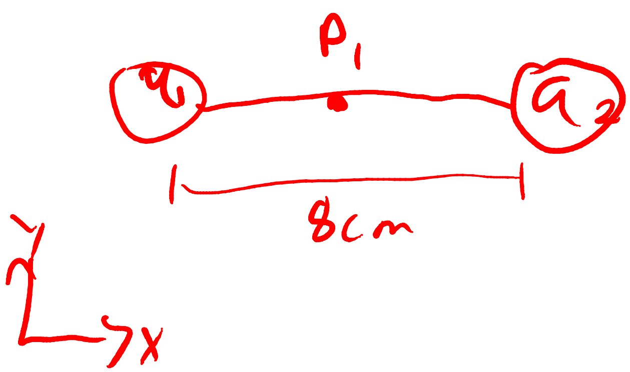

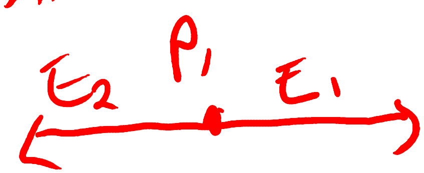

b) What's $\vec{E}(P_1)$?

each charge makes $\vec{E} = \frac{kq}{r^2}$ (away)

same q, same r, opposite directions:

so $\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2 = 0$

(even though $V(P_1) = 2250V$!)

You have two point charges, q1 and q2.
Each is +5nC, and they are 8cm apart on the x-axis.

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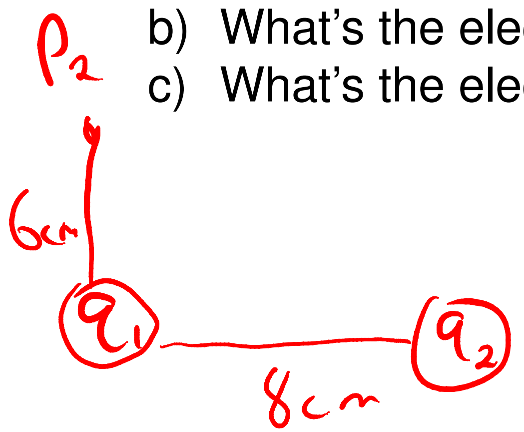


c) $V(P_2)$?

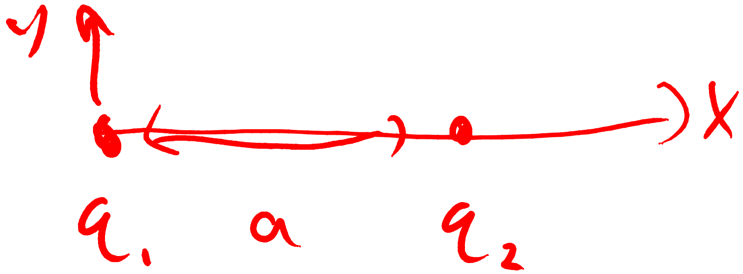
again, $V(P_2) = V_1 + V_2 = \frac{k q_1}{r_1} + \frac{k q_2}{r_2}$

q_1, q_2 same. $r_1 = 6cm$, $r_2 = \sqrt{6cm^2 + 8cm^2}$

so. $V = kq\left(\frac{1}{0.06} + \frac{1}{\sqrt{0.06^2 + 0.08^2}}\right)$

$V(P_2) = 1200V$

W00t! V's are not vectors, much easier to figure out than $\vec{E}(P_2)$.



What's $V(x)$?

$$V(x) = V_1 + V_2$$

What's r_1? $r_1 = |x|$

What's r_2? $r_2 = |x-a|$

$$V = \frac{kq_1}{|x|} + \frac{kq_2}{|x-a|}$$

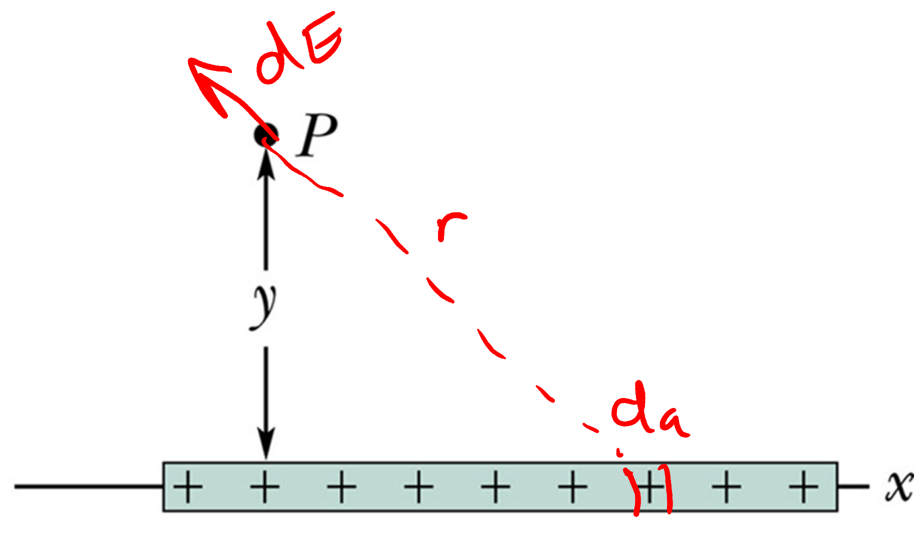
right of q_2: $|x| = x$ $\quad |x-a| = x-a$ $\quad$ so $V(x) = \frac{kq_1}{x} + \frac{kq_2}{x-a}$

middle: $|x| = x$ $\quad |x-a| = a-x$ $\quad$ so $V(x) = \frac{kq_1}{x} + \frac{kq_2}{a-x}$

left: $|x| = -x$ $\quad |x-a| = a-x$ $\quad$ so $V(x) = \frac{-kq_1}{x} + \frac{kq_2}{a-x}$


0
a
x

$$dV = \frac{k\,dq}{r}$$

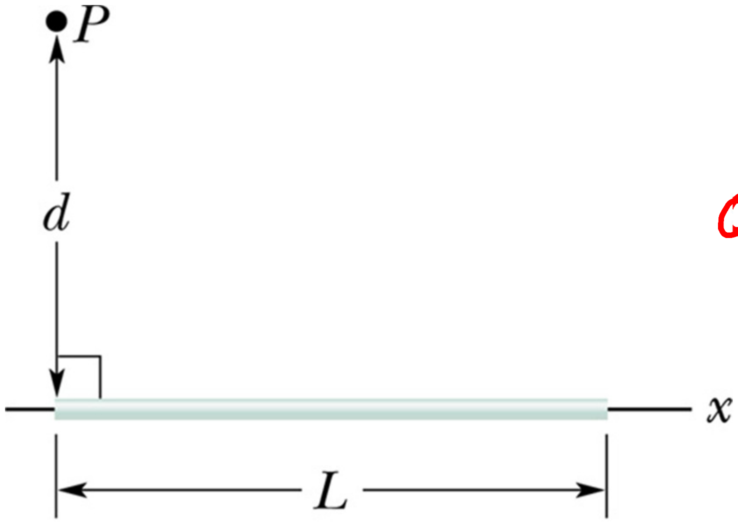


(c)

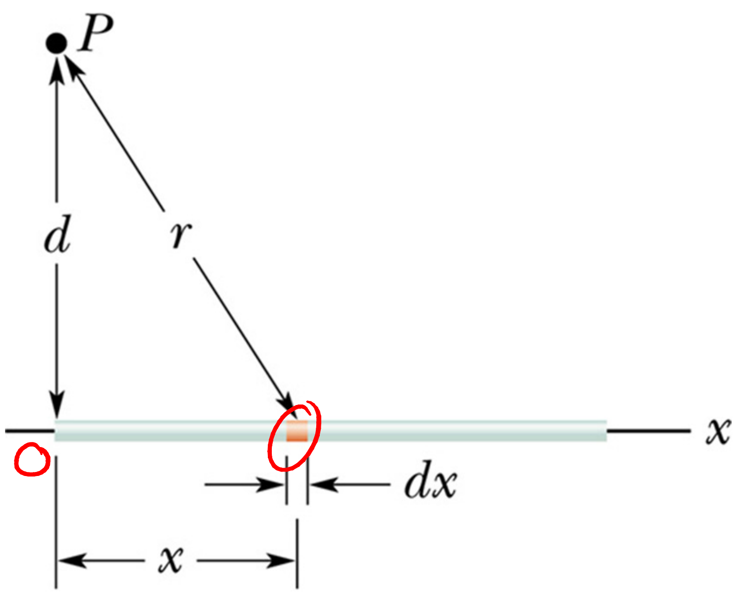
from uniform $\lambda = q_o/L$

$$dV = \frac{k\,dq}{r} \qquad dq = \lambda\,dx$$

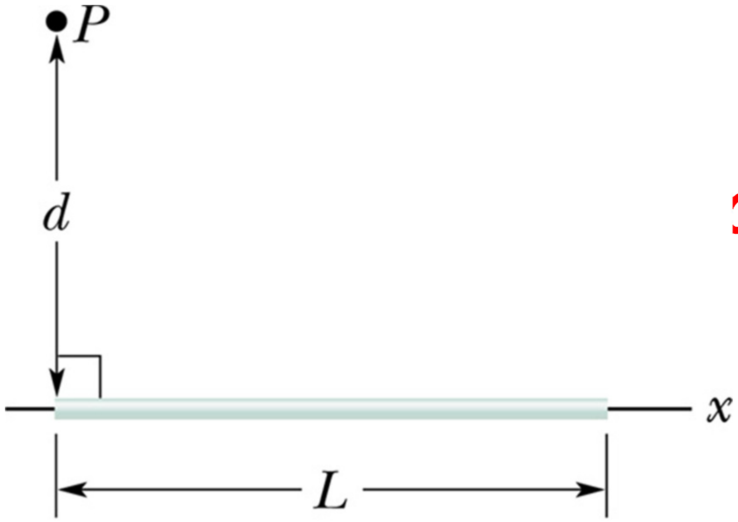
$$r = \sqrt{d^2 + x^2}$$



(a)



