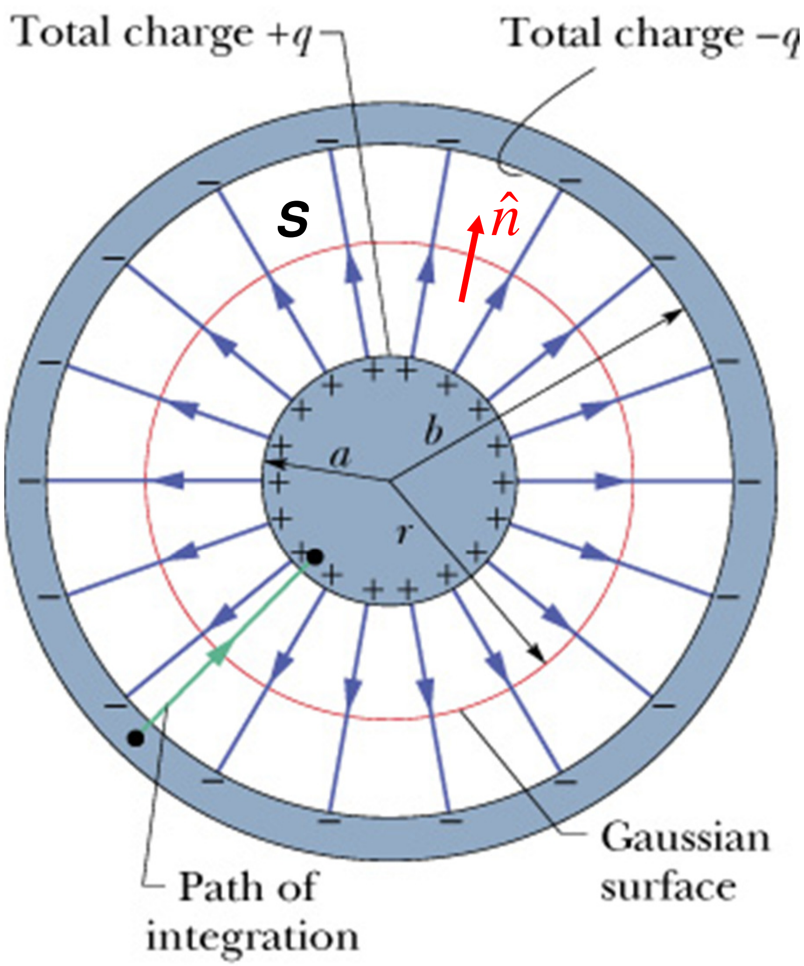
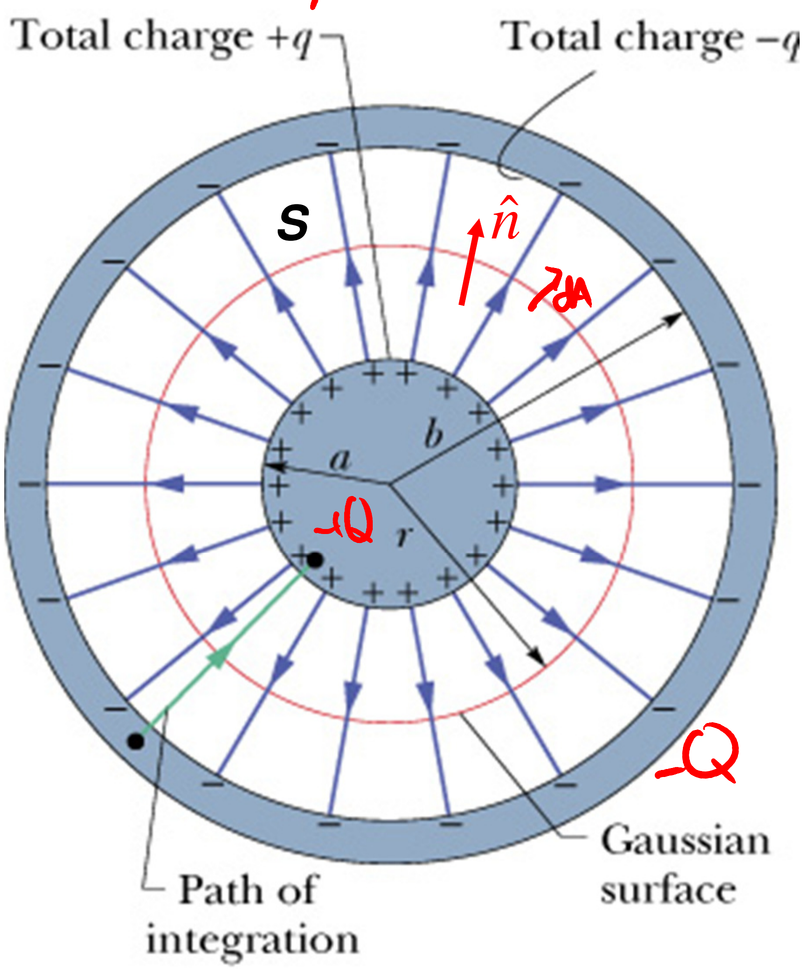
Now you try it: find C per length of that coax cable.
Use this end-on view.
Find E from the Q, then you can integrate to find V.



