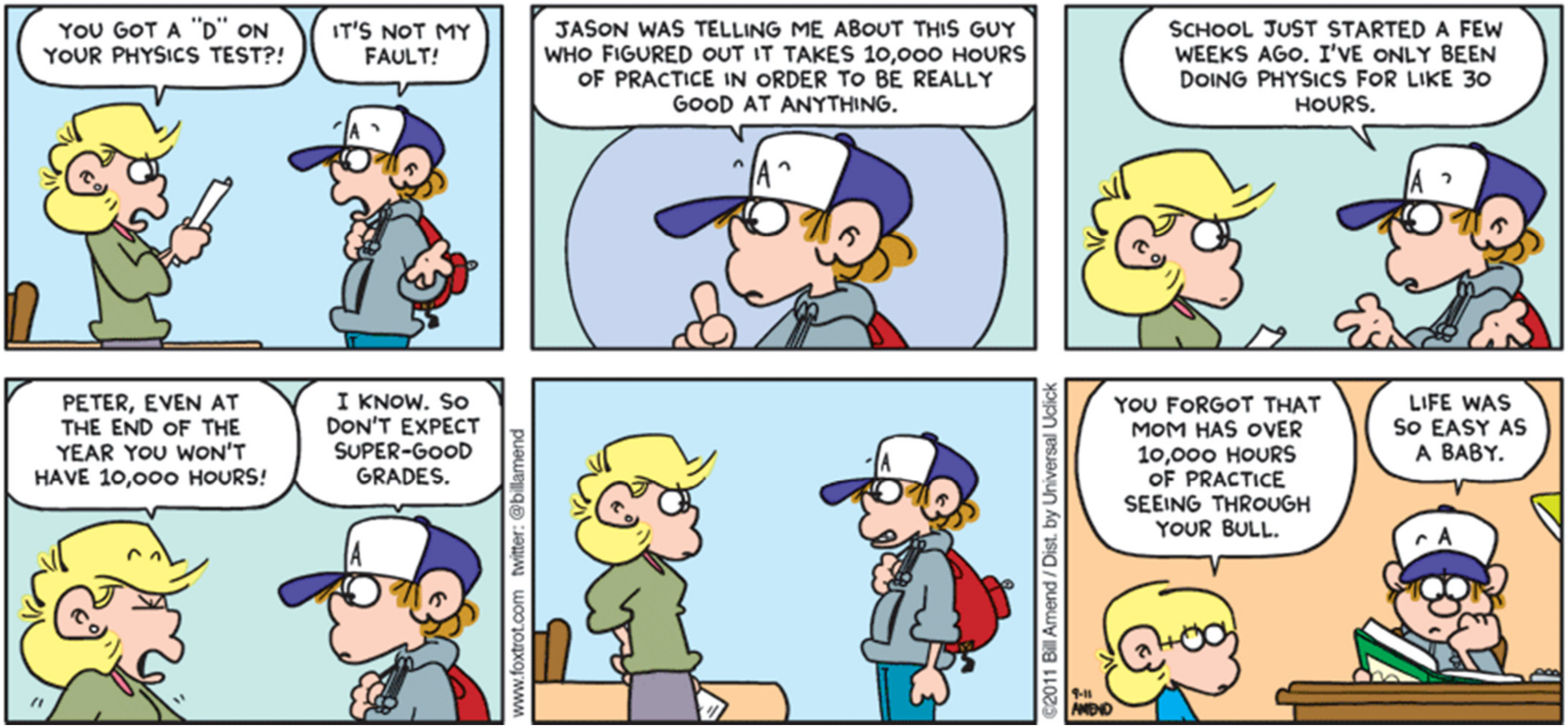
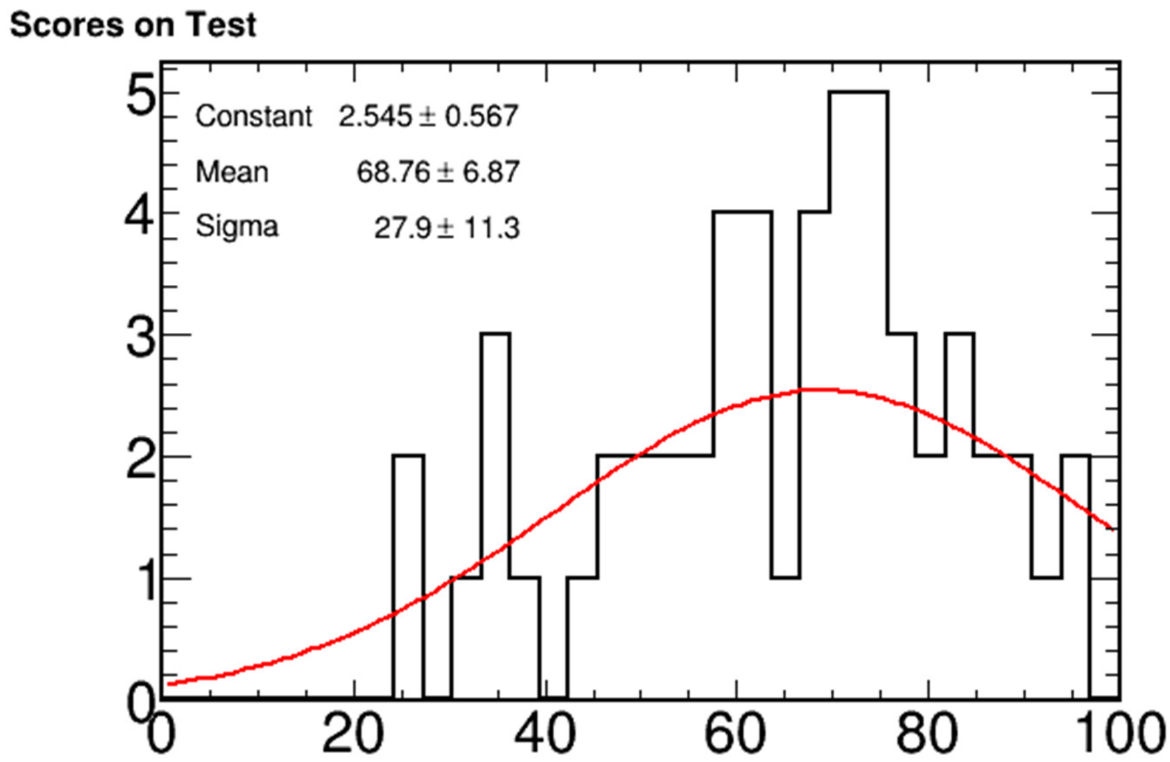
From the “Foxtrot” comic:



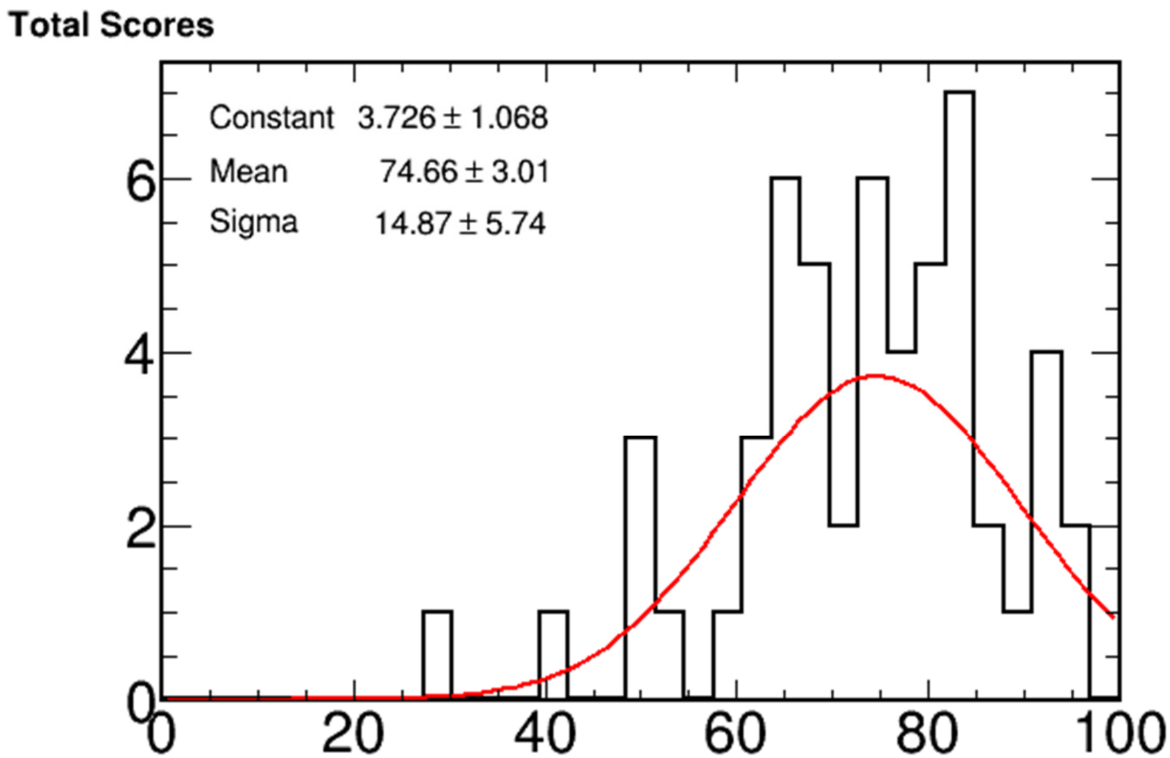
Fun fact: cartoonist Bill Amend took the unusual career path of turning a physics major into a cartooning career



- 70+: Doing fine!
- 60's: doing ok, some things you need to figure out
- 50's: meh
- Lower: time to worry

(*the red line and fit parameters show a Gaussian fit failing miserably*)

Best guess at a "curve" (*ie*, turning percents into letter grades) is online, and uses your total score (see next slide)

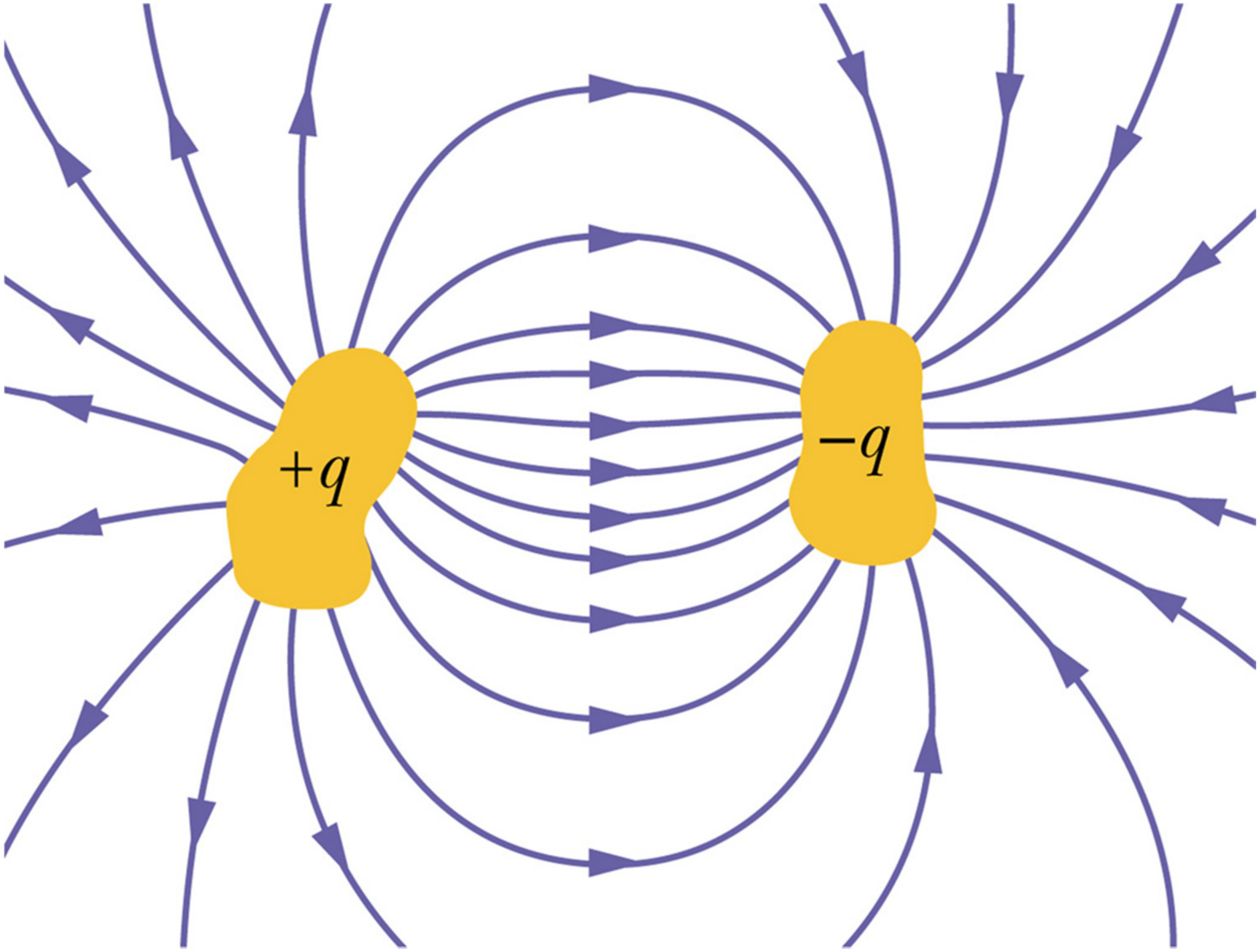


eGradebook total calculation done with:

• Dropping lowest thing in each of: HW, In class thing, clicker, online HW, online RQ

Following the recipe on the “grades” link, a letter grade is posted.
If the semester was done today, that’s what you’d get.

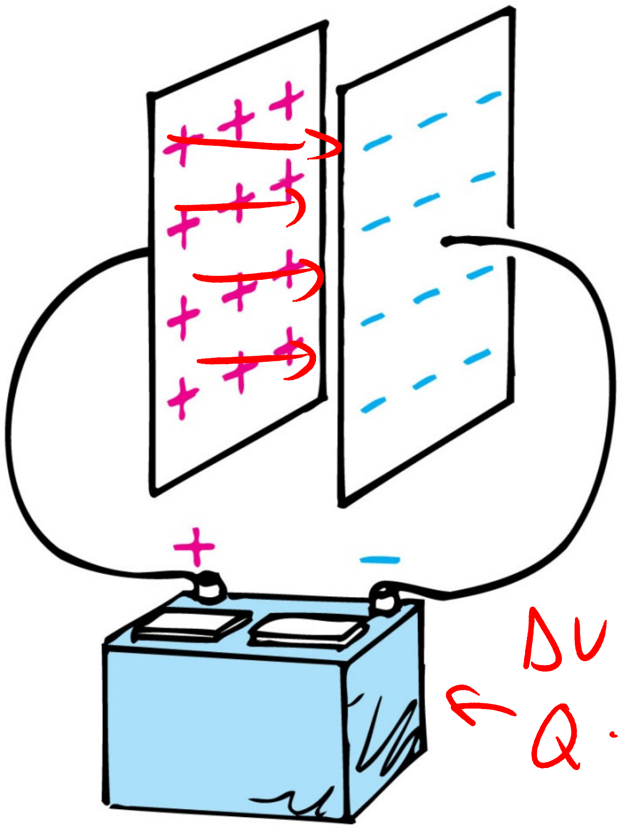
Someone will now ask: “Was this curved?”.
Answer: the very act of turning some percent grade into some letter grade is exactly what “curving” does. So, yes.