(b)



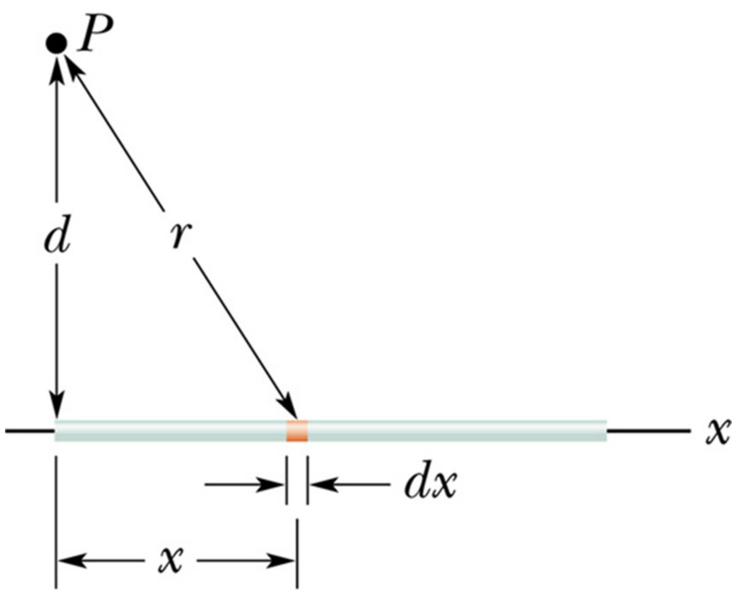
(a)

from uniform $\lambda = q_0/L$

$$dV = \frac{k\,dq}{r} \qquad dq = \lambda\,dx$$

$$r = \sqrt{d^2 + x^2}$$

$$dV = \frac{k\,\lambda\,dx}{(d^2 + x^2)^{1/2}}$$



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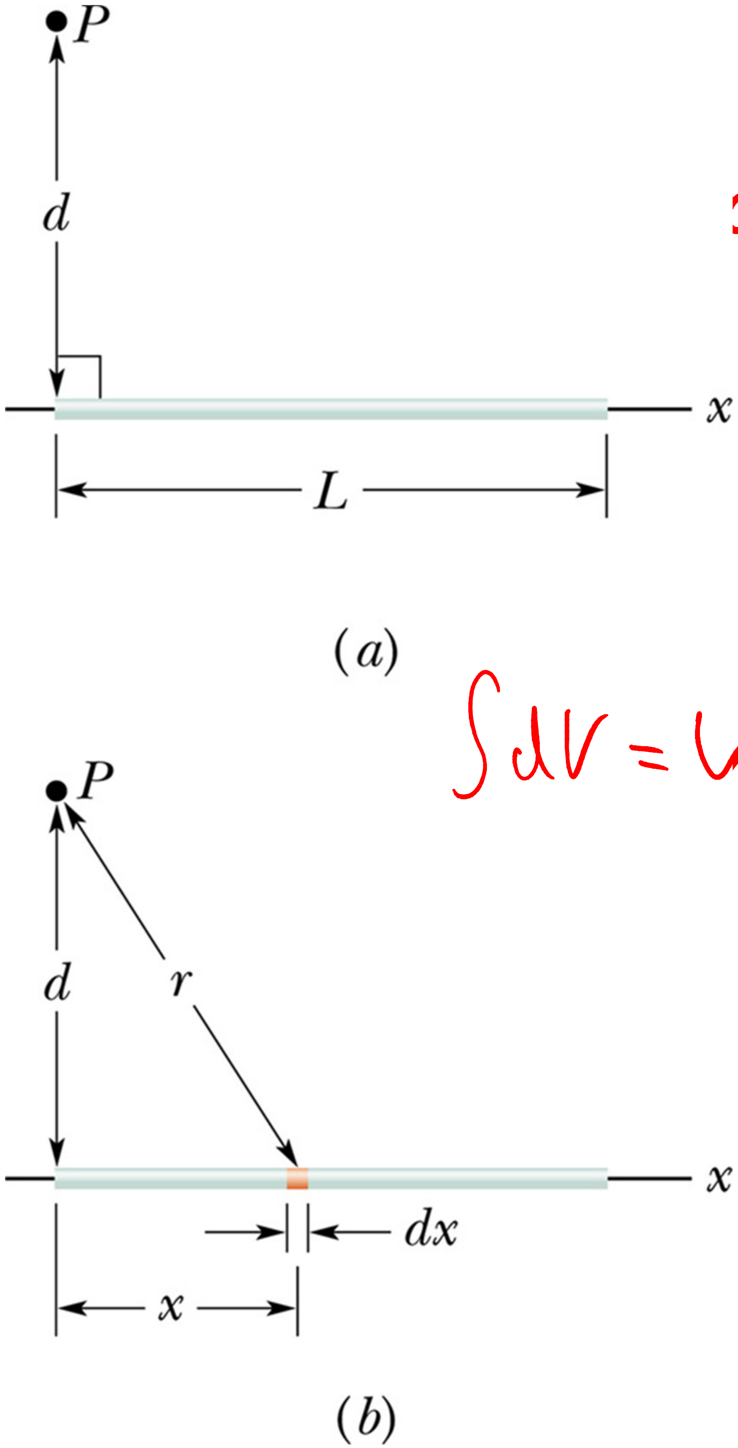


(a)

(b)

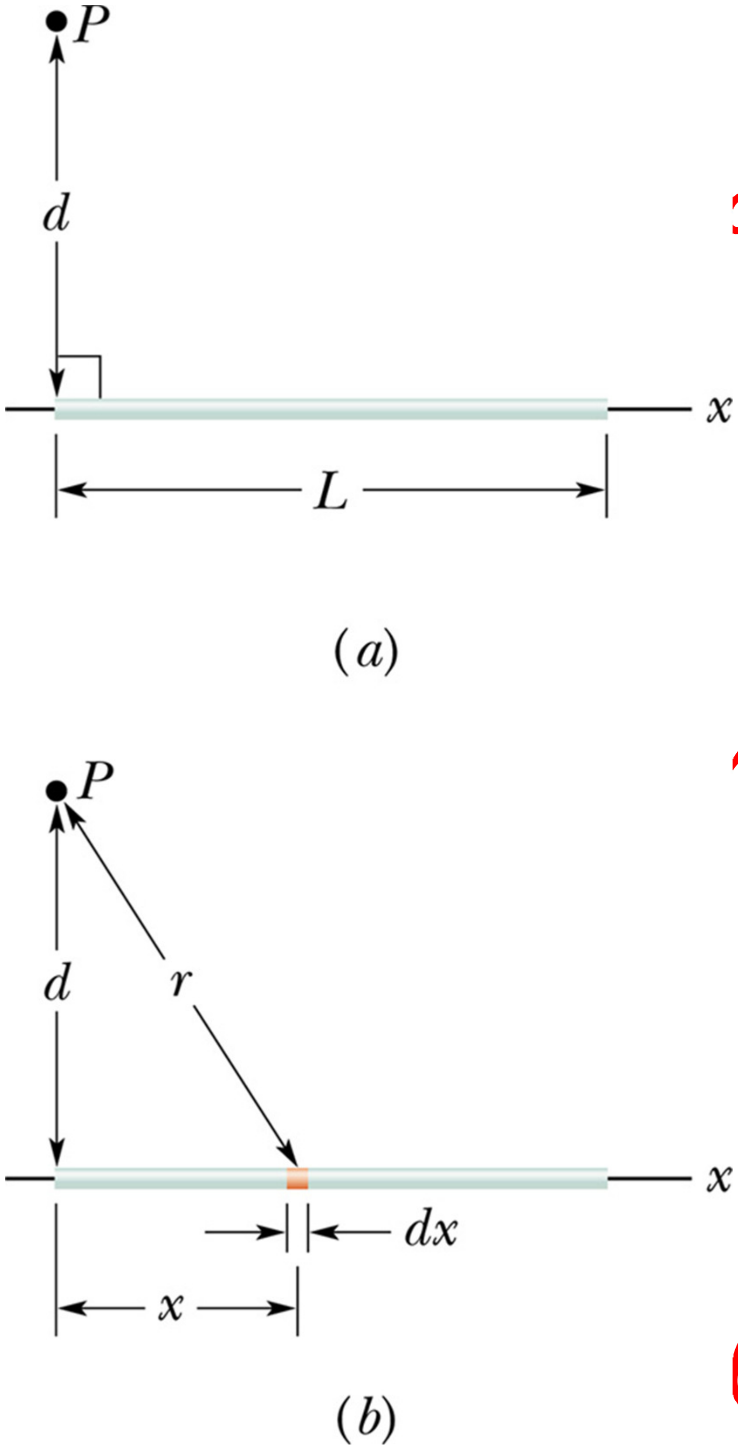
from uniform $\lambda = q_0/L$

$$dV = \frac{k\,dq}{r} \qquad dq = \lambda\,dx$$

$$r = \sqrt{d^2 + x^2}$$

$$dV = \frac{k\,\lambda\,dx}{(d^2+x^2)^{1/2}}$$

$$\int dV = V = \int_0^L \frac{k\,\lambda\,dx}{(d^2+x^2)^{1/2}} = k\lambda \int_0^L \frac{dx}{(d^2+x^2)^{1/2}}$$