Cylindrical and Spherical capacitor examples skipped in class, but here in the notes for reference

Red line is slice of a cylinder, length L (into page), radius r.



Top and bottom lids are ∥ to $\vec{E}$, so Φ_{top}, $\Phi_{bot} = 0$.

Side: ⊥ to $\vec{E}$ everywhere, so

$$\int \vec{E}\cdot d\vec{A} = \int |E||dA|\cos\cancel{\theta}\,0$$

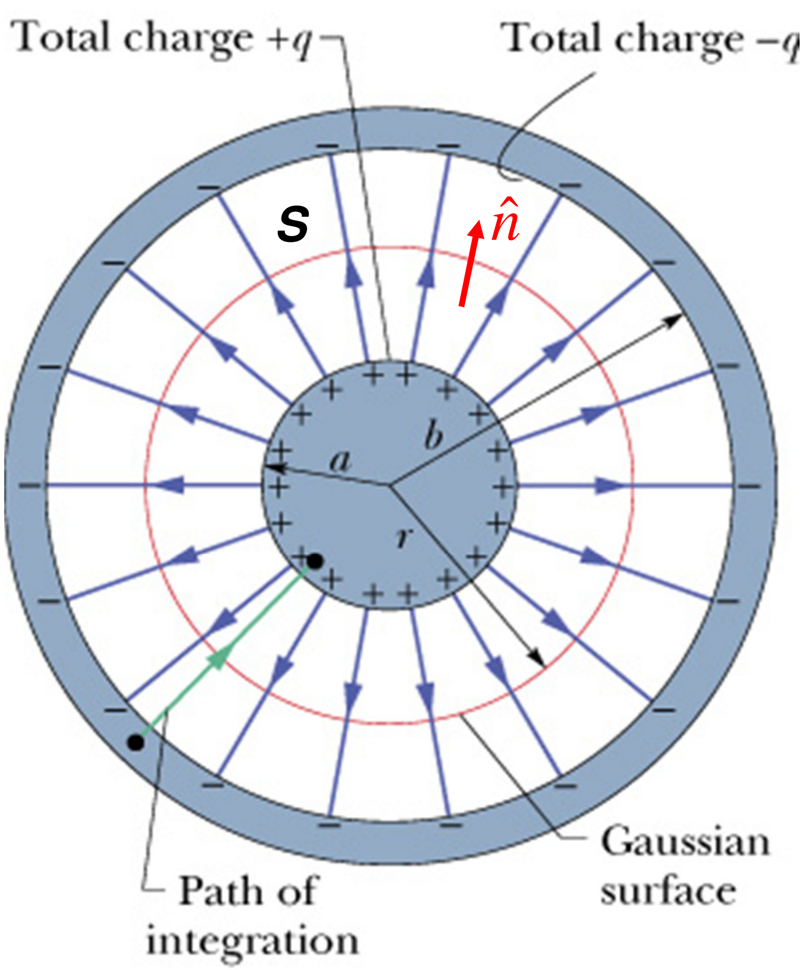
Each dA same distance from all charge, so E const on that surface