+q
−q

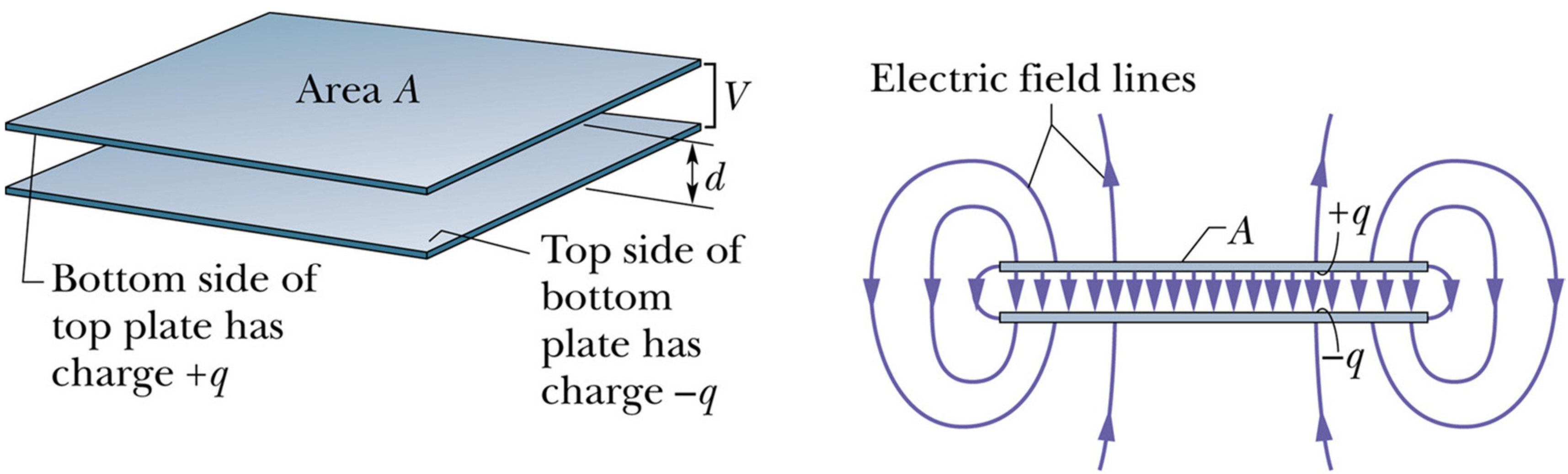
"Capacitance"

$q = CV$

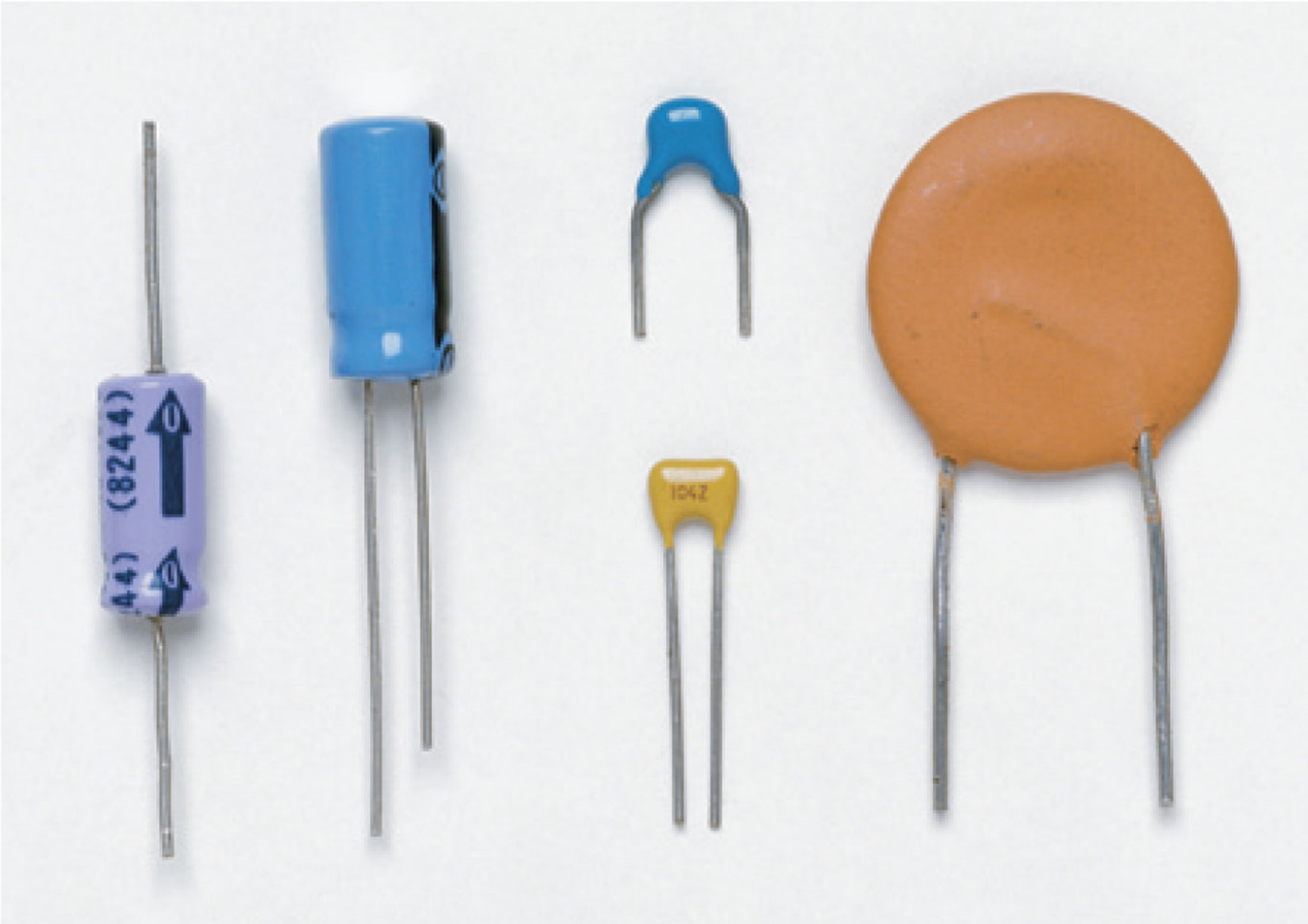
$1 \text{ Farad} = F = C/V$

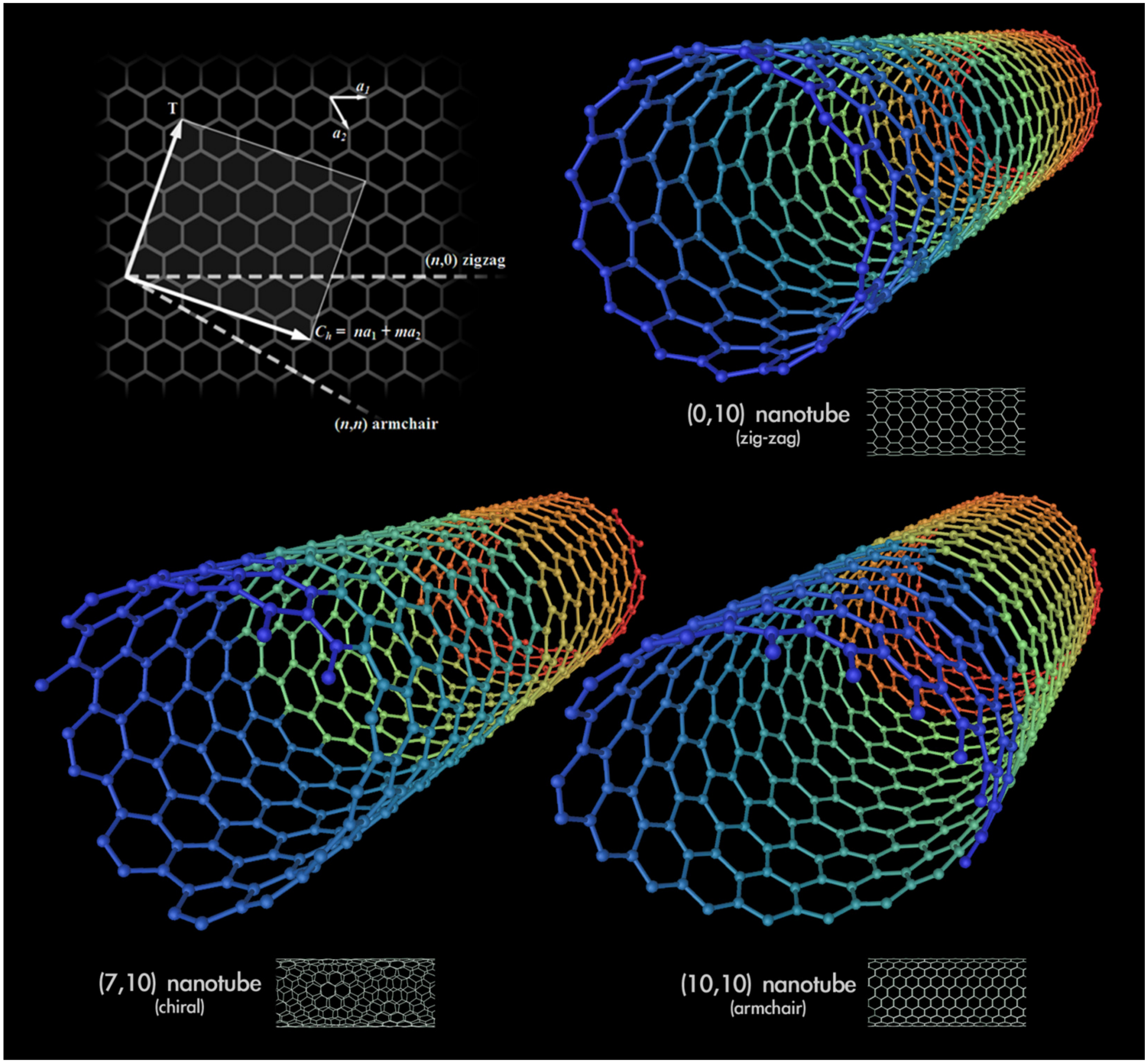


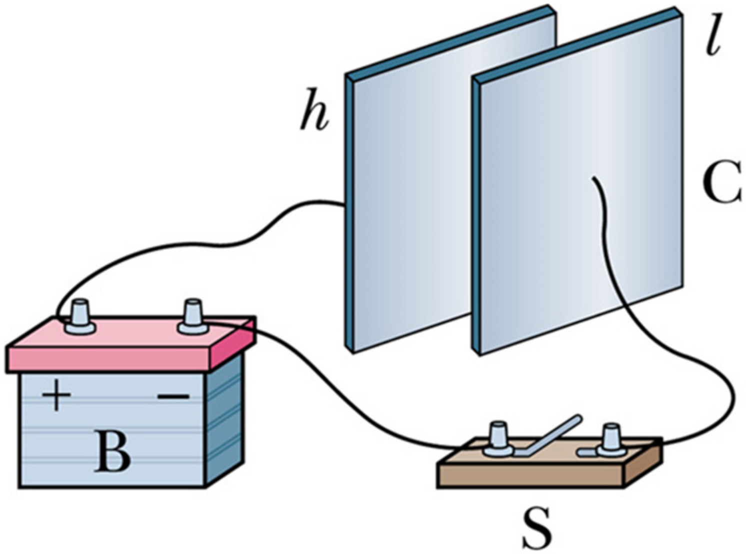
ΔV-holding Q-producing machine



(a) (b)


(8244)
104Z


T
a_1
a_2
$(n,0)$ zigzag
$C_h = na_1 + ma_2$
(n,n) armchair
(0,10) nanotube
(zig-zag)
(7,10) nanotube
(chiral)
(10,10) nanotube
(armchair)

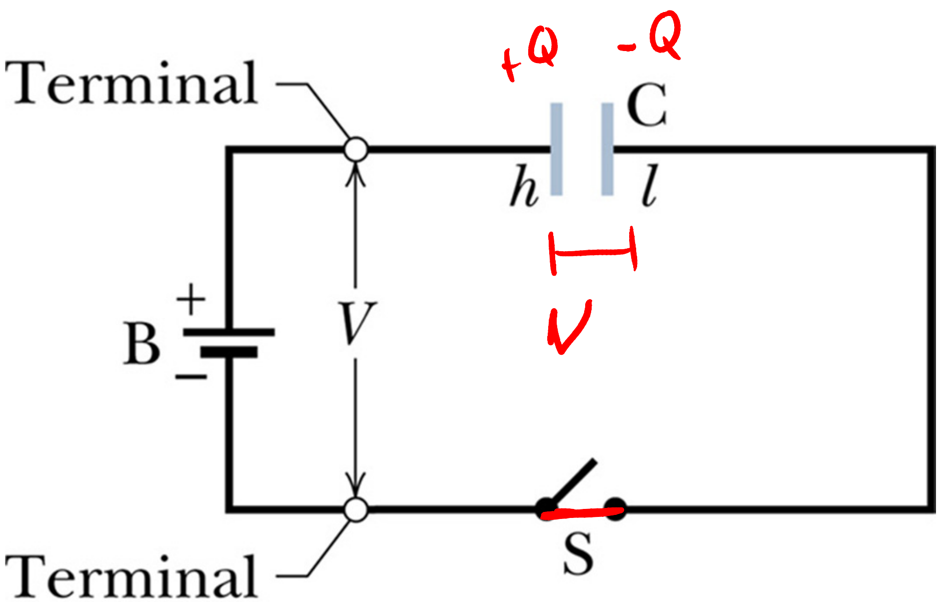


(*a*)