from uniform $\lambda = q_0/L$

$$dV = \frac{k\,dq}{r} \qquad dq = \lambda\,dx$$

$$r = \sqrt{d^2+x^2}$$

$$dV = \frac{k\,\lambda\,dx}{(d^2+x^2)^{1/2}}$$

$$V = \int_0^L \frac{k\lambda\,dx}{(d^2+x^2)^{1/2}} = k\lambda \int_0^L \frac{dx}{(d^2+x^2)^{1/2}}$$

$$= k\lambda \Big|_0^L \ln\left(d + (x^2+d^2)^{1/2}\right)$$

$$= k\lambda\left(\ln\left(L + (L^2+d^2)^{1/2}\right) - \ln d\right)$$

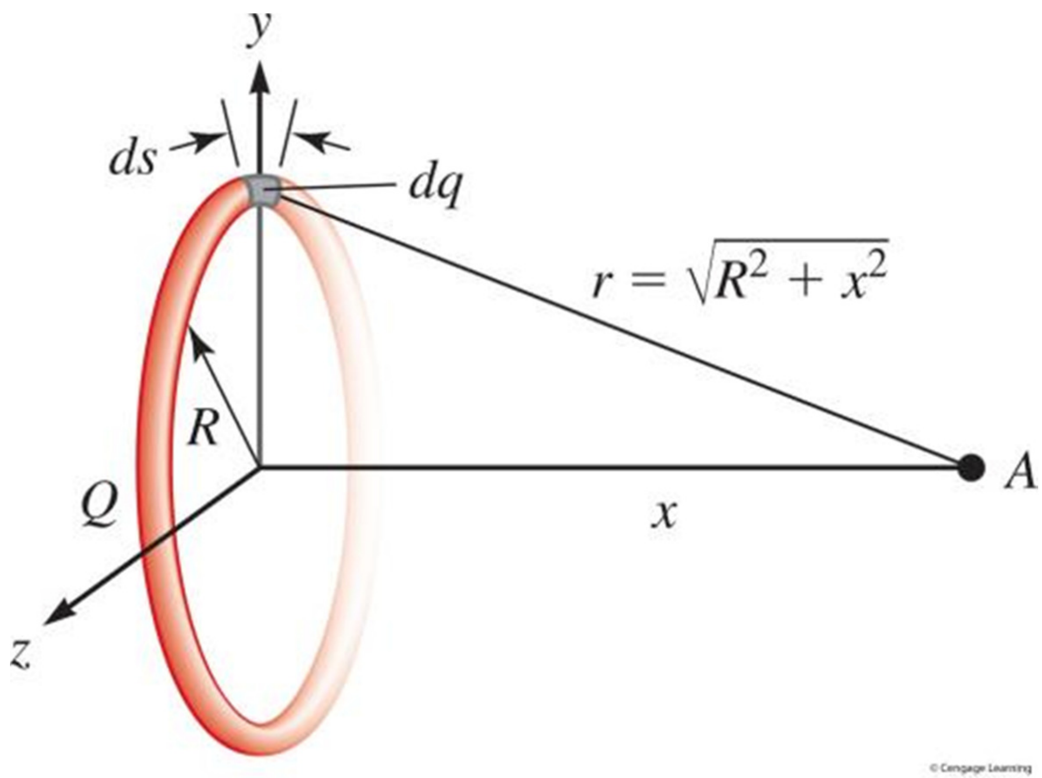
$$V(P) = k\lambda \ln\left(\frac{L + (L^2+d^2)^{1/2}}{d}\right)$$

How about other shapes?

$dV = \frac{k\,dq}{r}$

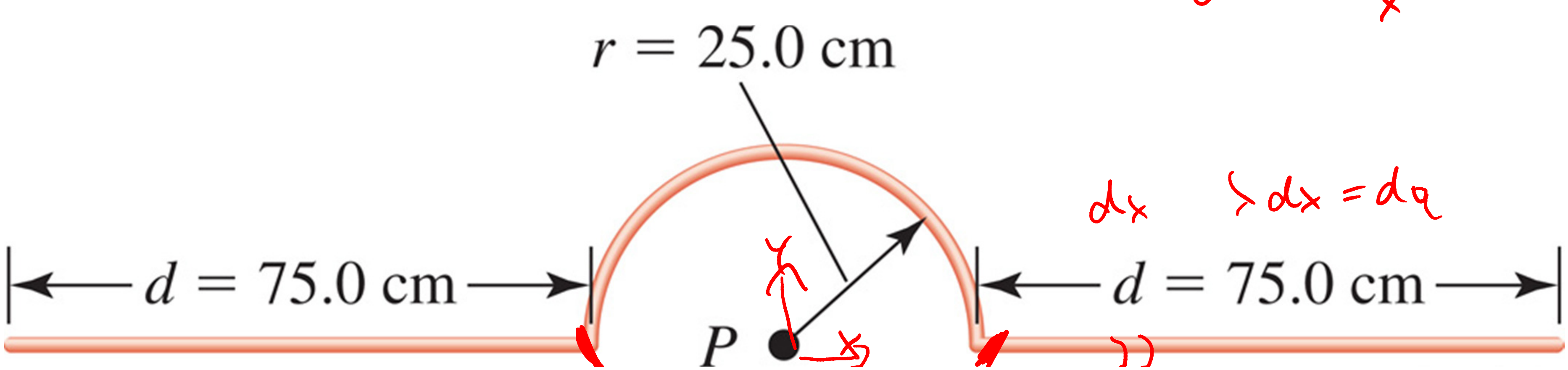
all dq's same r!

so, $V = \frac{kQ}{\sqrt{R^2+x^2}}$



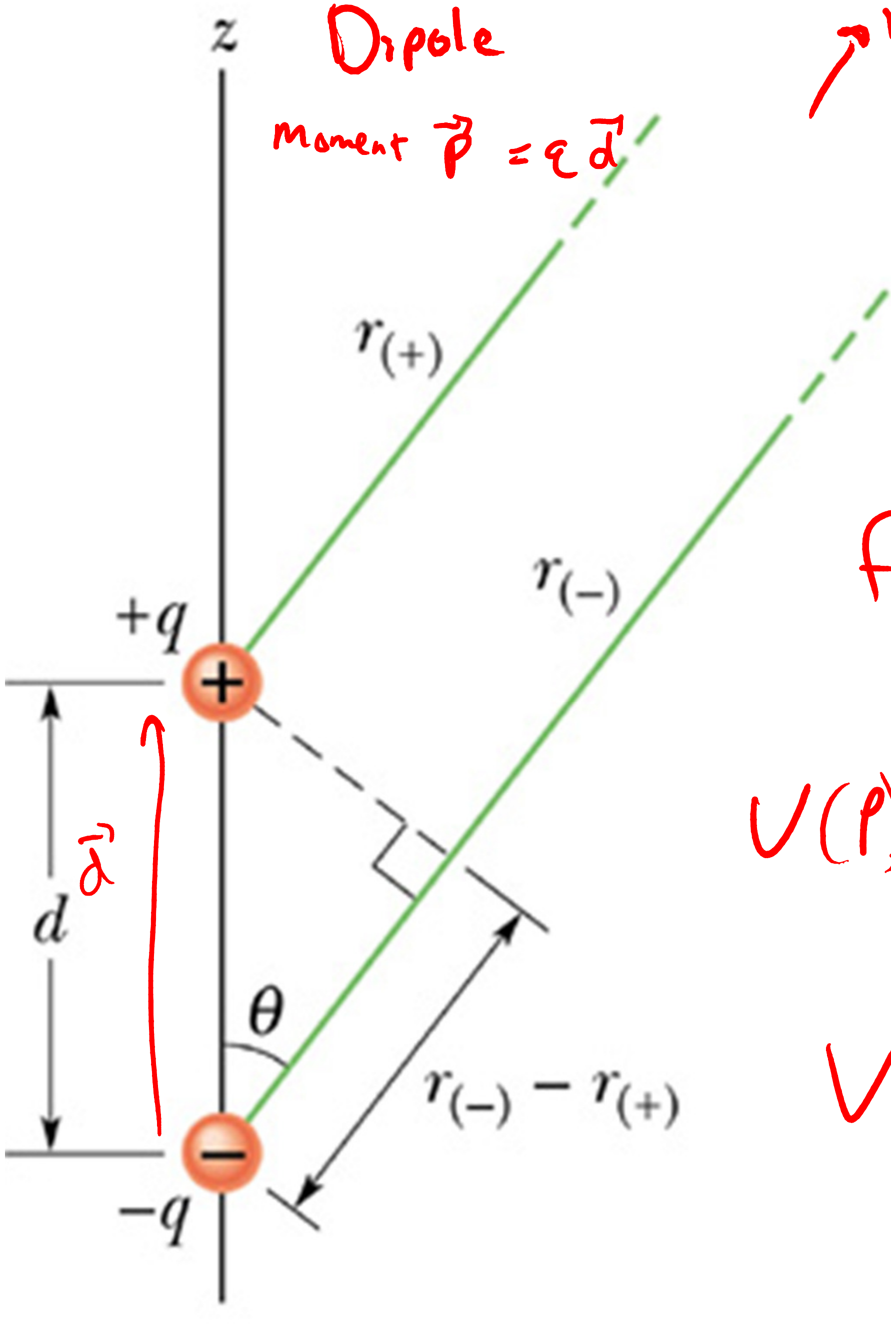
$dV = \frac{k(\lambda\,dx)}{x}$

$dx \quad \lambda dx = dq$



Dipole

moment $\vec{p} = q\vec{d}$



(b)