$$\Phi_{net} = E\int dA = EA = E2\pi rL = \frac{q_{enc}}{\varepsilon_0}$$

So $E = \dfrac{Q}{2\pi\varepsilon_0 rL}$

So, $E = \frac{Q}{2\pi\varepsilon_0 r L}$

find $V = -\int_{-}^{+} \vec{E}\cdot d\vec{s}$



$\vec{E}\cdot d\vec{s} = |E|\,|ds|\cos\theta$

opposite ~~Same~~ direction, $\cos\theta = -1$, $ds = dr$

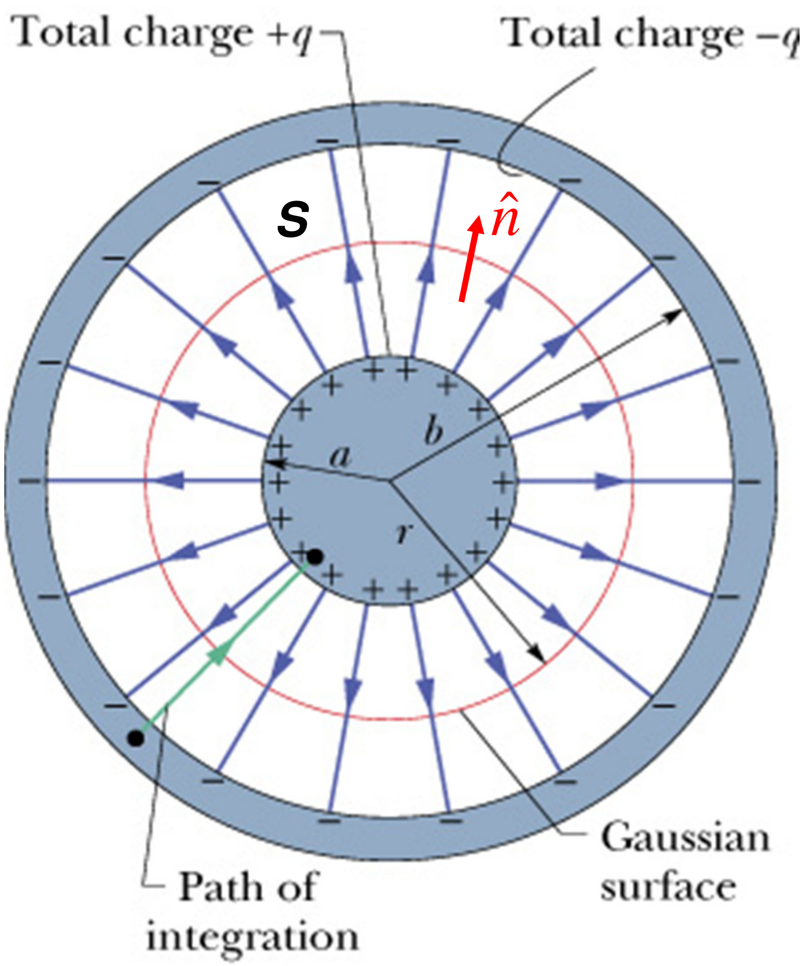
So $V = -\int_{b}^{a} E\,dr(-1)$

$= -\int_{a}^{b} \frac{Q}{2\pi\varepsilon_0 L r} = \frac{-Q}{2\pi\varepsilon_0 L}\int_{a}^{b} \frac{dr}{r}$