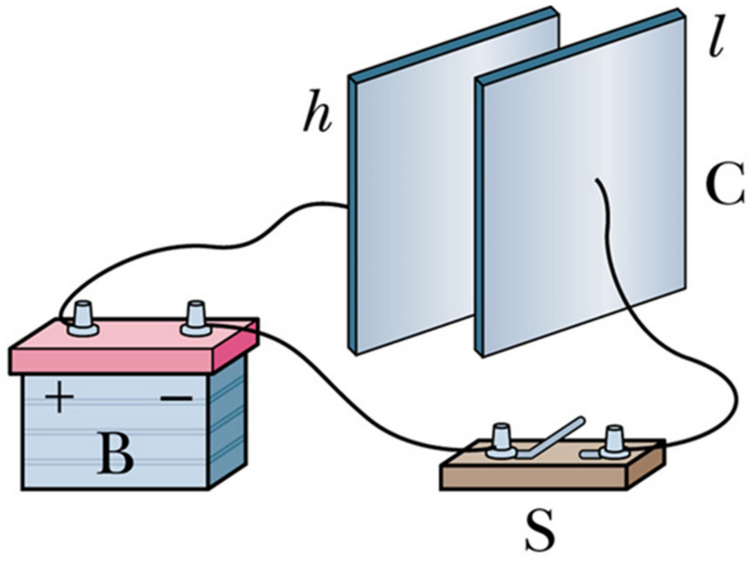
$q = CV$

C = Capacitance

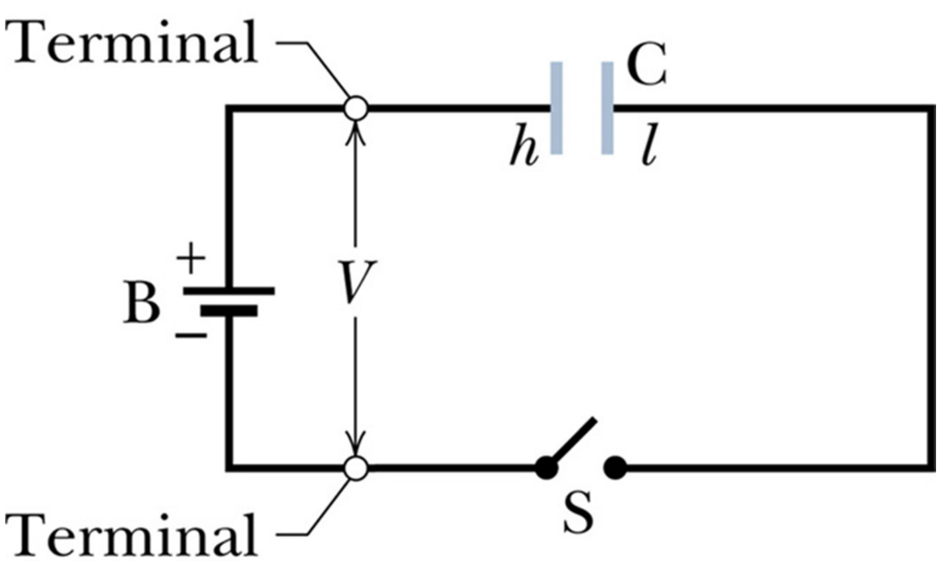
$$\text{Farad} = \frac{\text{Coulombs}}{\text{Volt}}$$



(*b*)

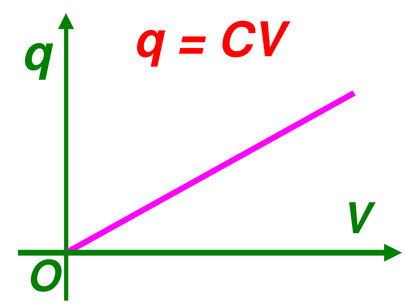


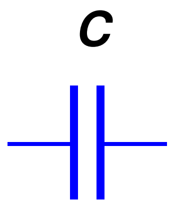
(*a*)

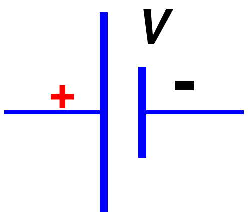


(*b*)

$$C = \frac{q}{V}$$




C


V
+
-

So, what is this proportionality constant C for this set of parallel plates?
Let's work it out using Gauss' Law

$C = Q/V$

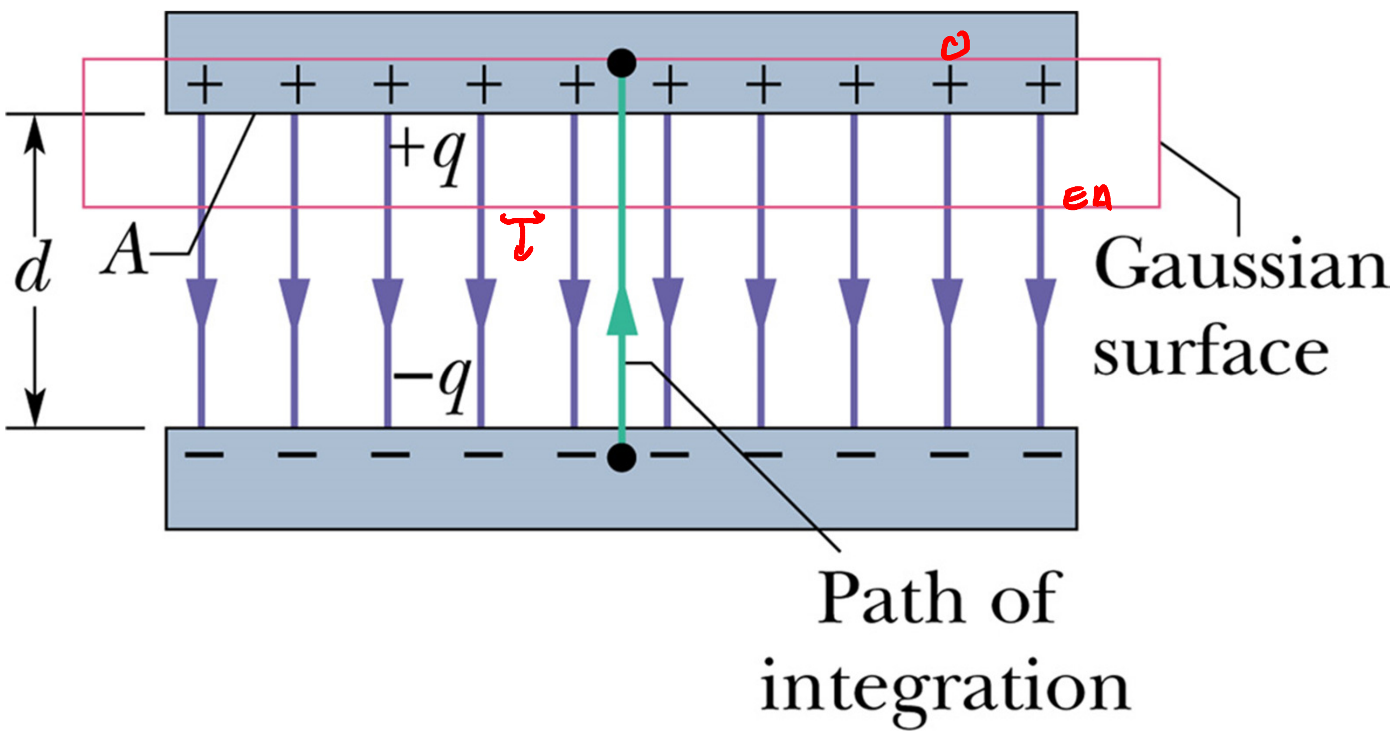
$\int \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$

$\int |E||dA| \cos\theta = q_{enc}/\varepsilon_0$

$E \int dA = EA = q_{enc}/\varepsilon_0$

$0 + 0 + 0 + 0 + 0 + EA = q_{enc}/\varepsilon_0$

$E = \frac{q}{\varepsilon_0 A}$



$$E = \frac{Q}{\varepsilon_0 A}$$

$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{s}$$

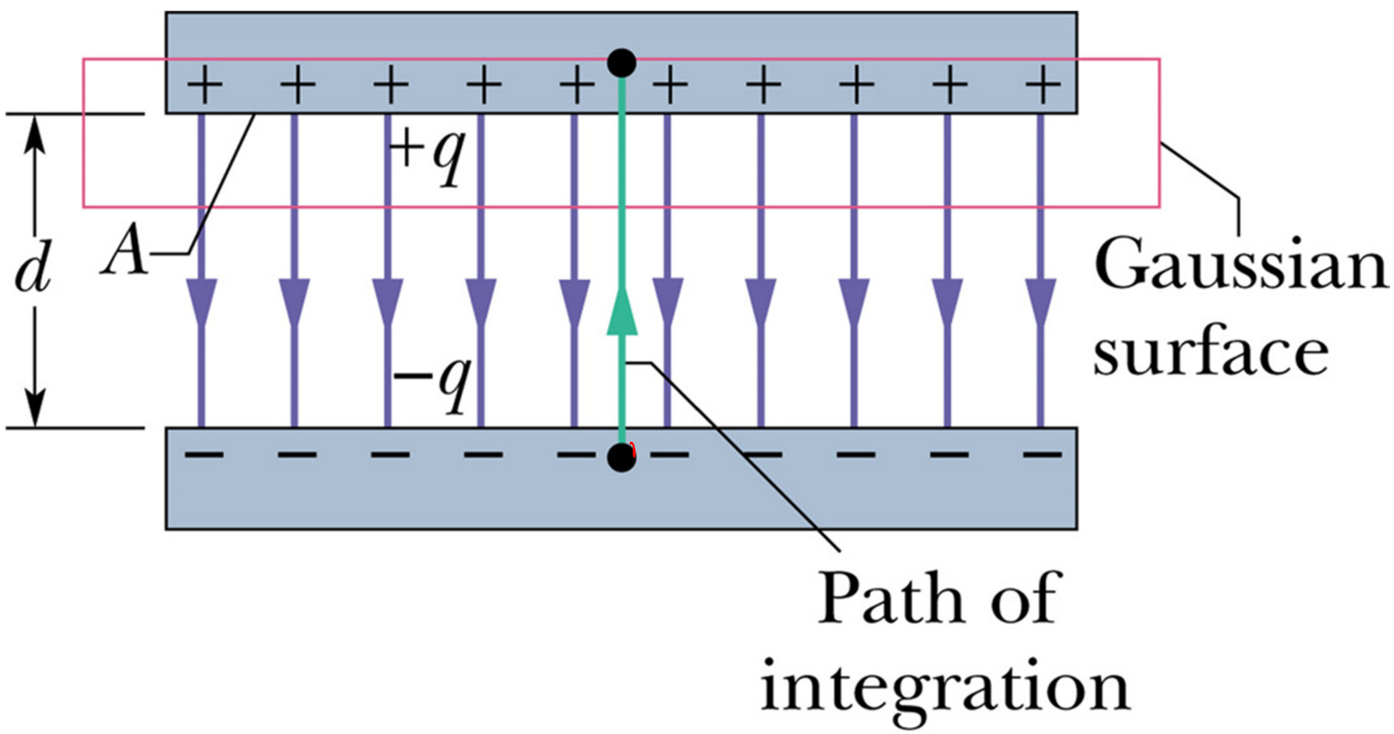
$$-\int_i^f |E||ds| \cos\theta$$

$$\Delta V = E\int_i^f ds = Ed$$

$$V = E\cdot d = \frac{Q\,d}{\varepsilon_0 A}$$

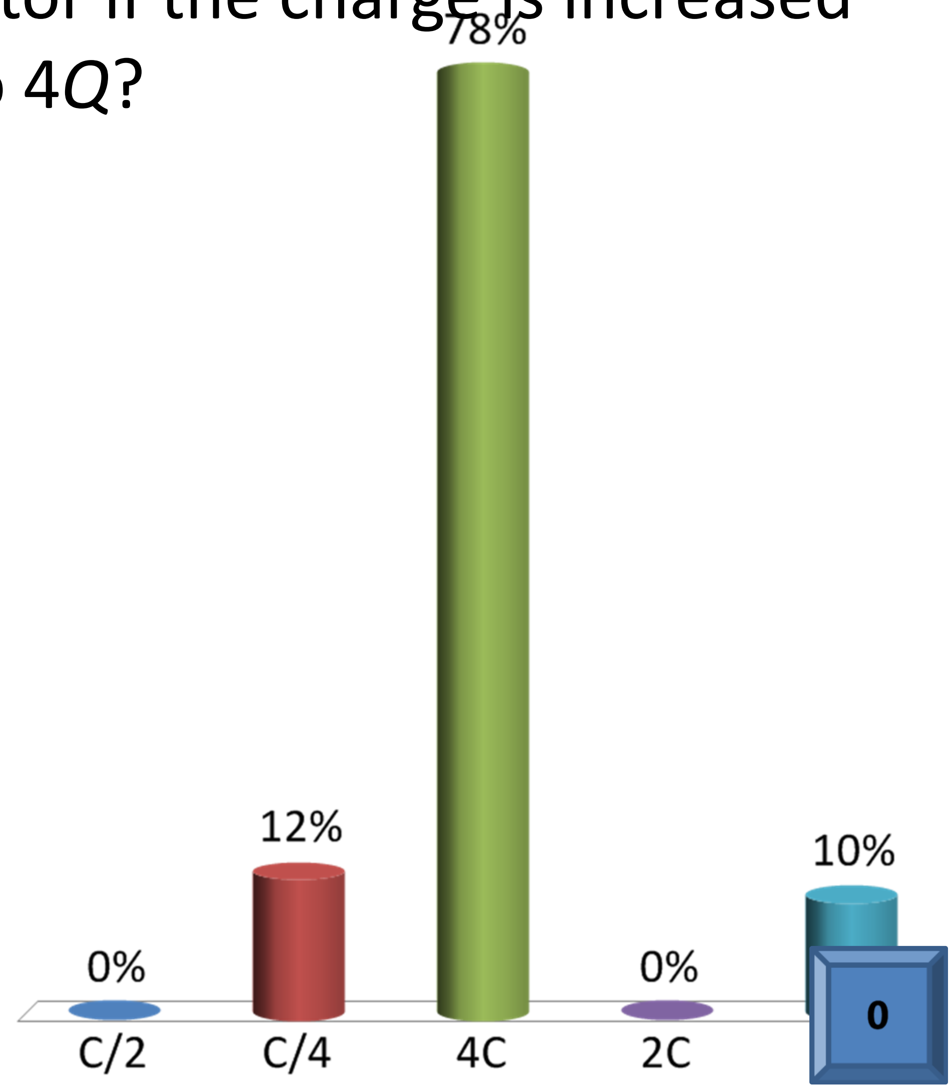
$$C = \frac{Q}{V}$$

$$C = \frac{\varepsilon_0 A}{d}$$



The plates of an isolated parallel plate capacitor with a capacitance *C* carry a charge *Q*. What is the capacitance of the capacitor if the charge is increased to 4*Q*?