$$V_{TOT} = V_+ + V_-$$

$$= \frac{k(-q)}{r_-} + \frac{kq}{r_+}$$

$$= kq\left(\frac{r_+ - r_-}{r_+ r_-}\right)$$

for $r \gg d$ $\quad (r_- - r_+) \simeq d\cos\theta$

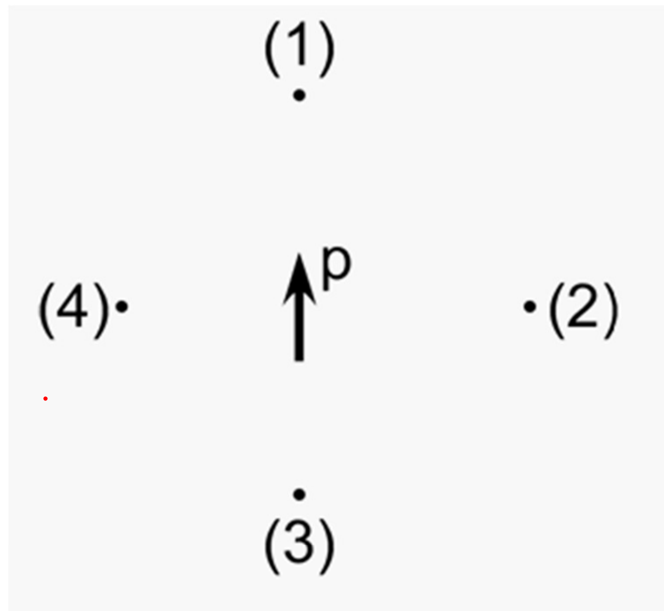
and $\quad r_- r_+ \simeq r^2$

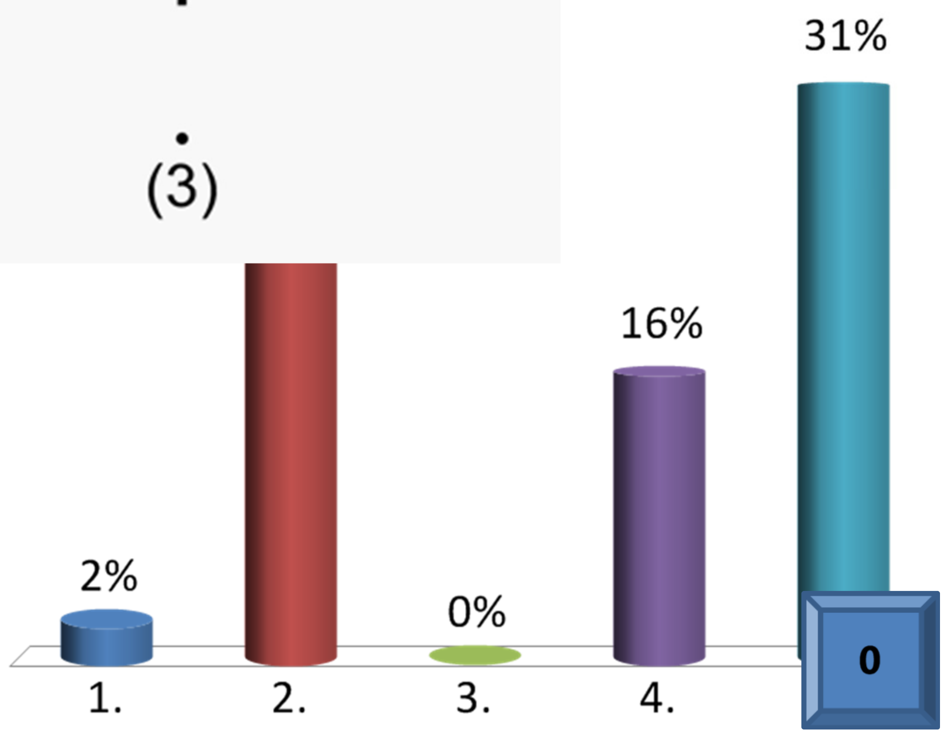
$$V(P) \simeq kq\frac{(d\cos\theta)}{r^2} = \frac{k|\vec{p}|\cos\theta}{r^2}$$

$$V = \frac{k\,\vec{p}\cdot\hat{r}}{r^2}$$

The drawing shows four points surrounding an electric dipole. Which one of the following expressions best ranks the electric potential at these four locations?

1. 1 > 2 > 3 > 4
2. ✓ 1 > 2 = 4 > 3
3. 3 > 2 > 4 > 1
4. 3 > 2 = 4 > 1
5. 2 = 4 > 1 = 3





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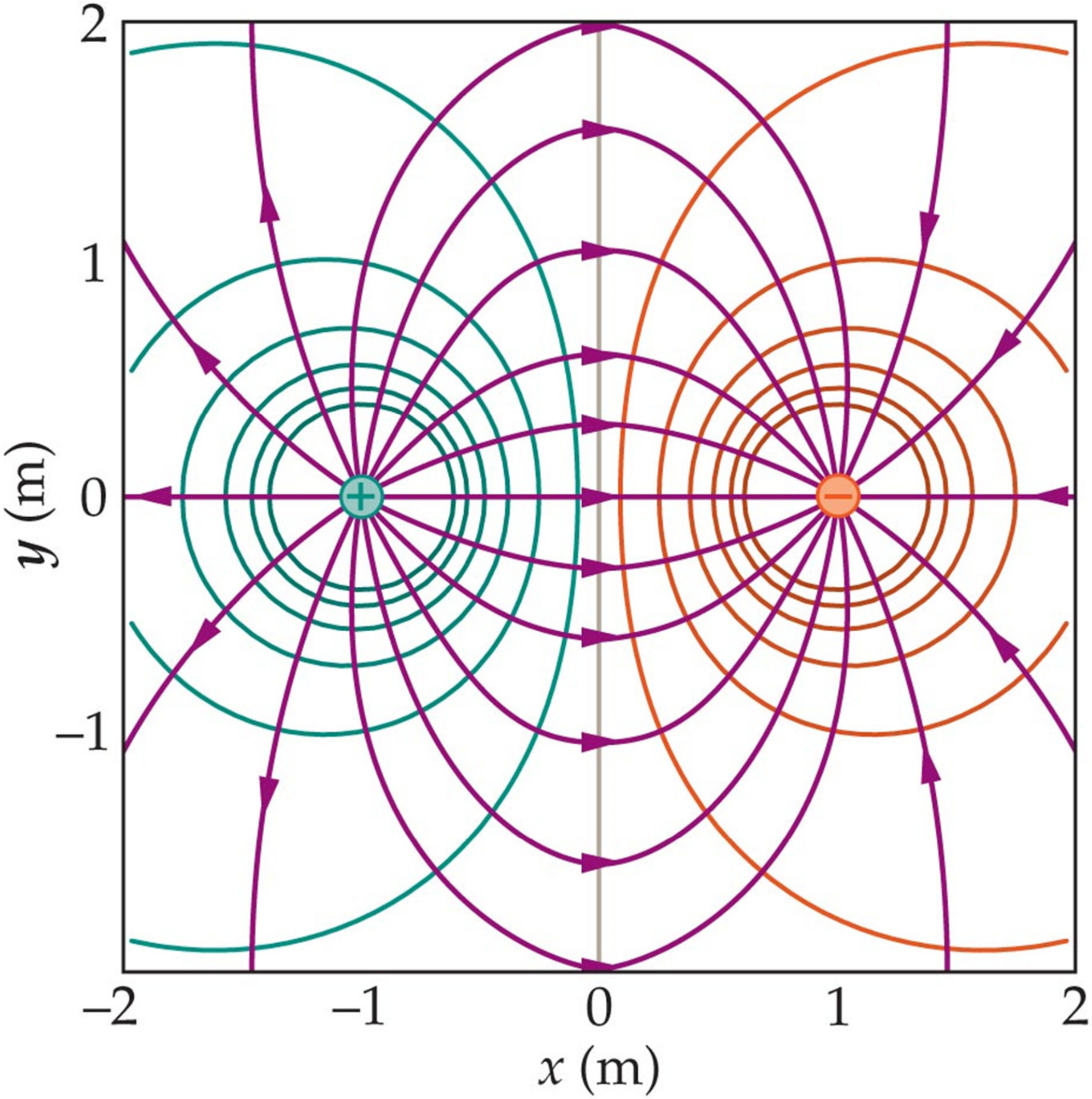
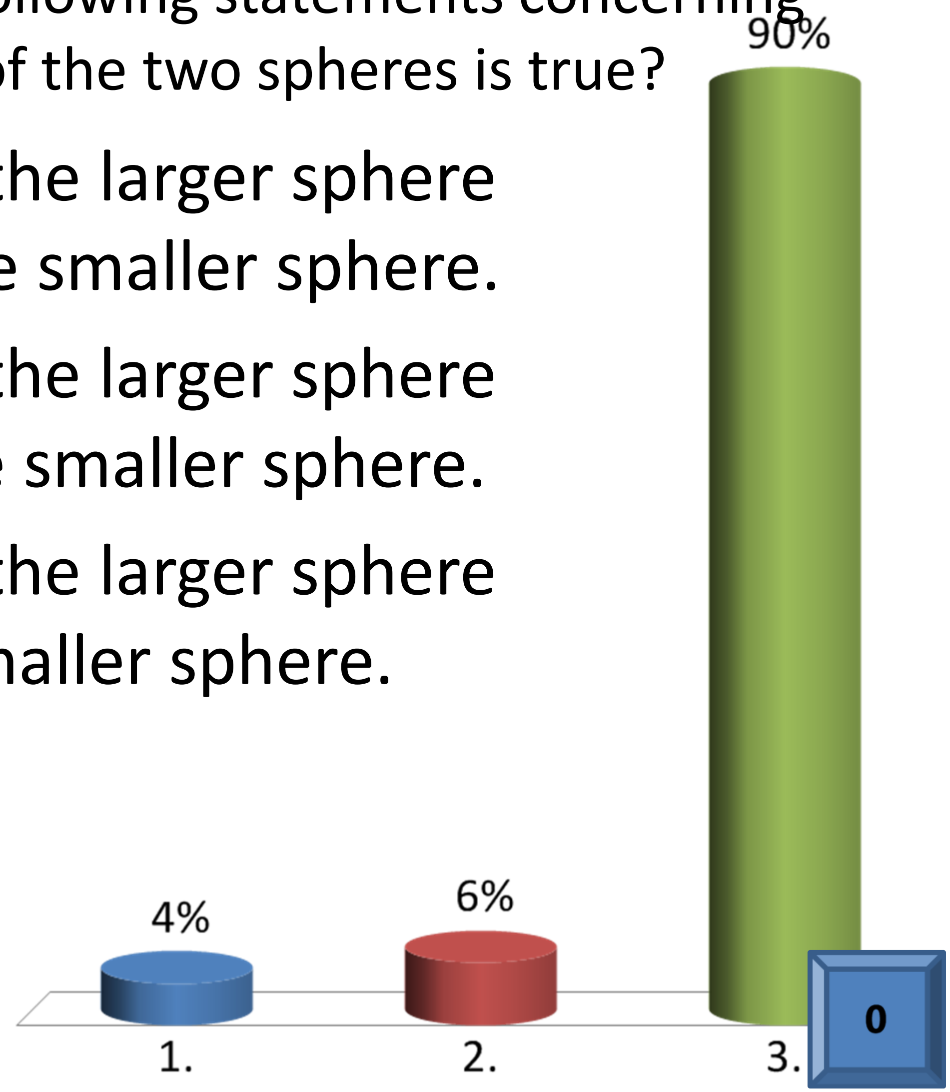