$V = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$

So, we know V

we know Q



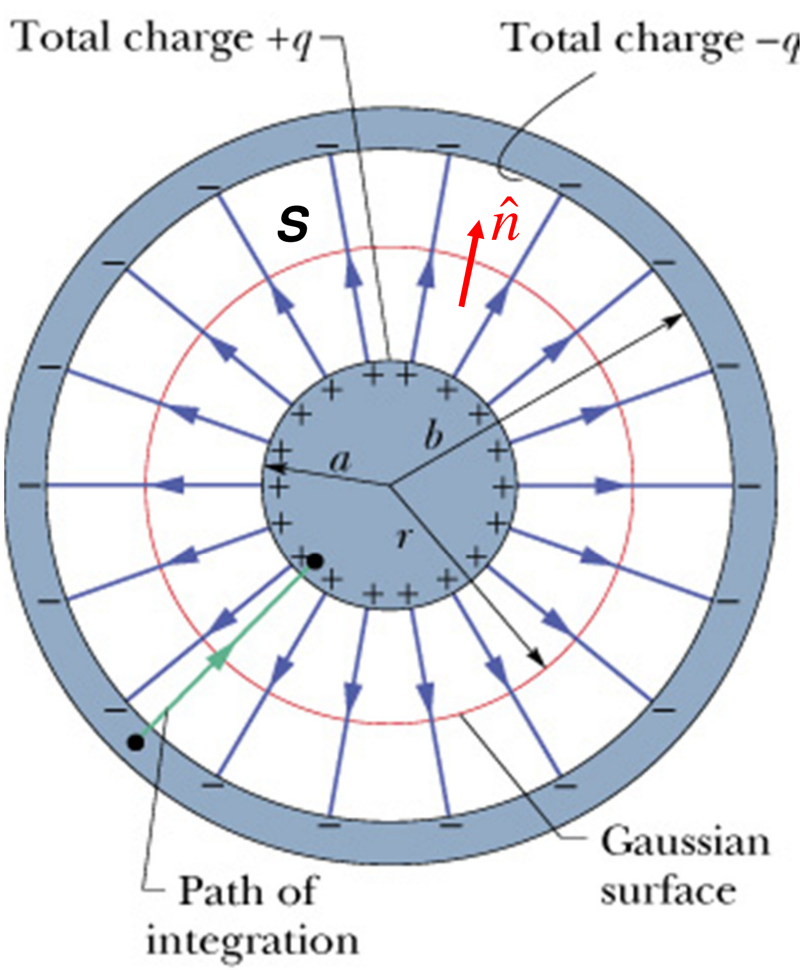
$$C = \frac{Q}{V}$$

$$V = \frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{Q/2\pi\varepsilon_0 L \ln(b/a)}$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

How bout a spherical shape?
Cross-section is the same picture, but this is now a sphere not a cable.