1. C/2
2. C/4
3. 4C
4. 2C
5. C



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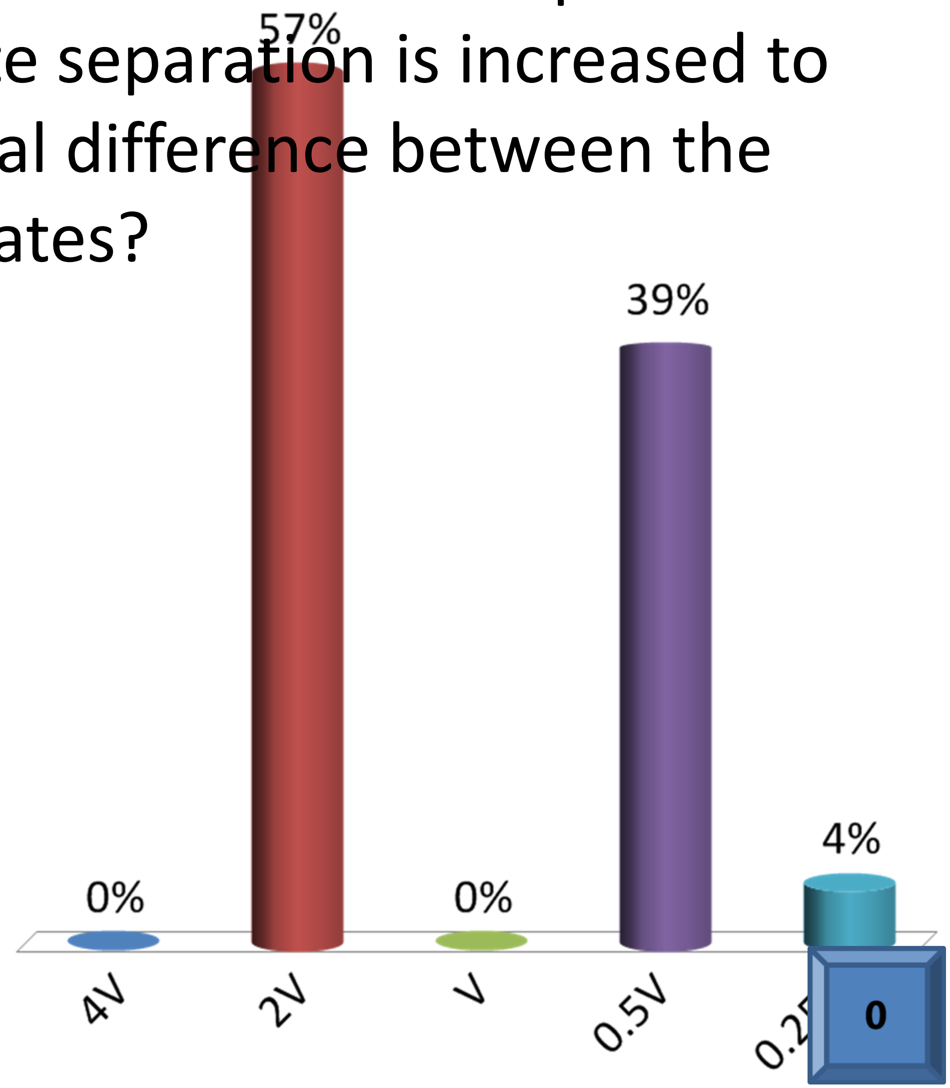
A parallel plate capacitor with plates of area *A* and plate separation *d* is charged so that the potential difference between its plates is *V*. If the capacitor is then isolated and its plate separation is increased to 2*d*, what is the potential difference between the plates?

1. 4V
2. 2V ✓
3. V
4. 0.5V
5. 0.25V

$C = \frac{Q}{V}$

$C = \frac{\varepsilon_0 A}{d}$

$\frac{\varepsilon_0 A}{d} = \frac{Q}{V}$



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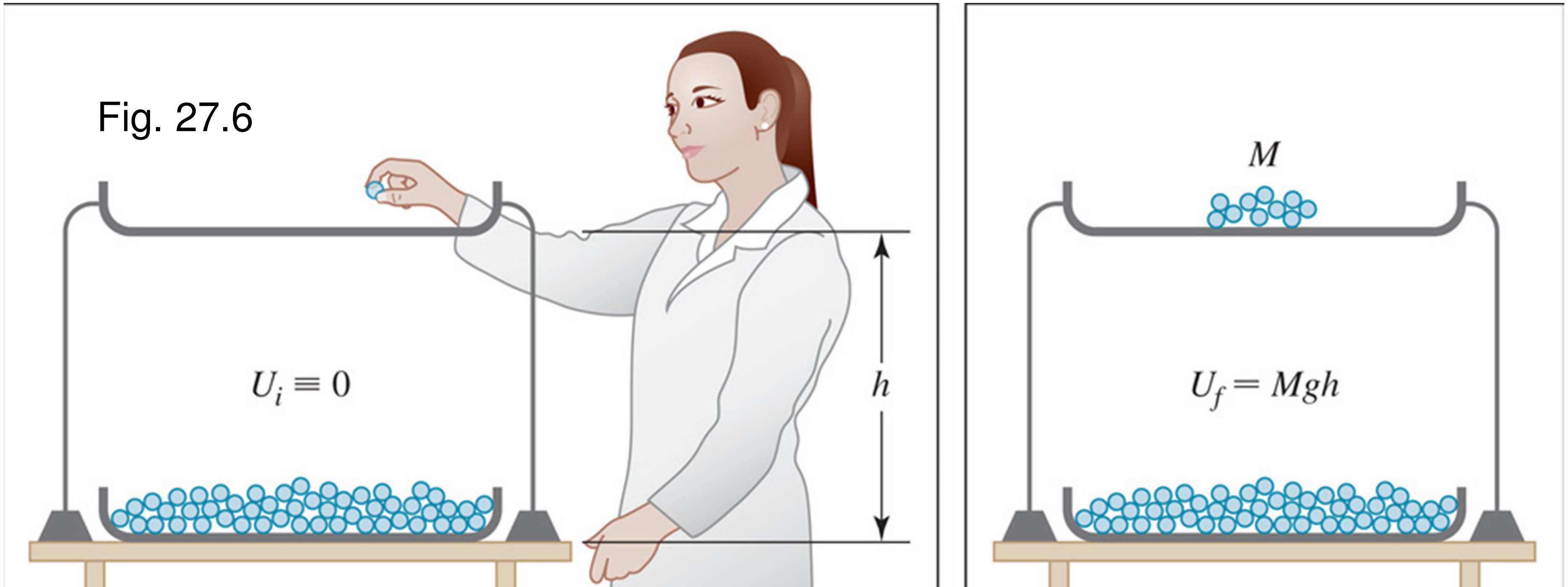
Energy Stored in a Capacitor

... and thus, Energy of an electric field itself.

Fig. 27.6



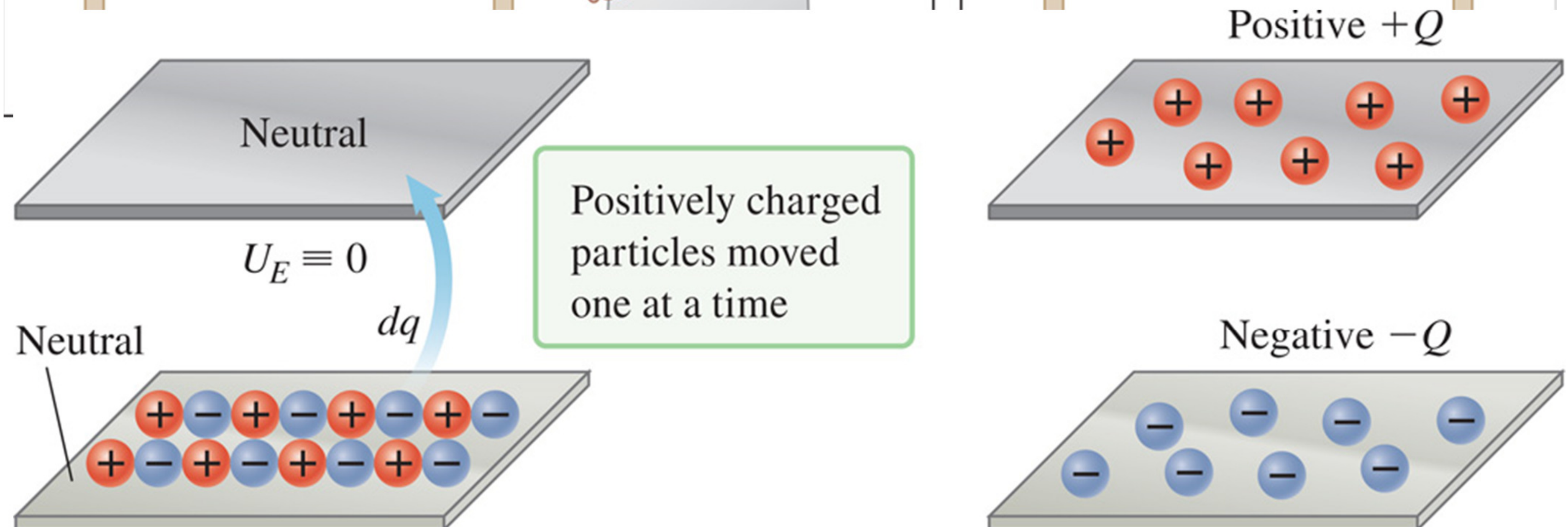
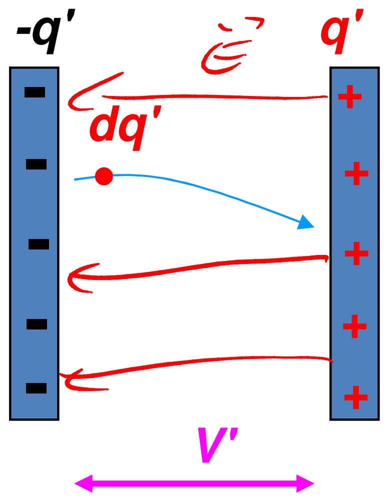
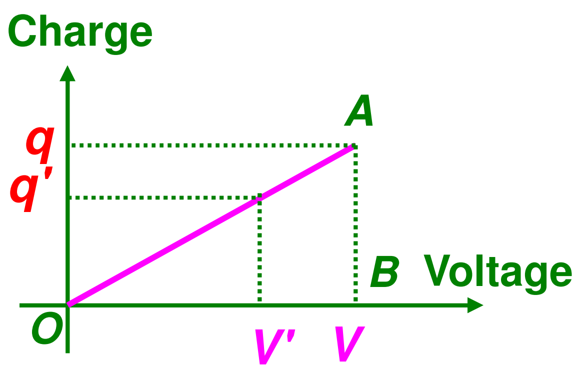


Fig. 27.7

A. Before

B. After





$dW = V' dq'$ $V' = \frac{q'}{C}$

$dW = \frac{q' dq'}{C}$

$\int dW = \int \frac{q' dq'}{C} = \frac{1}{C} \int_0^q q' dq'$

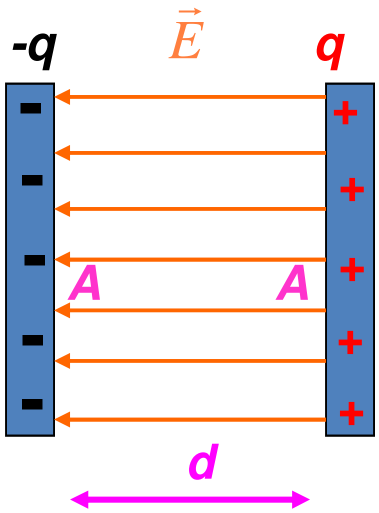
$W = \frac{1}{C} \Big|_0^q \frac{1}{2} q'^2$

$U_{cap} = \frac{q^2}{2C}$

$q = CV$

$U_{cap} = \frac{1}{2} C V^2$

→ energy in Cap.



$\vec{E}$ is Uniform

U/volume? $\quad J/m^3$

u = energy density = U/volume

$$u = \frac{U}{A \cdot d} = \frac{CV^2}{2Ad}$$

$$C = \frac{\varepsilon_0 A}{d}$$ (for a ∥-plate cap)

get $u = \frac{1}{2}\varepsilon_0 \left(V/d\right)^2$

$V/d = E$ if E was constant

$$u = \frac{1}{2}\varepsilon_0 E^2$$

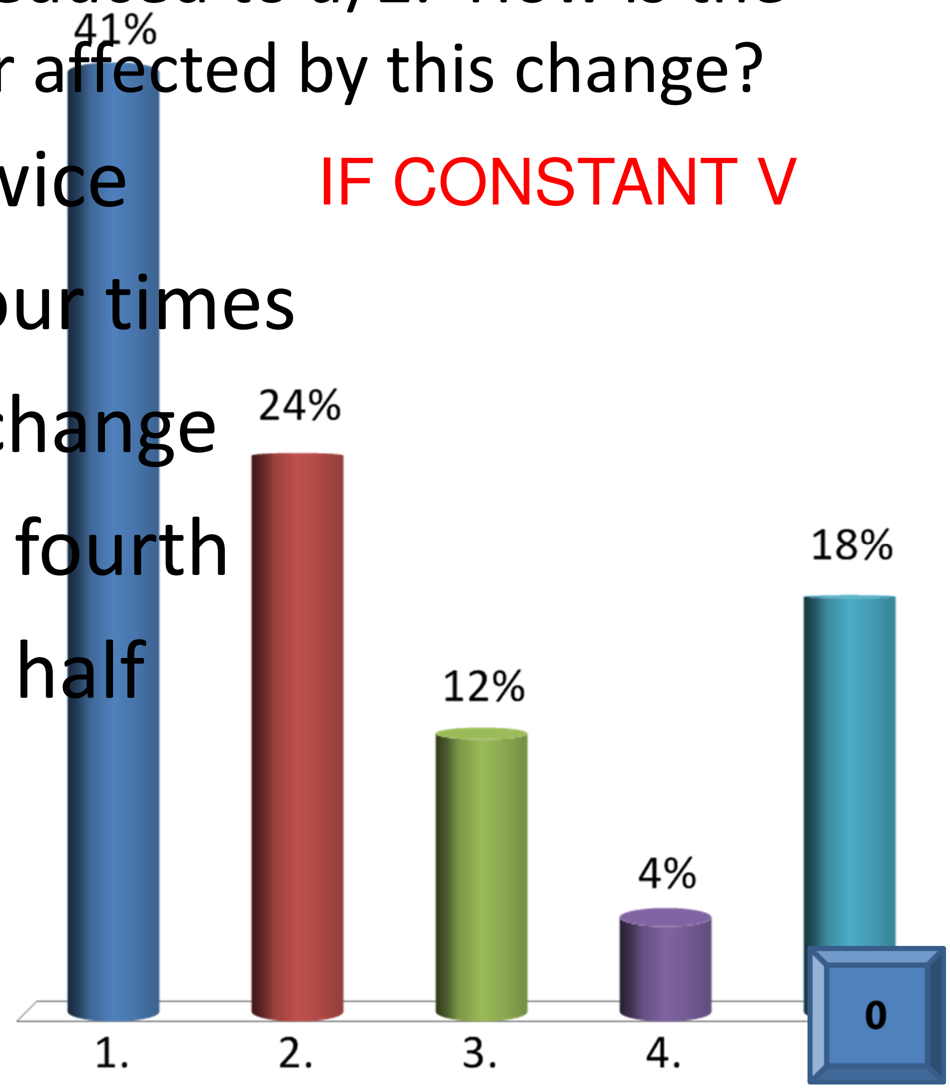
The plates of parallel plate capacitor are separated by a distance *d* and carry a charge of *q*. The distance between the plates is reduced to *d*/2. How is the energy in the capacitor affected by this change?

1. Energy increases by twice ✓
2. Energy increases by four times
3. The energy does not change
4. Energy decreases to a fourth
5. Energy decreases to a half

IF CONSTANT V

$U = \frac{1}{2} C V^2$

$C = \frac{\varepsilon_0 A}{d}$ $C \rightarrow 2C$



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The plates of parallel plate capacitor are separated by a distance *d* and carry a charge of *q*. The distance between the plates is reduced to *d*/2. How is the energy in the capacitor affected by this change?