(b) Equipotentials for a dipole

Consider two conducting spheres with one having a larger radius than the other. Both spheres carry the same amount of excess charge. Which one of the following statements concerning the electric potential of the two spheres is true?

1. The electric potential of the larger sphere is greater than that of the smaller sphere.
2. The electric potential of the larger sphere is the same as that of the smaller sphere.
3. ✓ The electric potential of the larger sphere is less than that of the smaller sphere.



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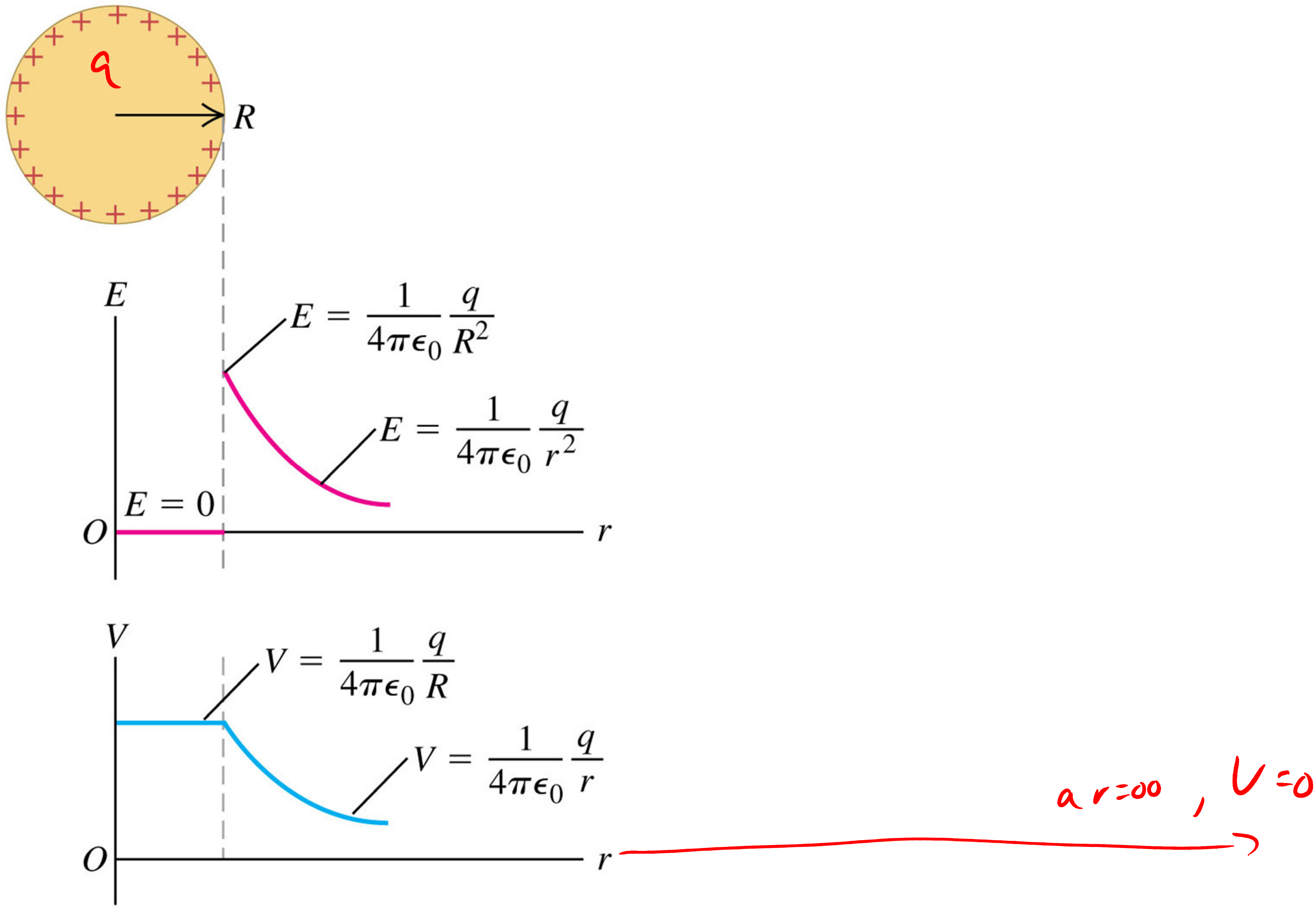
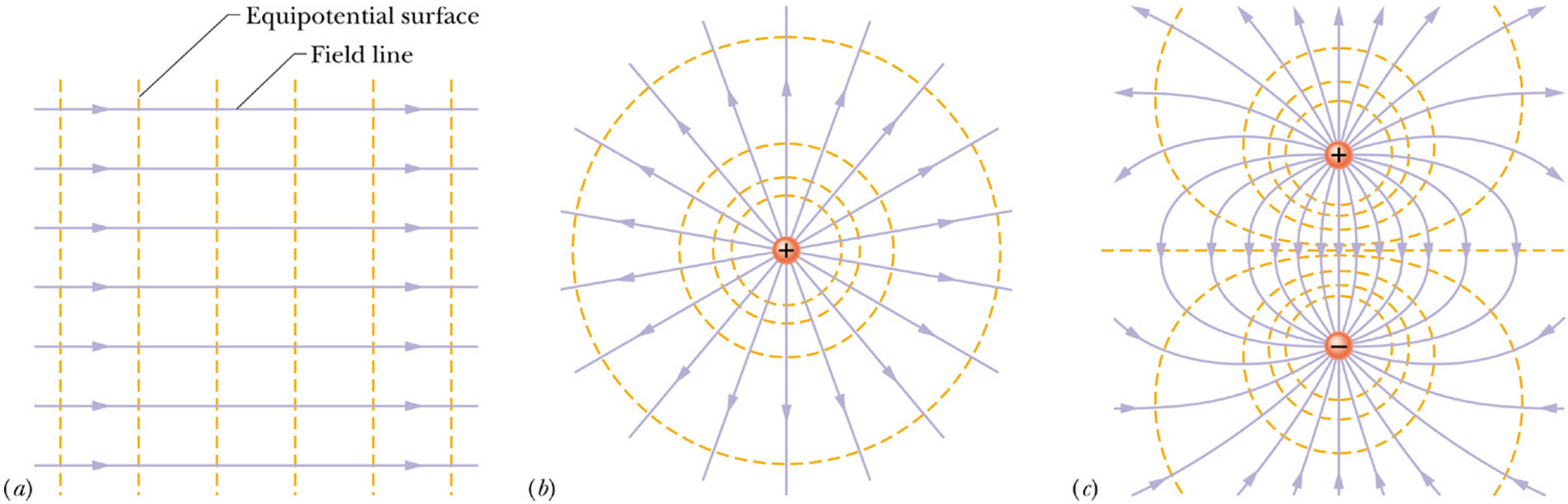
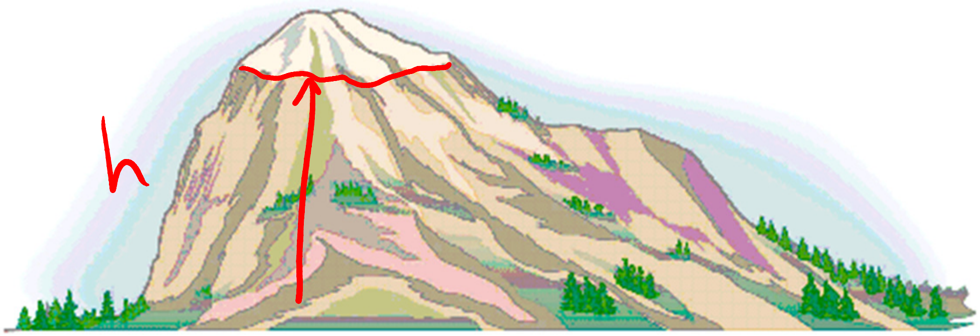
What if we charge a conducting sphere?