Spheres

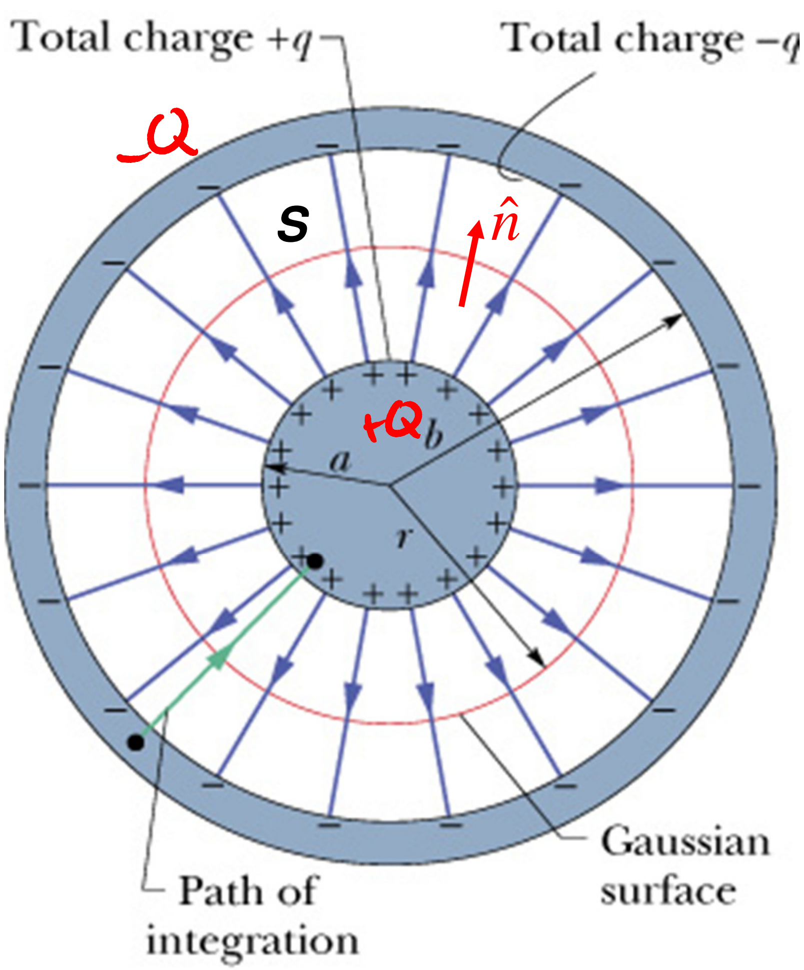
$C = Q/V$

G.S. red line sphere around ball

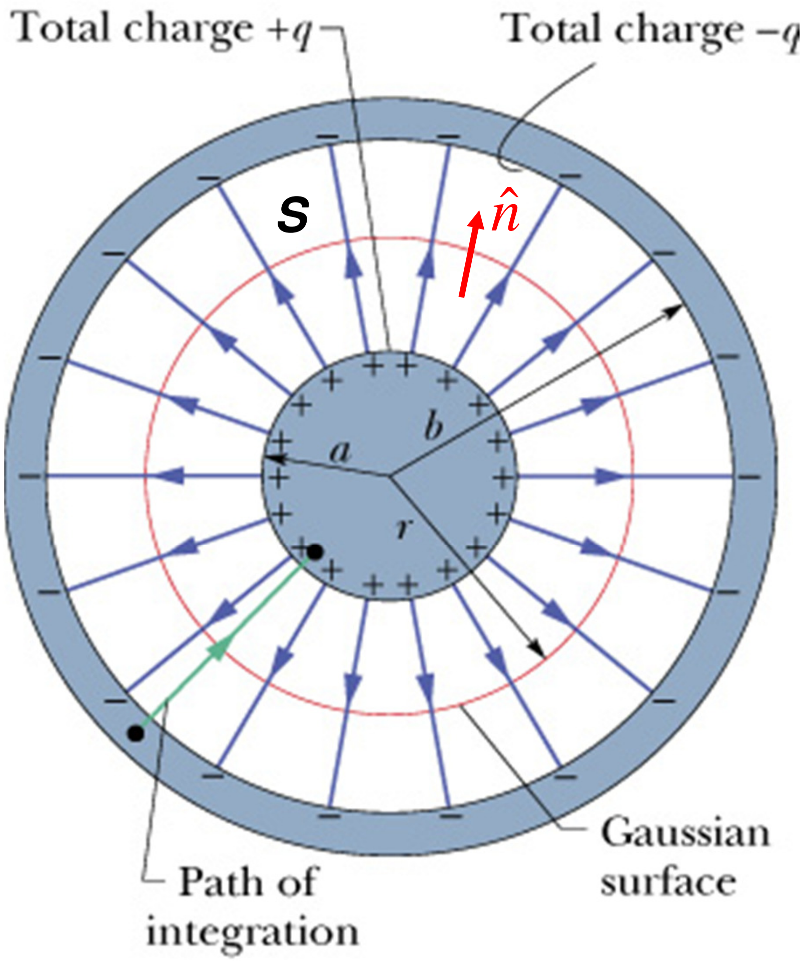
$\Phi = \int \vec{E}\cdot d\vec{A} = \cancel{q_{enc}}/\varepsilon_0$