1. Energy increases by twice
2. Energy increases by four times
3. The energy does not change
4. Energy decreases to a fourth
5. Energy decreases to a half ✓

IF CONSTANT Q

$U = \frac{Q^2}{2C}$

$C \Rightarrow 2\times$

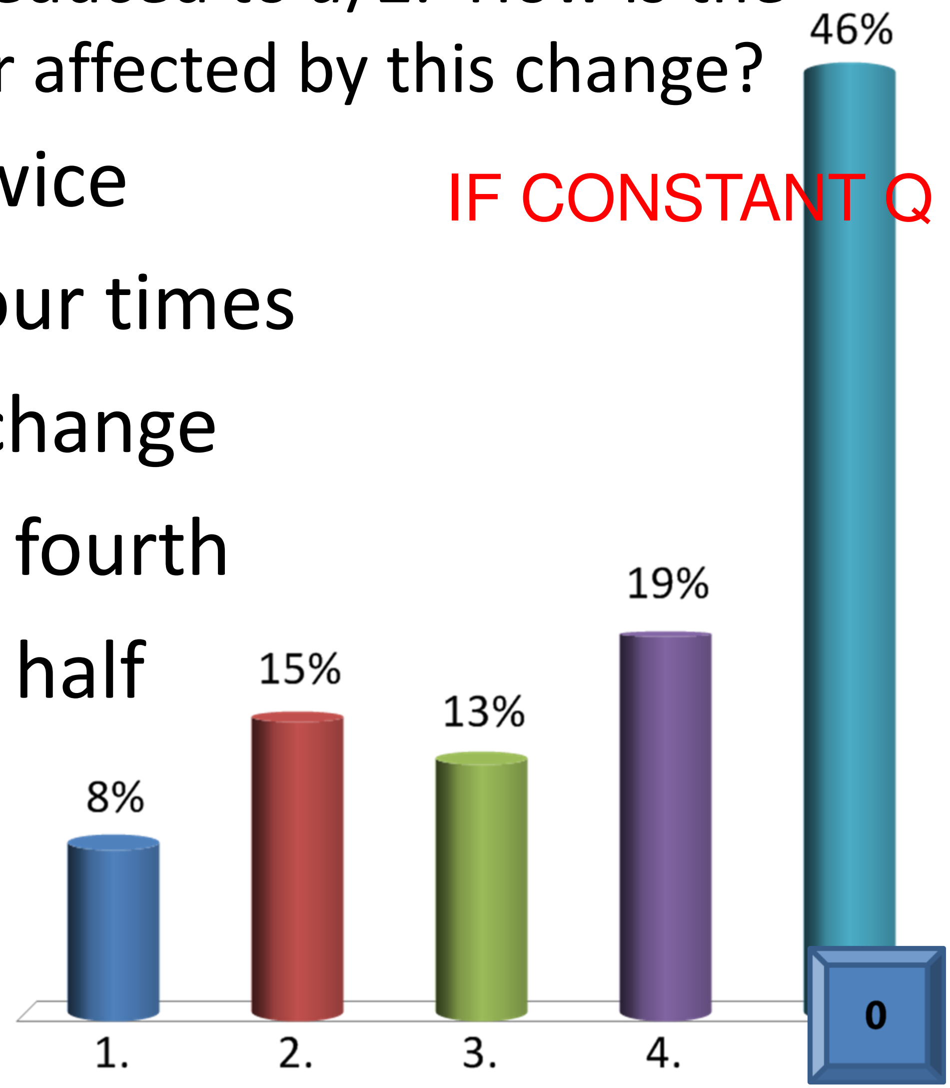

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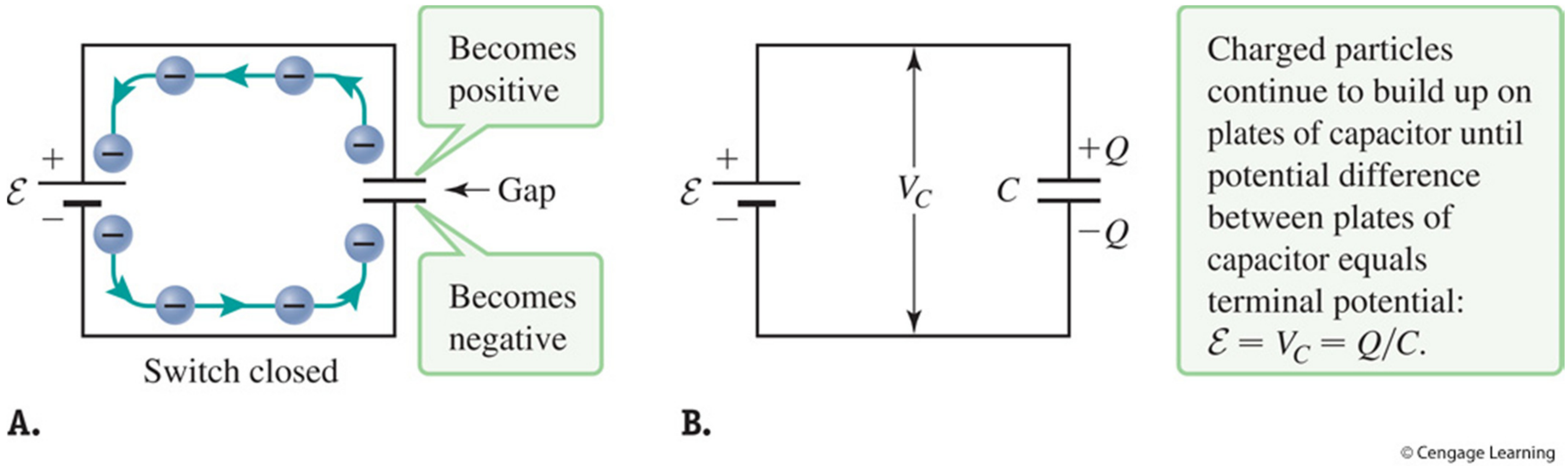
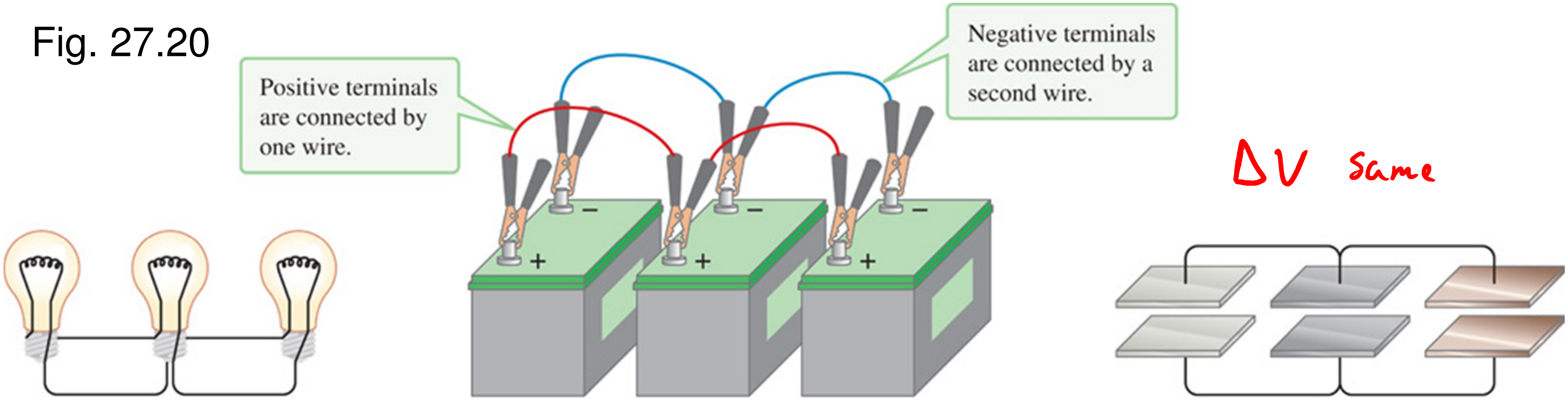
Fig. 27.14



Becomes positive
Gap
Becomes negative
Switch closed
A.
+Q
−Q
C
B.
Charged particles continue to build up on plates of capacitor until potential difference between plates of capacitor equals terminal potential: ℰ = V_C = Q/C.
© Cengage Learning

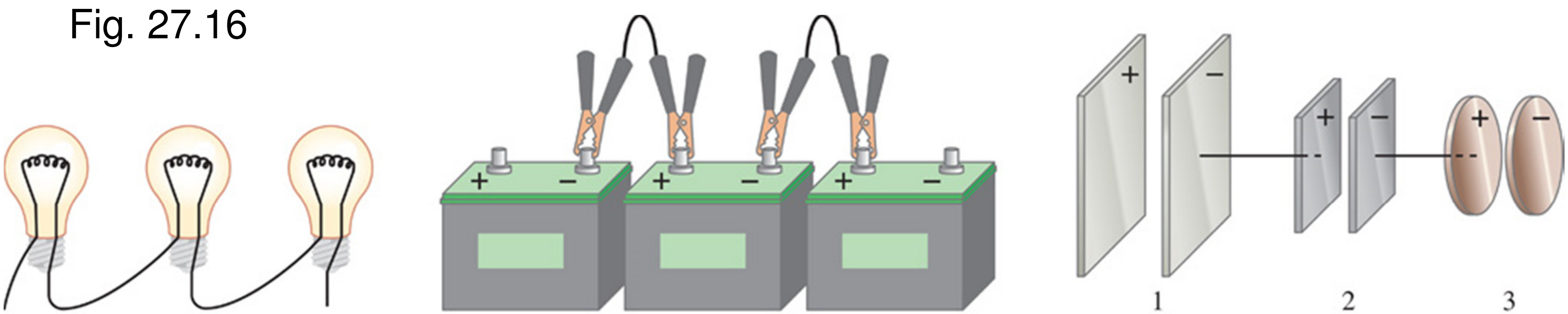
Fig. 27.20



A. Lightbulbs in parallel **B. Batteries in parallel** **C. Capacitors in parallel**

ΔV same

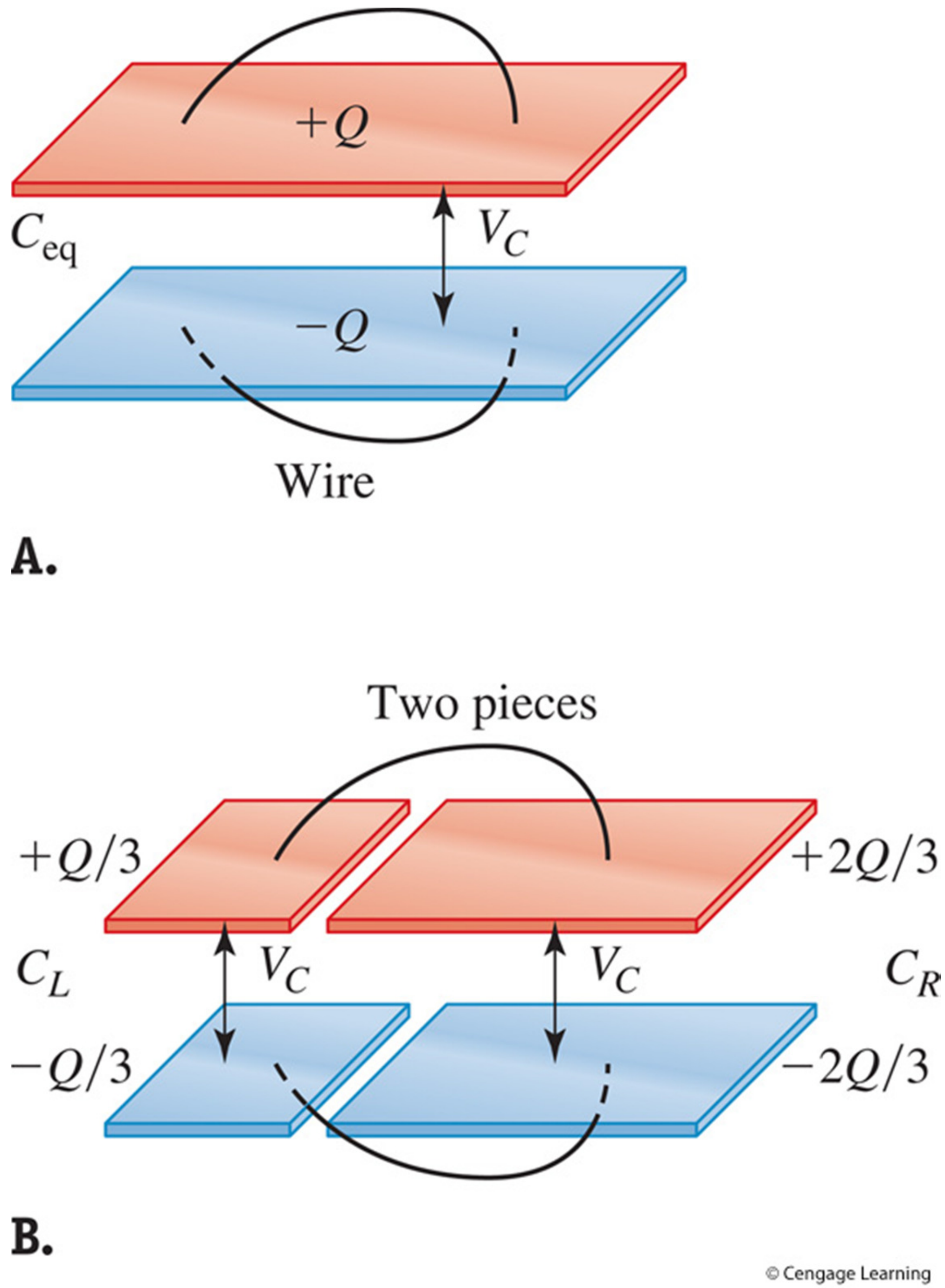
Fig. 27.16



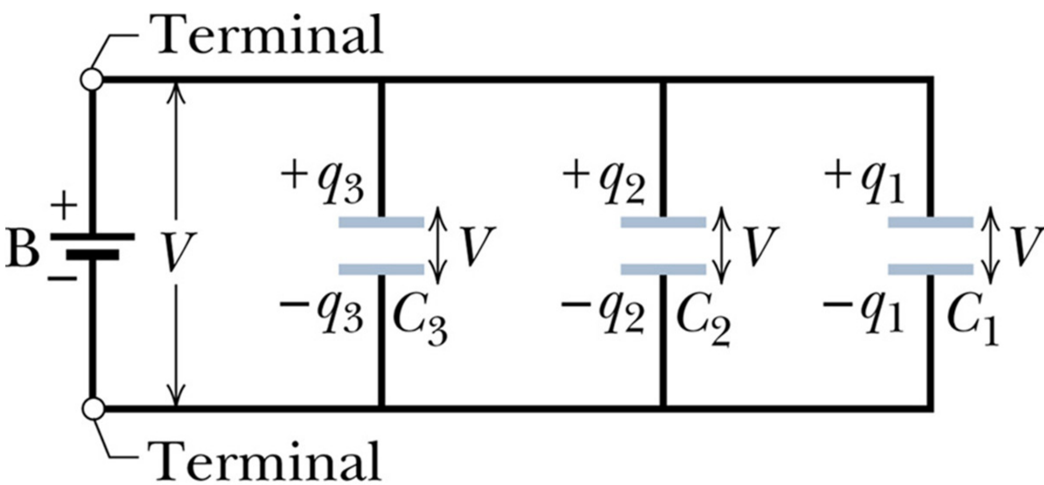
A. Lightbulbs in series **B. Batteries in series** **C. Capacitors in series**

Q same

Fig. 27.21


+Q
C_{eq}
V_C
−Q
Wire
A.
Two pieces
+Q/3
+2Q/3
C_L
V_C
V_C
C_R
−Q/3
−2Q/3
B.
© Cengage Learning

What’s the total C of this parallel set of caps?