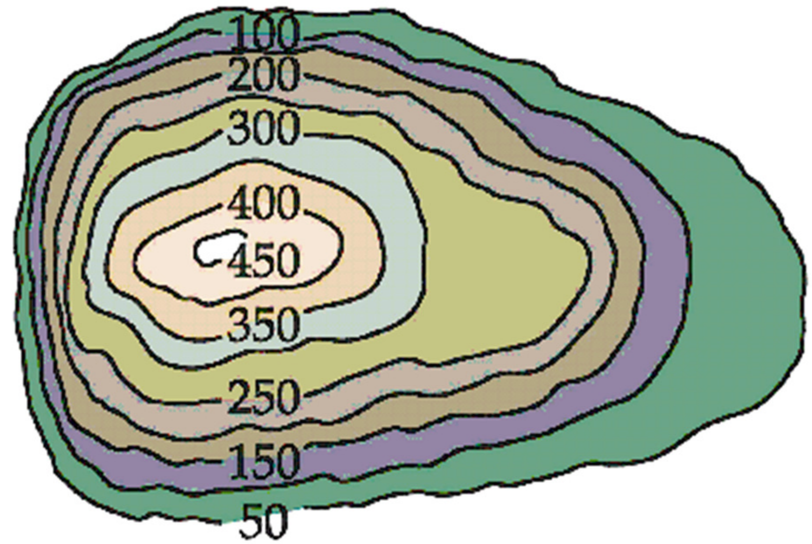


See class website “links” page for some simulations to play with



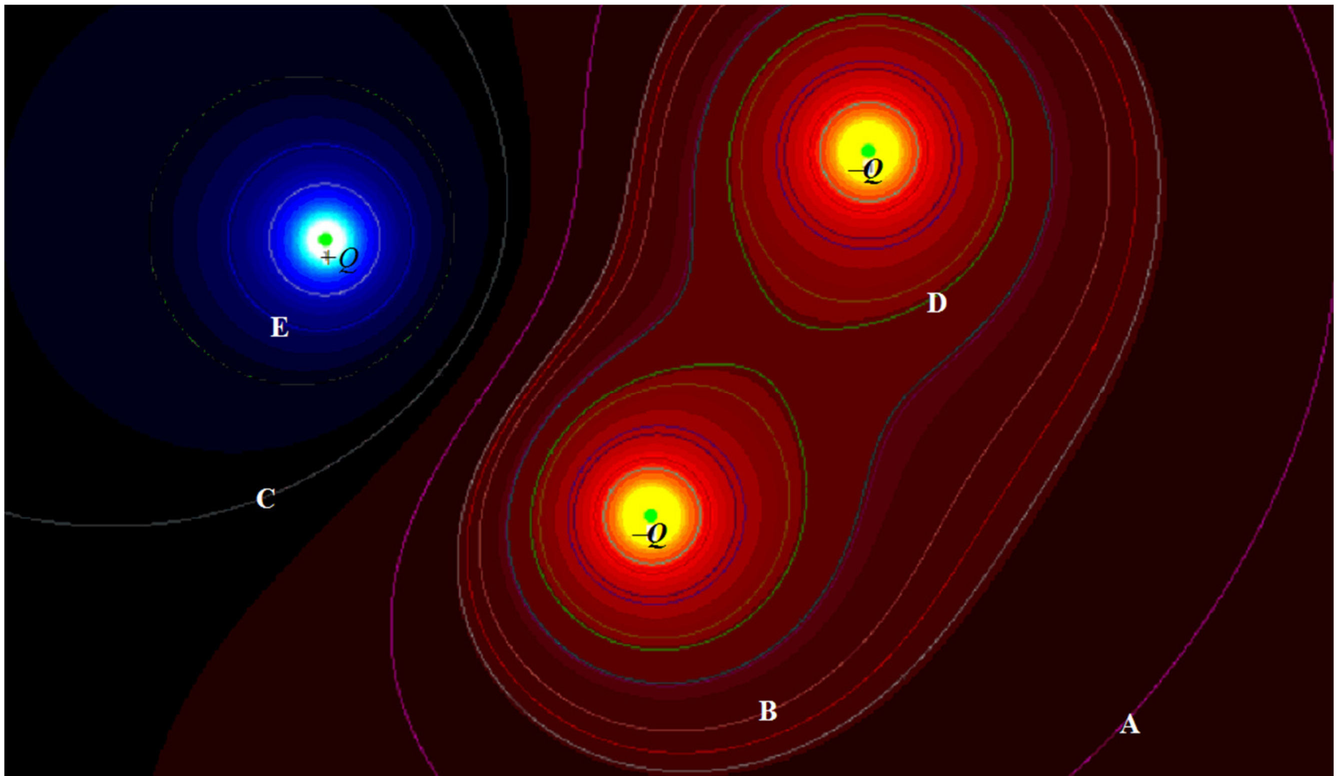
$U_g = m g h$

"U_g" $= g h$



Three point charges are of equal magnitude, but one is positive (blue) and two are negative (yellow). Some equipotential lines surrounding these charges are shown. At which of the points will an electron have the greatest electric potential energy?

1. A
2. B
3. C
4. D ✓
5. E



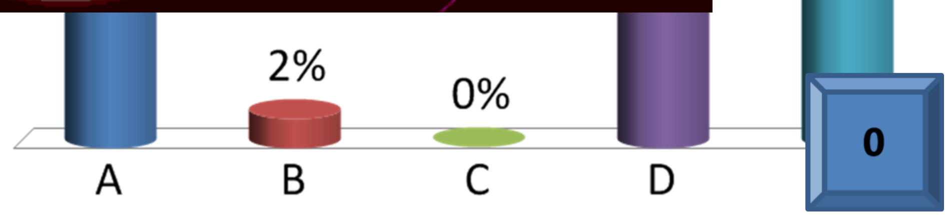


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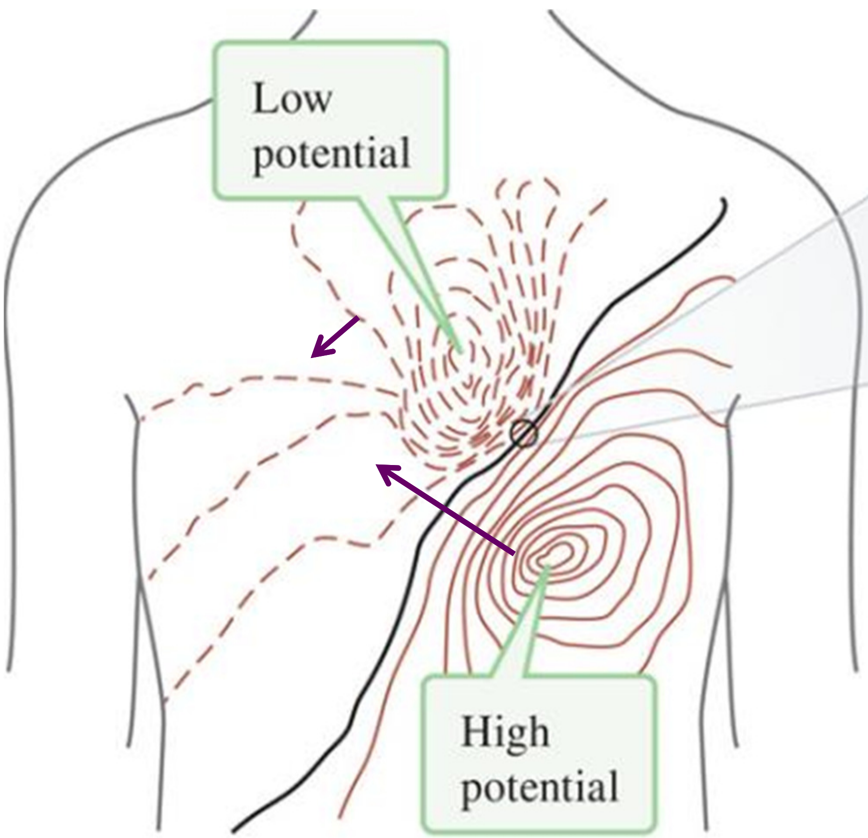
Fig.26.31



$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

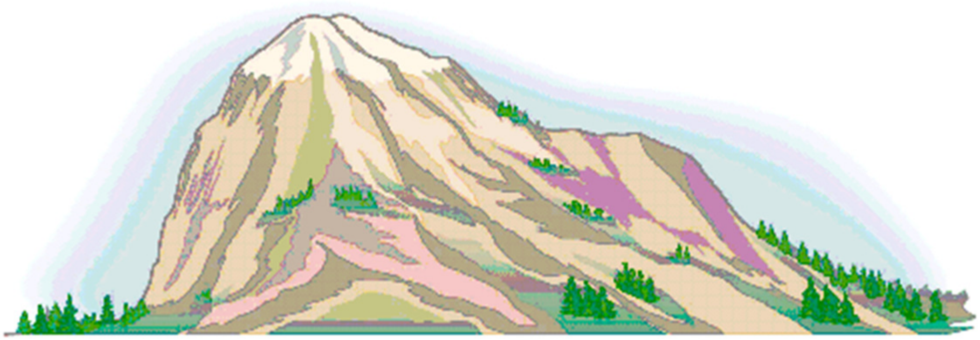
$$E_z = -\frac{\partial V}{\partial z}$$

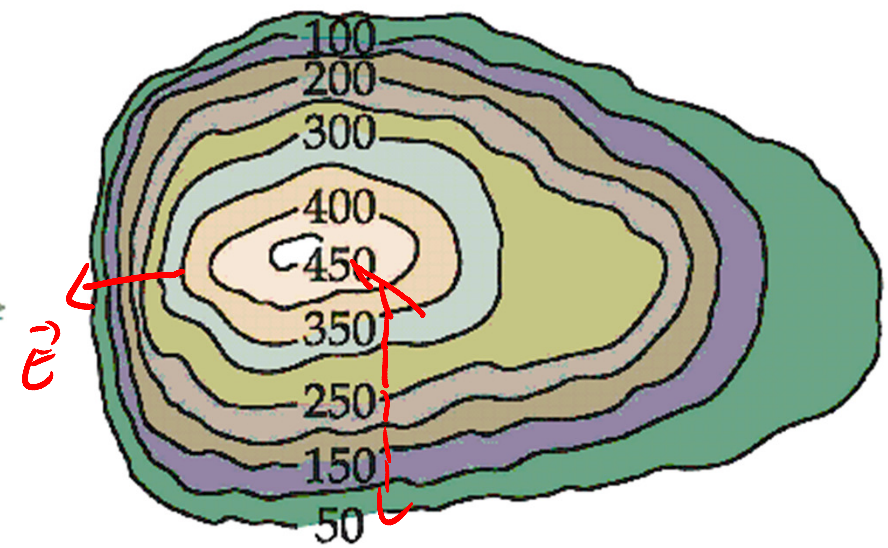
Take the derivative with respect to one variable, holding the others fixed.