$q_1 = C_1 V \quad q_2 = C_2 V$

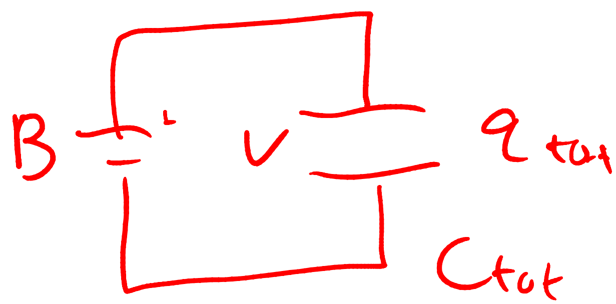
$q_3 = C_3 V$

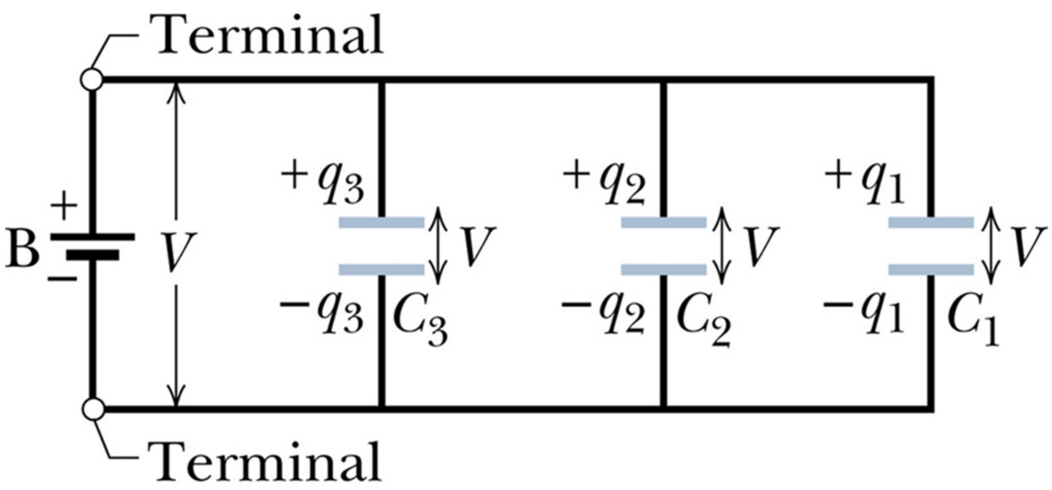
$q_{tot} = q_1 + q_2 + q_3$

$q_{tot} = C_1 V + C_2 V + C_3 V$

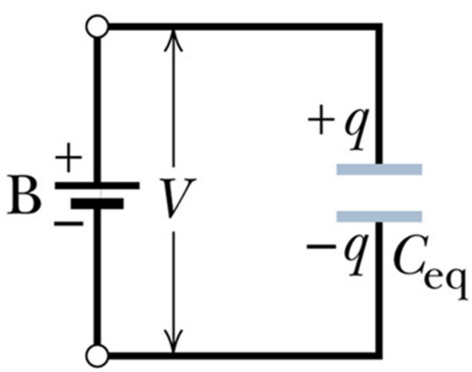
$q_{tot} = V(C_1 + C_2 + C_3)$

C_{tot}





(a)

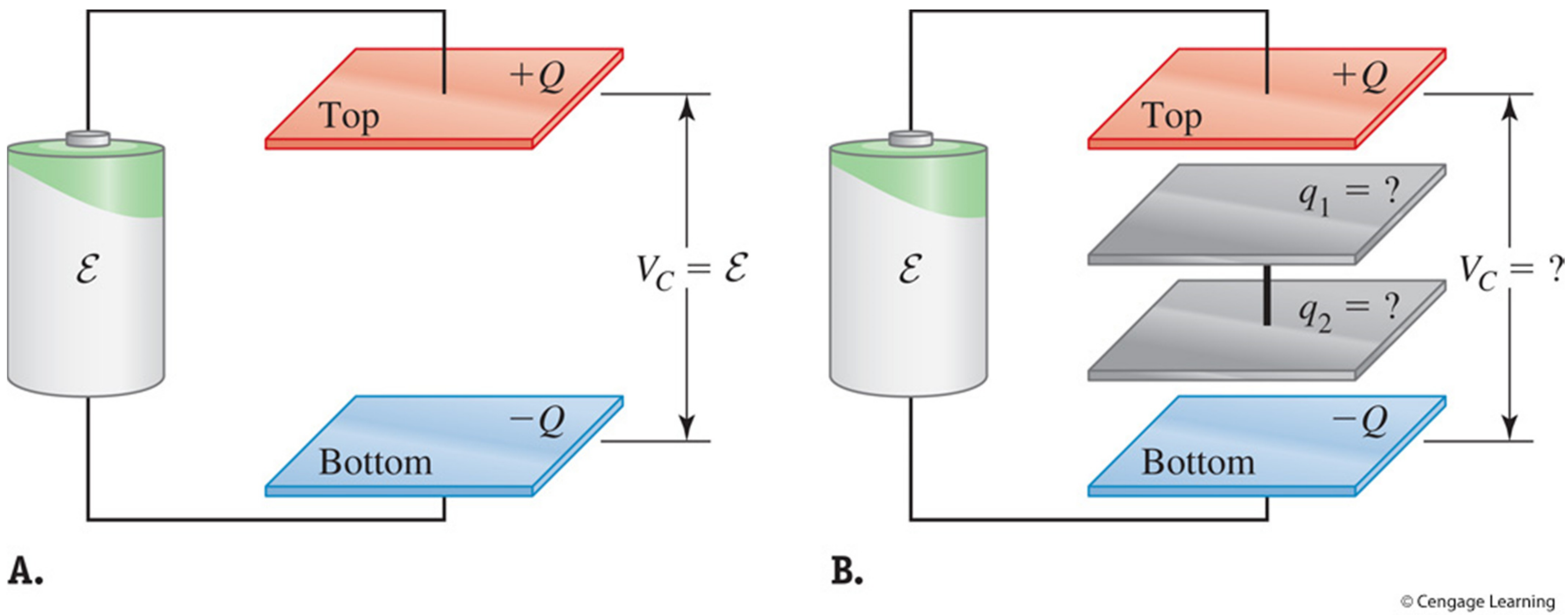


(b)

$= C_{TOT} = C_1 + C_2 + C_3$

$C_{TOT} = \sum C_n$

Fig. 27.15



Top
+Q
$\mathcal{E}$
$V_C = \mathcal{E}$
Bottom
−Q
A.
Top
+Q
$q_1 = ?$
$\mathcal{E}$
$V_C = ?$
$q_2 = ?$
Bottom
−Q
B.
© Cengage Learning

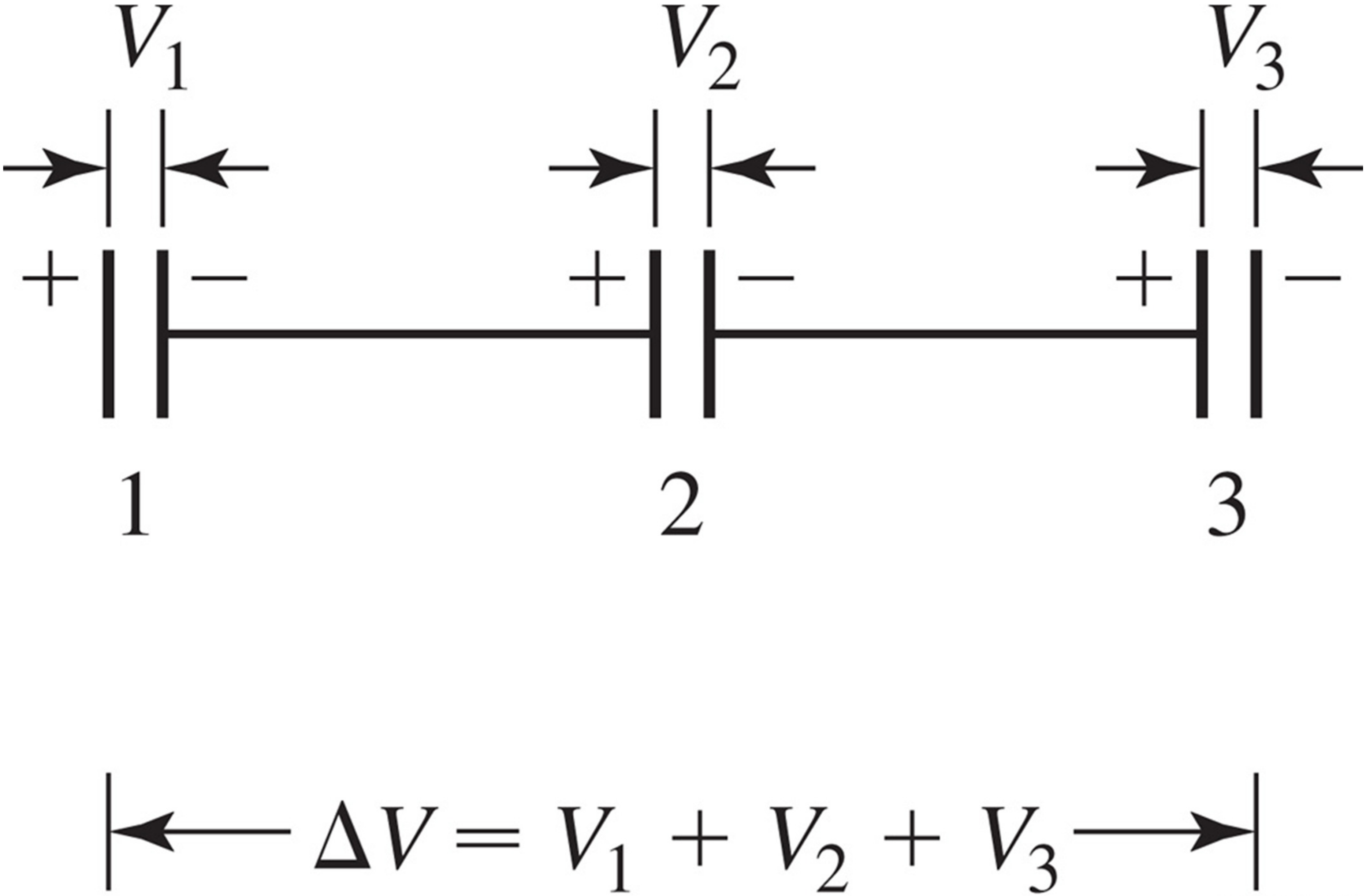
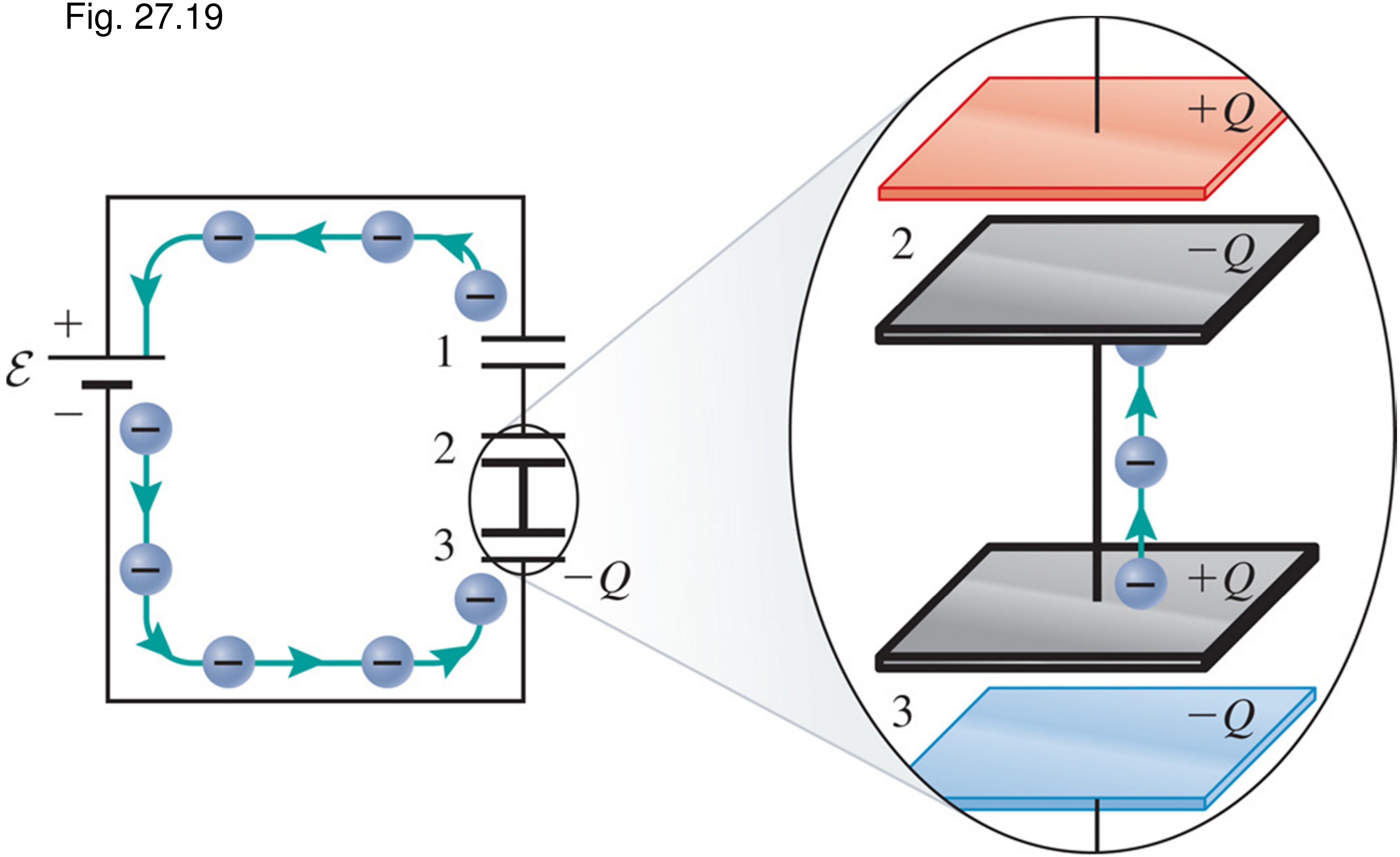


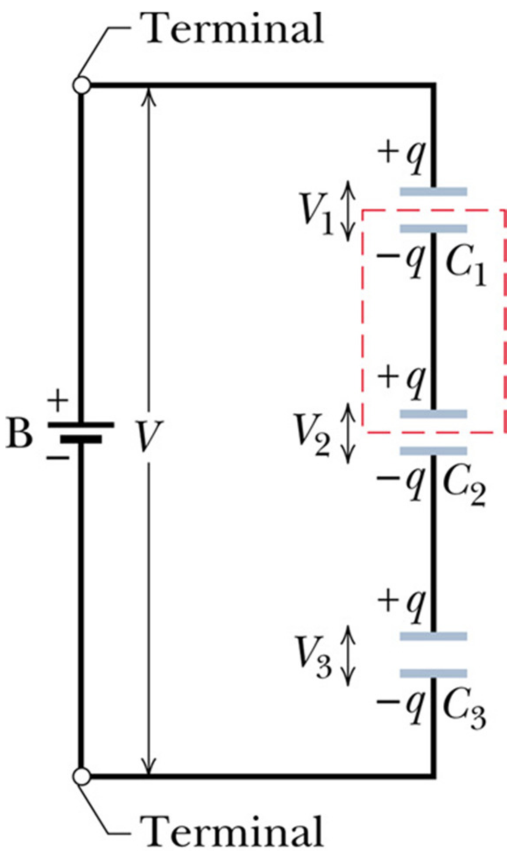
Fig. 27.17

Fig. 27.19

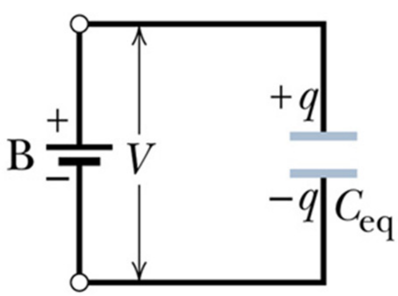


What's the total C of this series set of caps?

$$V_1 = \frac{q}{C_1} \quad V_2 = \frac{q}{C_2} \quad V_3 = \frac{q}{C_3}$$



(a)



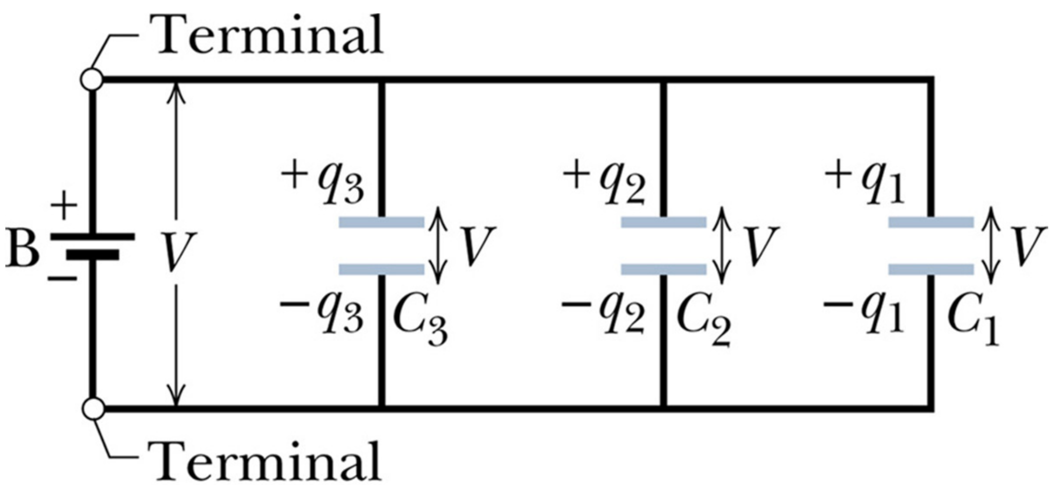
(b)

$$V = V_1 + V_2 + V_3$$

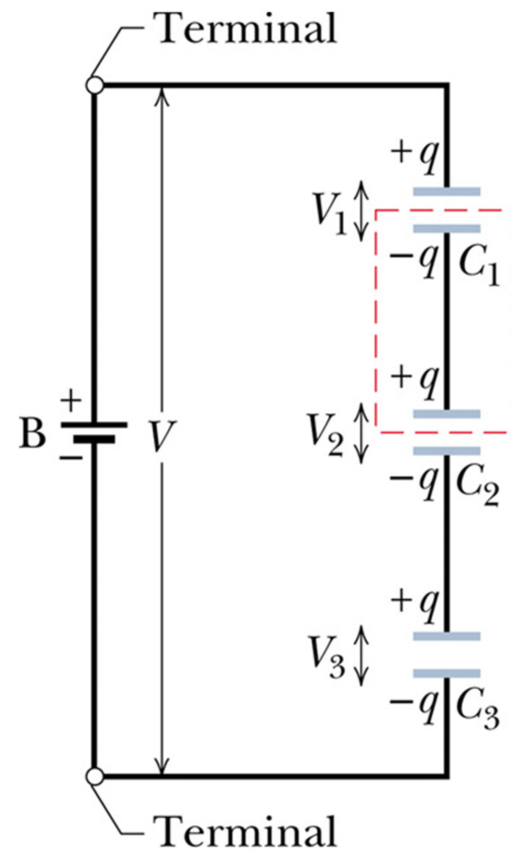
$$V = \left(\frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}\right)$$

$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

Or: $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$



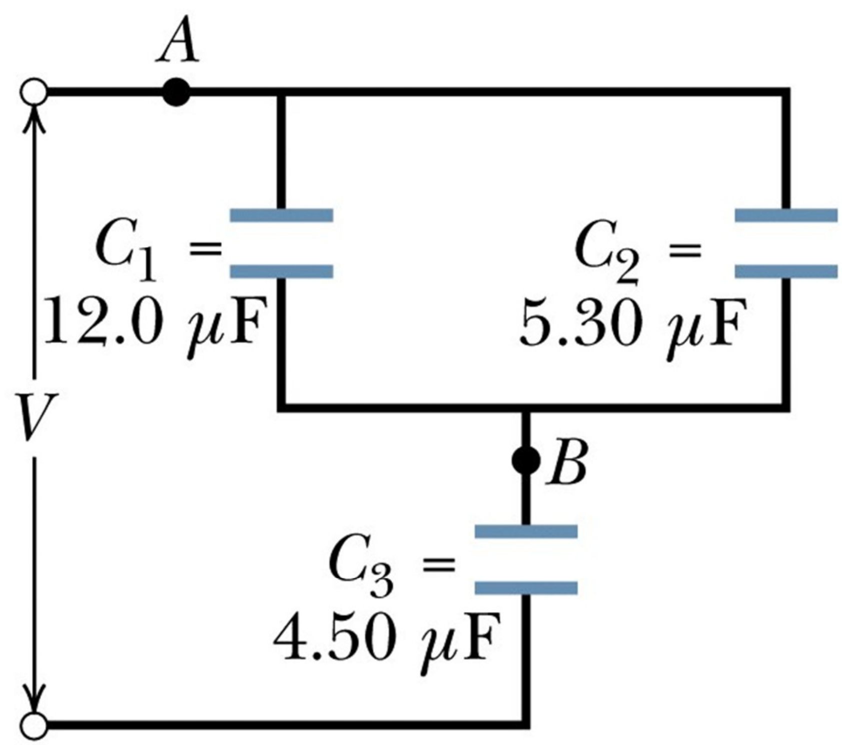
parallel connection



Series Connection

What are charges and voltages everywhere if We hook this up to A 12.5V car battery?

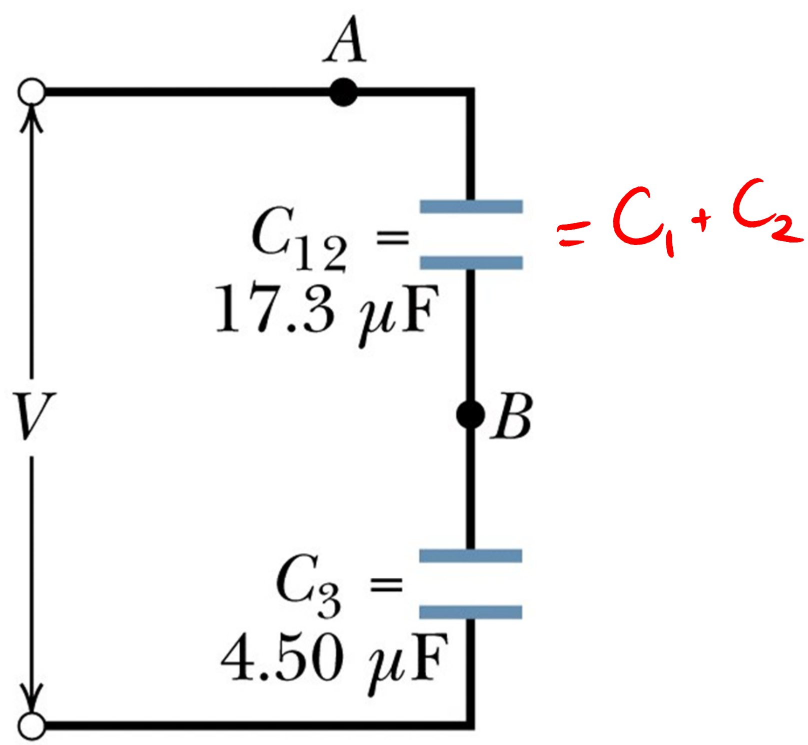
We first reduce the circuit to a single capacitor.



(a)

halliday_9e_fig_25_10a

The equivalent of parallel capacitors is larger.

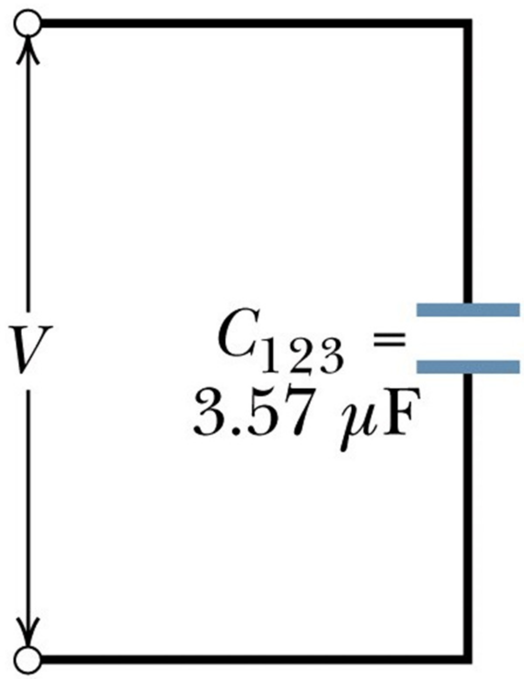


(b)

halliday_9e_fig_25_10b

The equivalent of series capacitors is smaller.

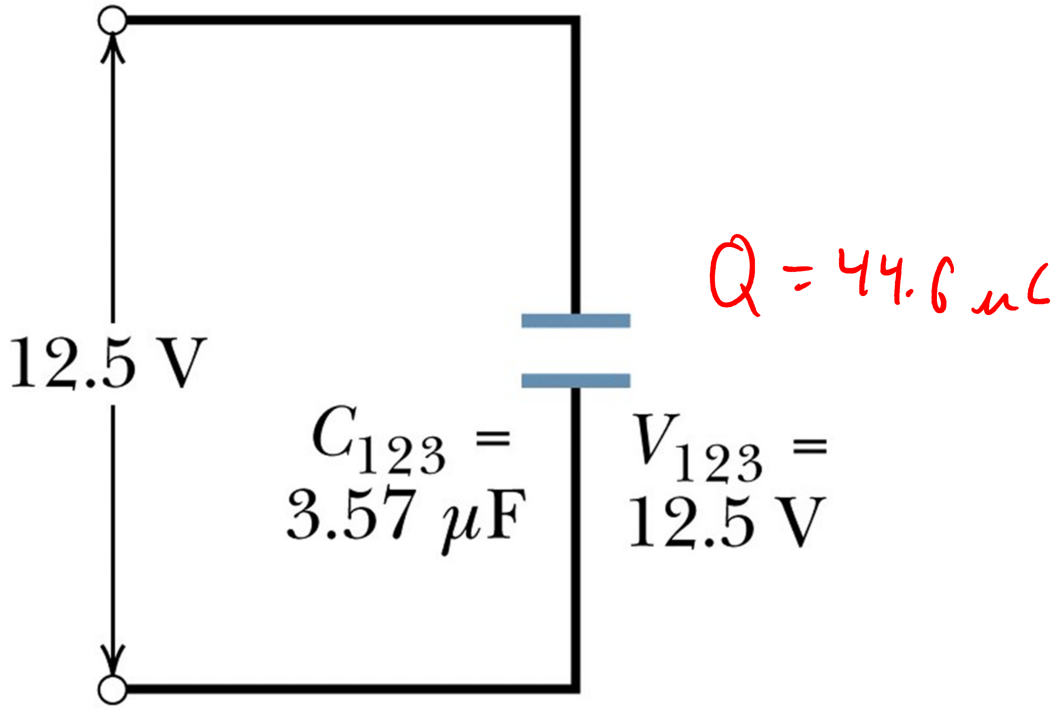
$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$



(c)

halliday_9e_fig_25_10c

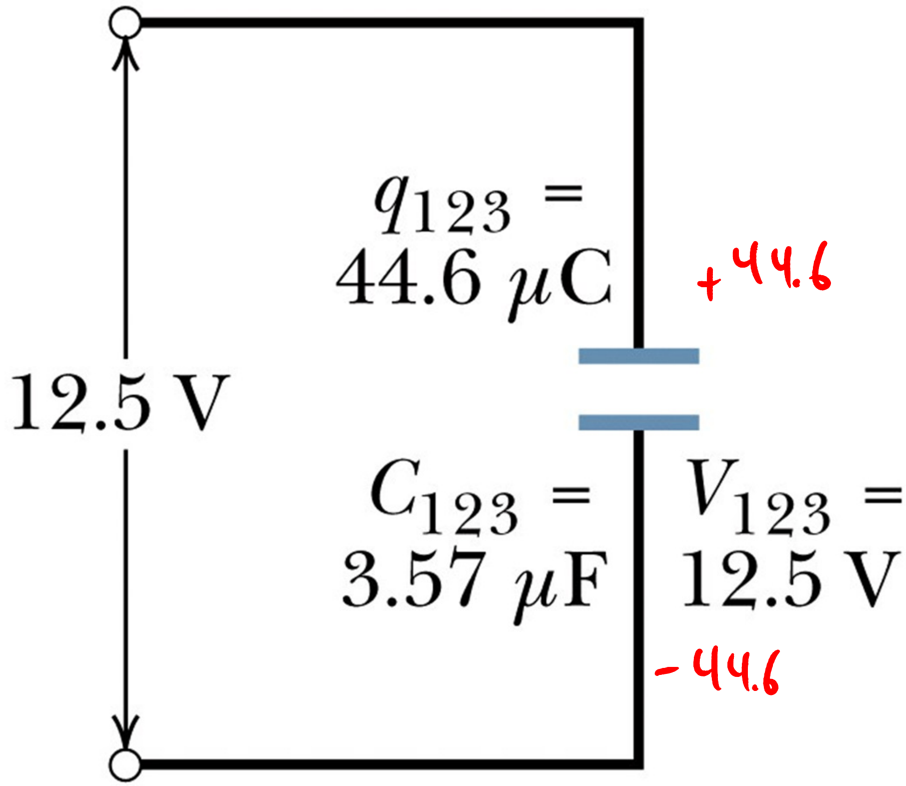
Next, we work backwards to the desired capacitor.