Result: a vector perpendicular to equipotentials, pointing toward lower V

For radial symmetry, $E_r = -\frac{\partial V}{\partial r}$

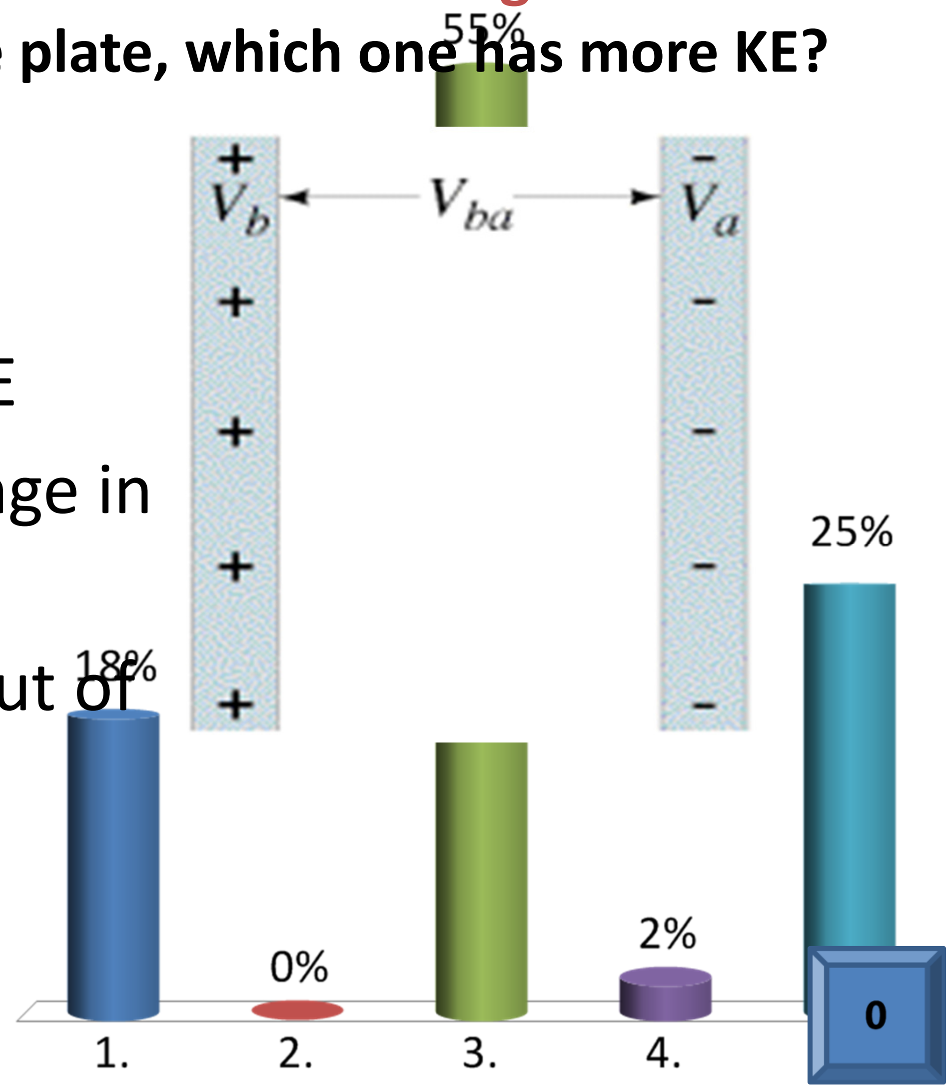
$\vec{F} = \vec{E} \cdot q$





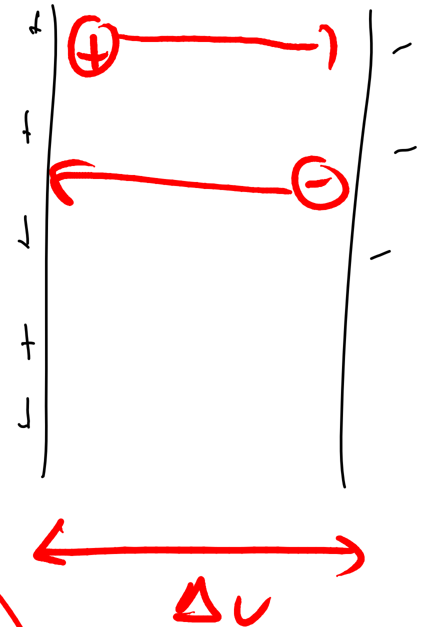
A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. When it strikes the opposite plate, which one has more KE?

1. Proton
2. Electron
3. Both acquire the same KE
4. Neither – there's no change in KE
5. Both have the same KE but of different signs



$\Delta U = \Delta V \cdot q$

conserve energy $\Delta U \rightarrow \Delta K = \frac{1}{2} m v^2$



for proton $\Delta V = V_f - V_i = -\Delta V$

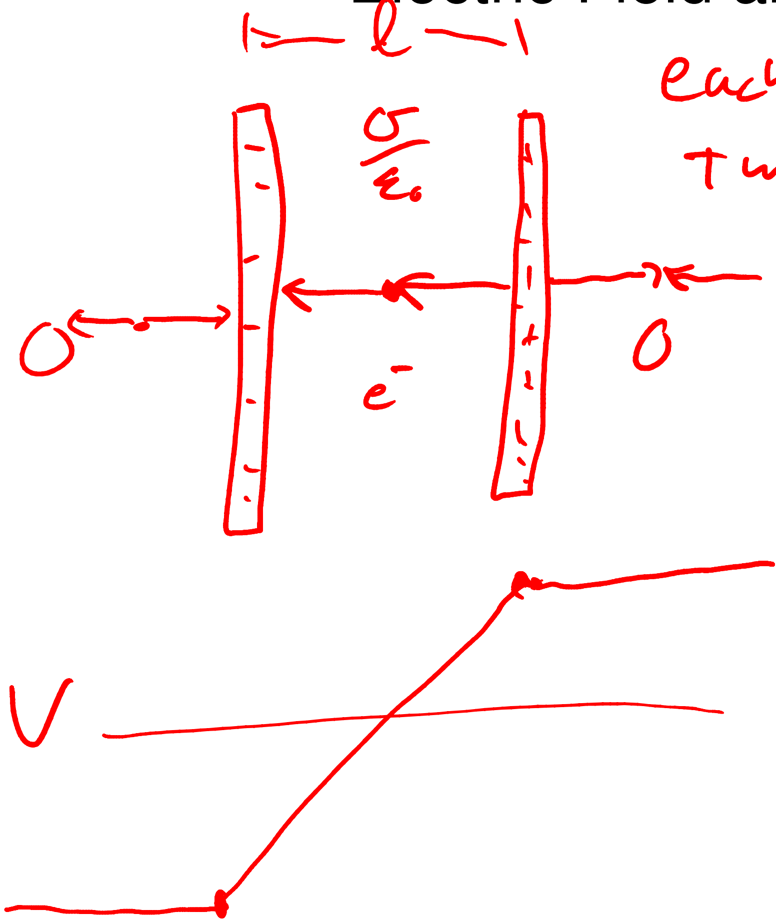
$\Delta U = -\Delta V(+e)$

for electron $\Delta = V_f - V_i = +\Delta V$

$\Delta U = \Delta V(-e) = -\Delta V e$

$\Delta U \rightarrow \Delta K = \frac{1}{2} m v^2$

Electric Field and Potential worksheet



each sheet makes $\frac{\sigma}{2\varepsilon_0} = E$

Two vectors add inside, $E_{tot} = \frac{\sigma}{\varepsilon_0}$

Two vectors cancel outside, $E = 0$

$$\Delta V \cdot q = \Delta U = -W = \int \vec{F} d\vec{\ell} = \int \vec{E} \cdot q \, d\vec{\ell}$$

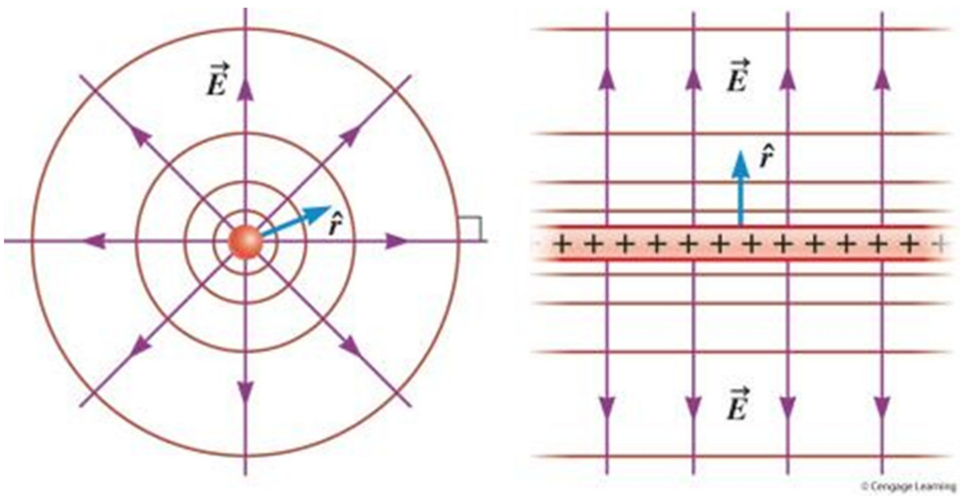
$$\Delta V = \int \vec{E} \cdot d\vec{\ell} = \frac{\sigma}{\varepsilon_0} \int d\ell = \frac{\sigma}{\varepsilon_0} \ell$$