(d)

halliday_9e_fig_25_10d

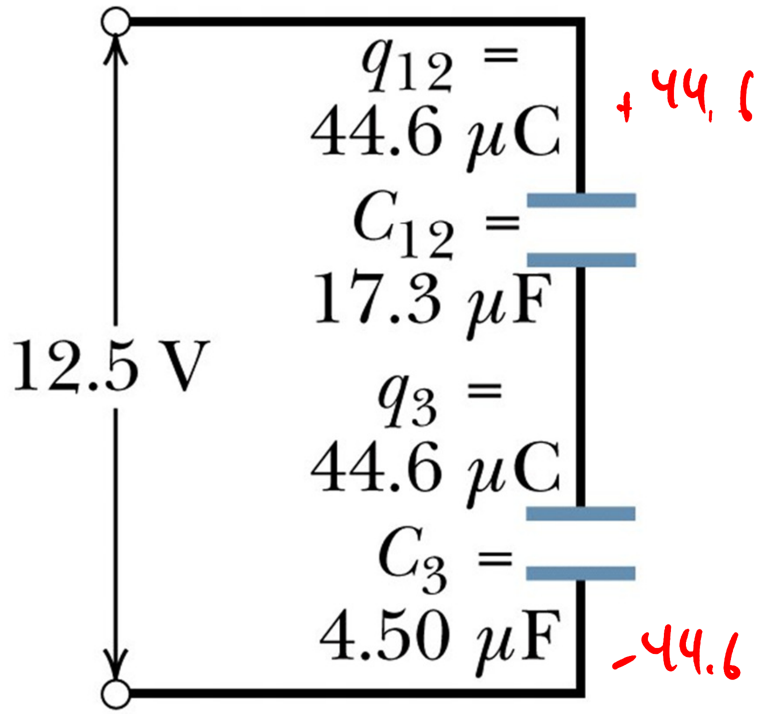
Applying $q = CV$ yields the charge.



(e)

halliday_9e_fig_25_10e

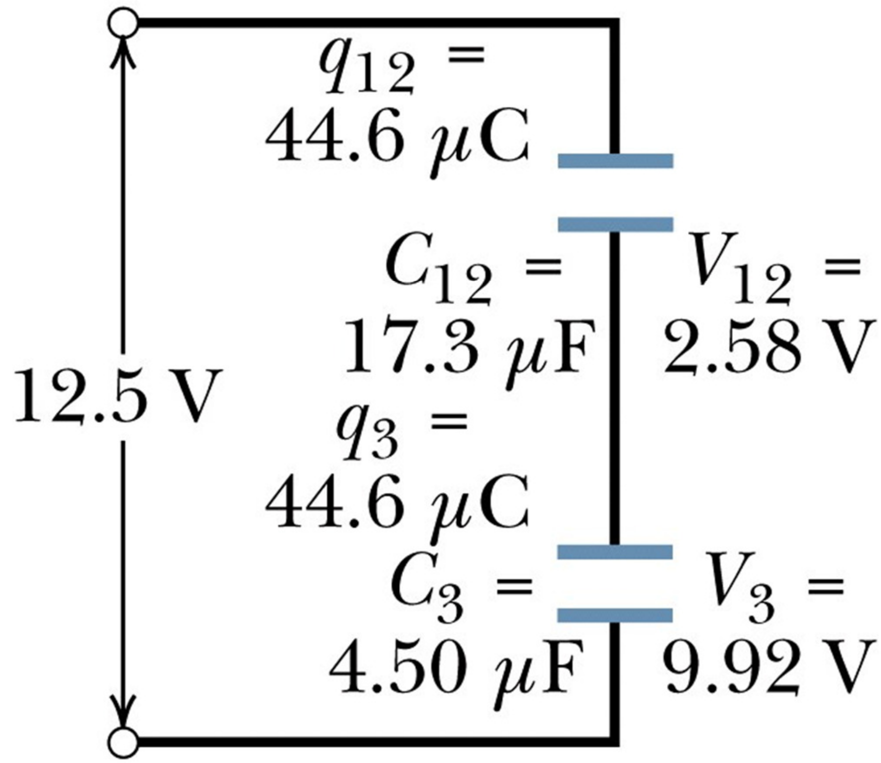
Series capacitors and their equivalent have the same *q* ("seri-q").



(*f*)

halliday_9e_fig_25_10f

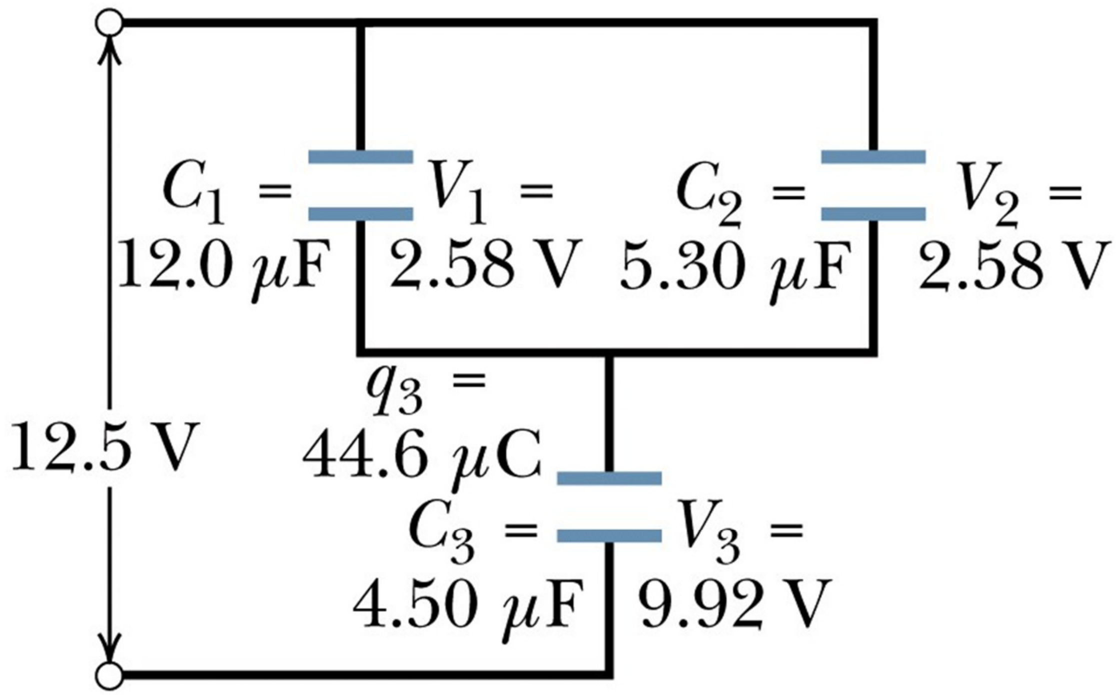
Applying $V = q/C$ yields the potential difference.



(g)

halliday_9e_fig_25_10g

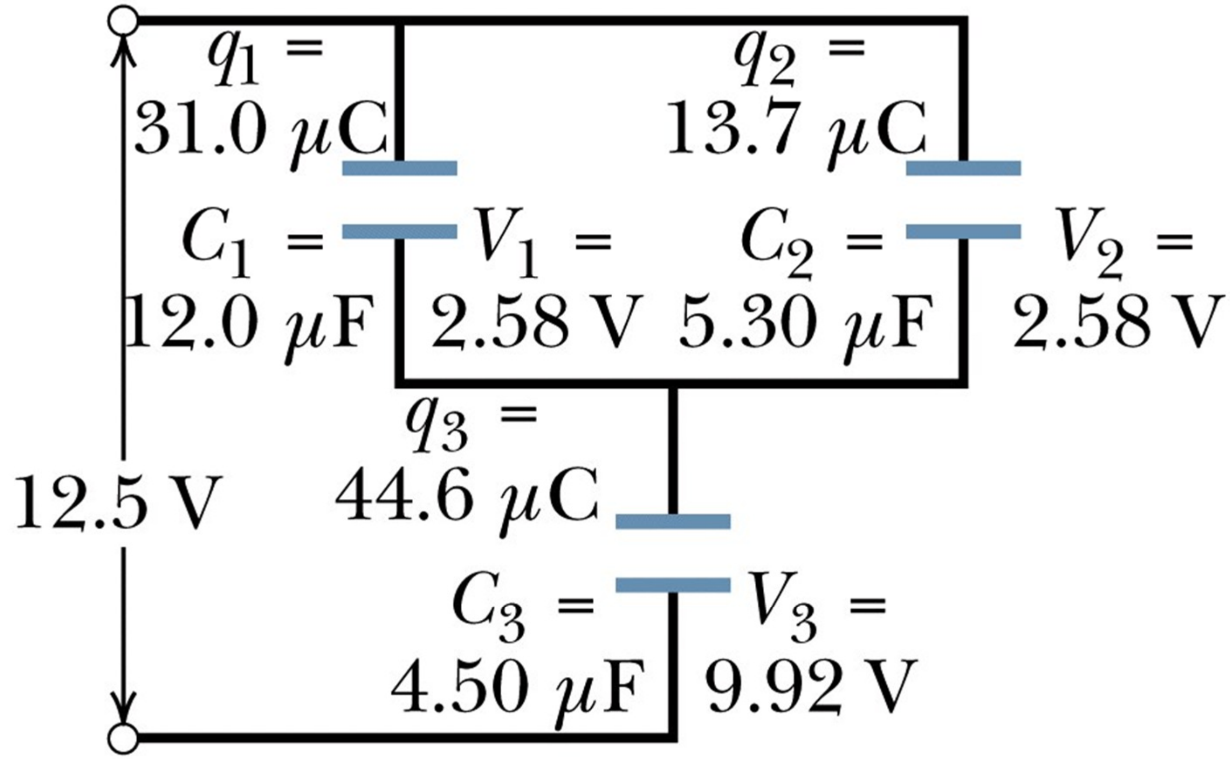
Parallel capacitors and their equivalent have the same *V* (“par-V”).



(*h*)

 halliday_9e_fig_25_10h

Applying $q = CV$ yields the charge.



(*i*)

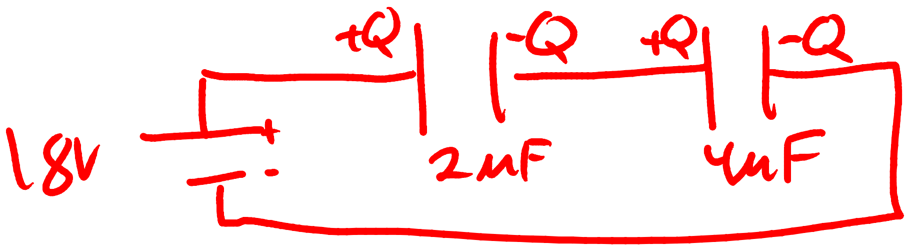
halliday_9e_fig_25_10i

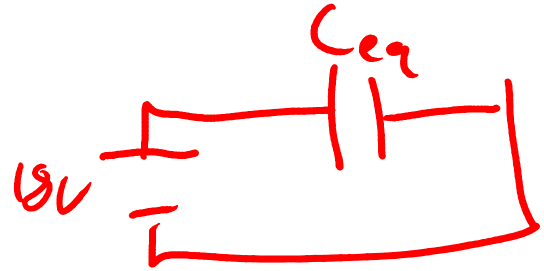
You work out this example:

A 2 μF and a 4 μF capacitor are connected in series with an 18V battery.

What are Q and V across each capacitor?

a 2μF and a 4μF cap connected in series w/ an 18 V battery. What are Q + V on each?





in series. $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2\mu F} + \frac{1}{4\mu F}$

$C_{eq} = 4/3\ \mu F$ $V = 18V$ $Q = CV = (4/3\,\mu F)(18) = 24\mu C$

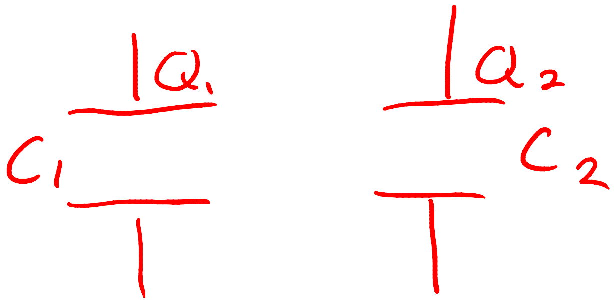
$Q_1 = C_1 V$ $Q_2 = C_2 V$

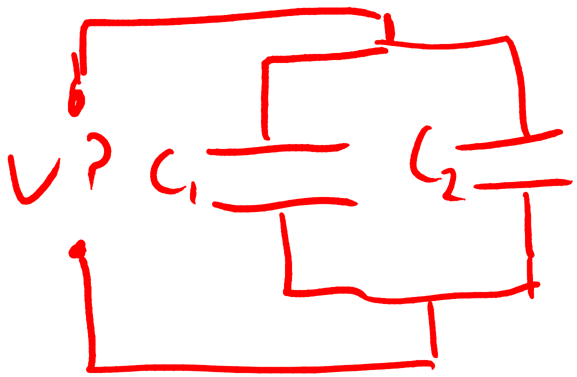
$V_1 = Q/C_1 = \frac{24\mu C}{2\mu F} = 12V$ $V_2 = \frac{Q}{C_2} = \frac{24\mu C}{4\mu F} = 6V$

$12V + 6V = 18V$ ✓

Now take those same capacitors, unplug them from the battery carefully so as not to lose their charge, and connect them together in parallel instead.

What are Q and V across each capacitor?



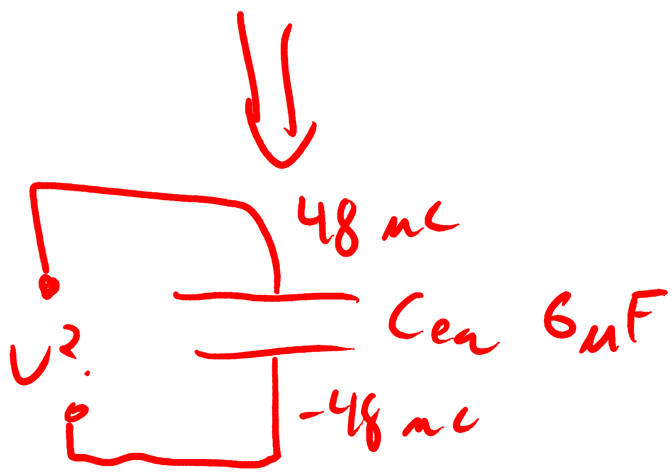


24 μC on each before unplugged

so 48 μC total

C's in || $C_{eq} = C_1 + C_2 = 6\mu F$

$V = \frac{Q}{C_{eq}} = \frac{48\mu C}{6\mu F} = 8V$



$Q_1 = C_1 V$ $Q_2 = C_2 V$

$= 2\mu F \cdot 8V$ $= 4\mu F\, 8V$

$Q_1 = 16\mu C$ $Q_2 = 32\mu C$

$Q_{TOT} = 48\mu C = 16 + 32$

Do Equivalent Capacitance Handout