Example from Book:

Finding a V from an E

Electric potential near a long thin charged rod is related to its electric field

EXAMPLE 26.12 Linear Symmetry



Find an expression for electric potential using the electric field, $\vec{E}(r) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$

INTERPRET and ANTICIPATE

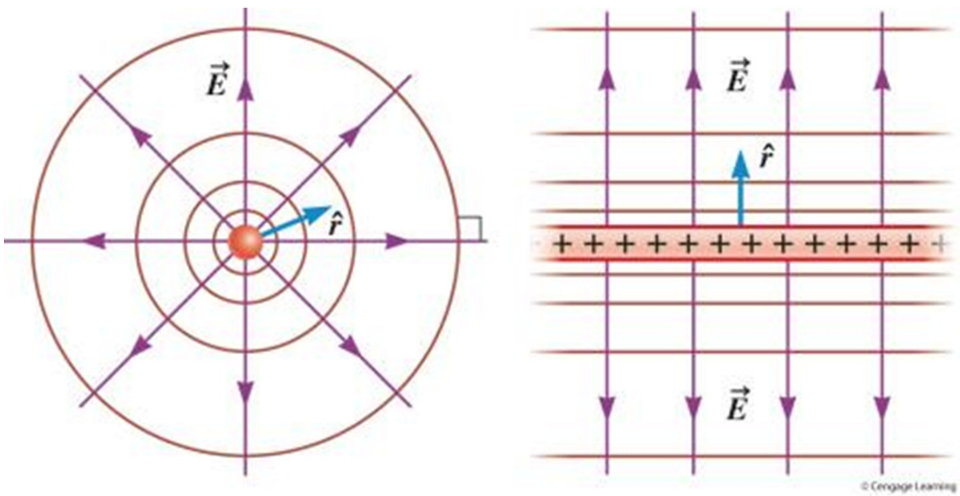
Field lines and equipotential surfaces are sketched—choose a path along E Start at rod's radius R, call that $V = 0$. (All further potentials are negative.)

SOLVE

$$\Delta V = -\int_{r_i=R}^{r_f} \vec{E} \bullet d\vec{r} \qquad \text{Choose } d\vec{r} = dr\hat{r}$$

Electric potential near a long thin charged rod is related to its electric field

EXAMPLE 26.12 **Linear Symmetry**



SOLVE

$$\vec{E}(r) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{\boldsymbol{r}}$$

$$\Delta V = -\int_{r_i=R}^{r_f} \vec{E} \bullet d\vec{r} \qquad d\vec{r} = dr\hat{\boldsymbol{r}}$$

$$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_i=R}^{r_f}\frac{1}{r}\hat{\boldsymbol{r}}\cdot dr\hat{\boldsymbol{r}}$$

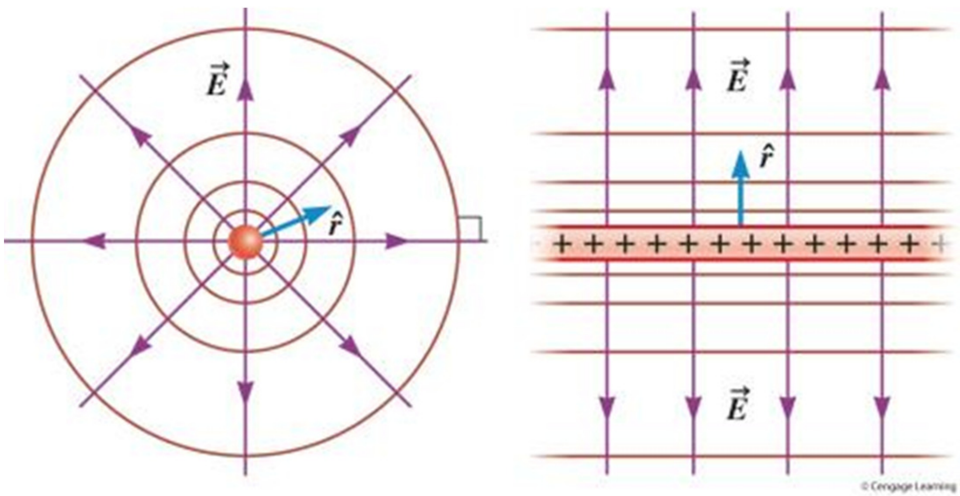
$$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_i=R}^{r_f}\frac{1}{r}dr\left(\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}}\right)$$

$$V\left(r_f\right) - 0 = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(r_f\right) - \ln\left(R\right)\right]$$

$$V\left(r\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R}{r}\right)$$

Electric potential near a long thin charged rod is related to its electric field

EXAMPLE 26.12 Linear Symmetry



CHECK and THINK

$$V(r) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R}{r}\right)$$

Find electric field… is it $\vec{E}(r) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$?

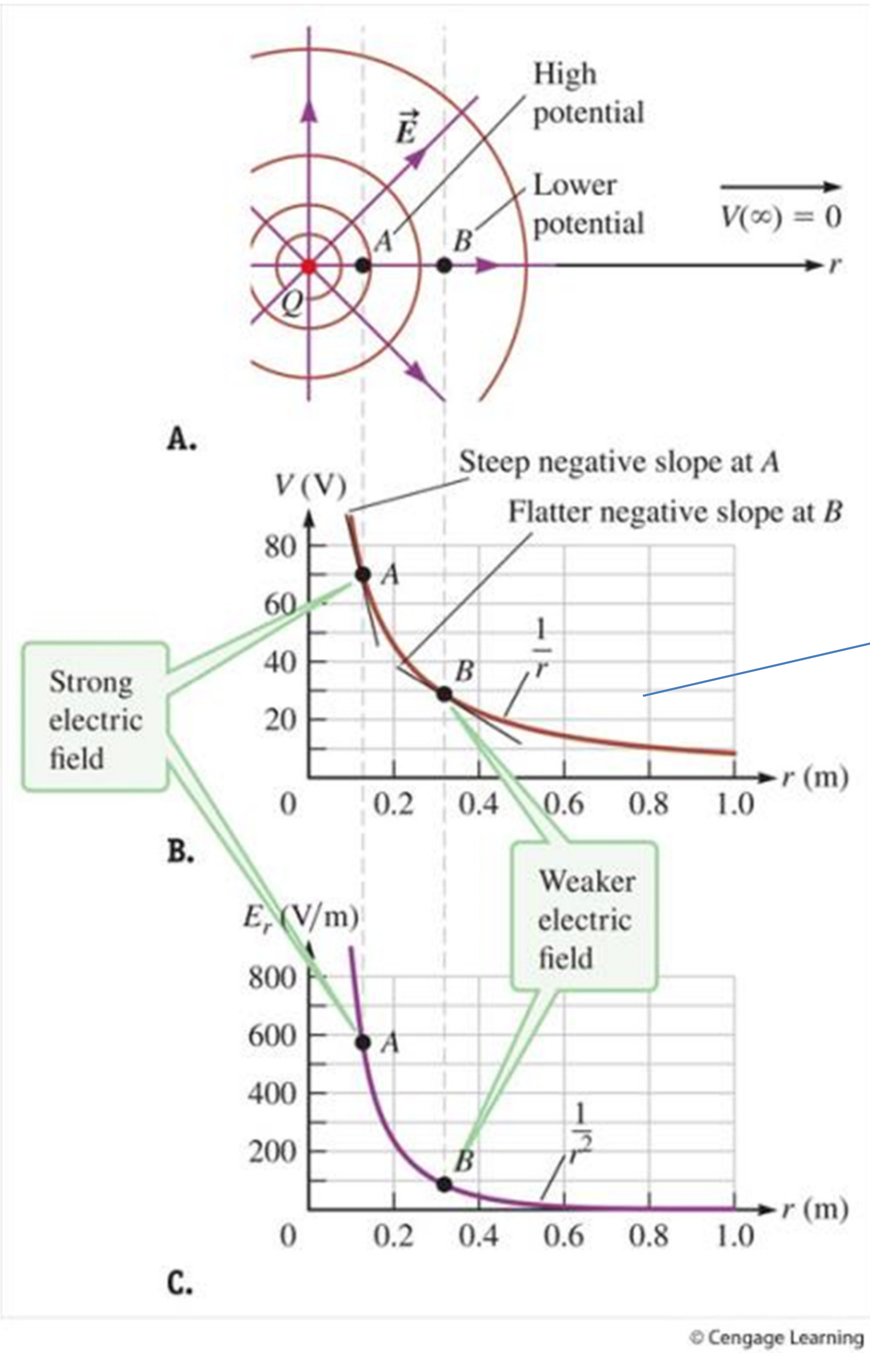
$$E_r = -\frac{\partial V(r)}{\partial r} = -\frac{\partial}{\partial r}\left[\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{R}{r}\right)\right]$$

$$E_r = -\frac{\lambda}{2\pi\varepsilon_0}\frac{\partial}{\partial r}\left[\ln(R) - \ln(r)\right]$$

$$E_r = -\frac{\lambda}{2\pi\varepsilon_0}\left[0 - \frac{1}{r}\right]$$

$$E_r = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}$$

Field and potential are paired by calculus, as seen on graphs



Slope of the potential versus *r* graph is the negative of the electric field