$= \int |\vec{E}||d\vec{A}| = E\int dA = EA$

$E(4\pi r^2) = \cancel{q_{enc}}/\varepsilon_0 \Rightarrow E = \dfrac{+Q}{4\pi\varepsilon_0 r^2}$



$$V = \int_{-}^{+} \vec{E}\cdot d\vec{s} \qquad ds = dr \qquad dr\downarrow \; E\uparrow \qquad \cos\theta = -1$$



$$V = -\int_b^a \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= \frac{-Q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2}$$

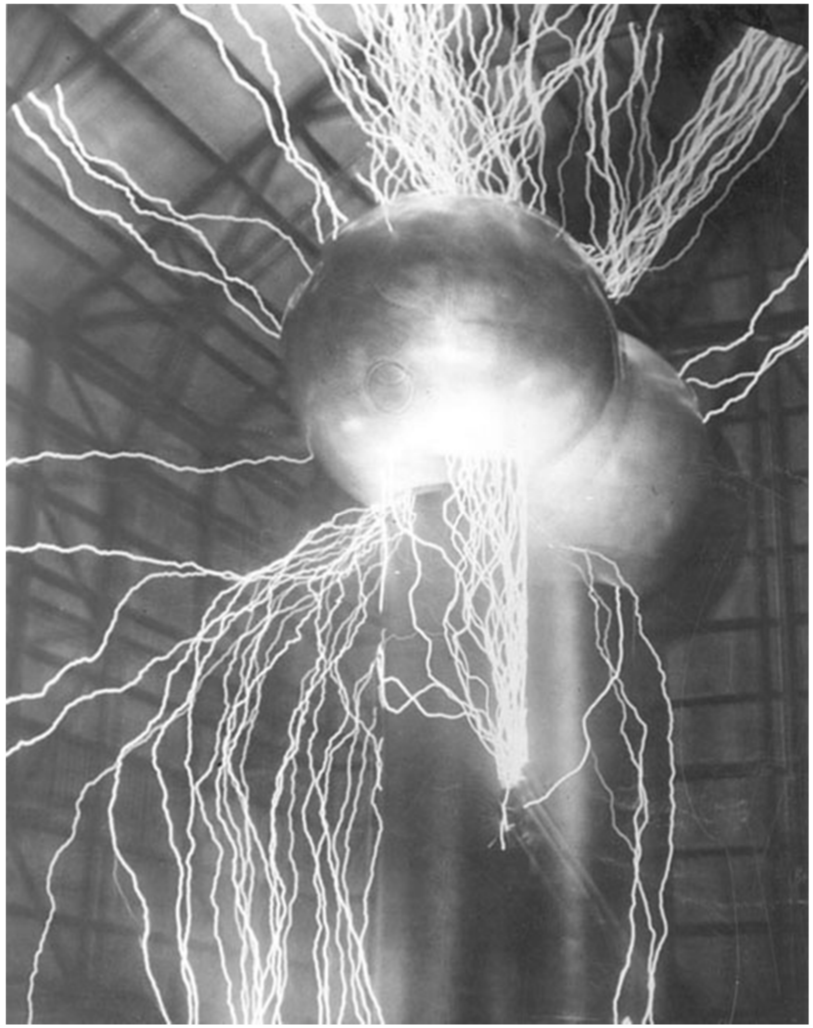
$$= \frac{Q}{4\pi\varepsilon_0}\Big|_b^a \left(\frac{1}{r}\right)$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = Q/V = \frac{Q}{Q/4\pi\varepsilon_0\,(1/a - 1/b)}$$

$$C = \frac{4\pi\varepsilon_0}{1/a - 1/b}$$

$$C = \frac{4\pi\varepsilon_0}{1/a - 1/b} = 4\pi\varepsilon_0 \frac{ab}{b-a} = 4\pi\varepsilon_0 \frac{a}{1-a/b}$$



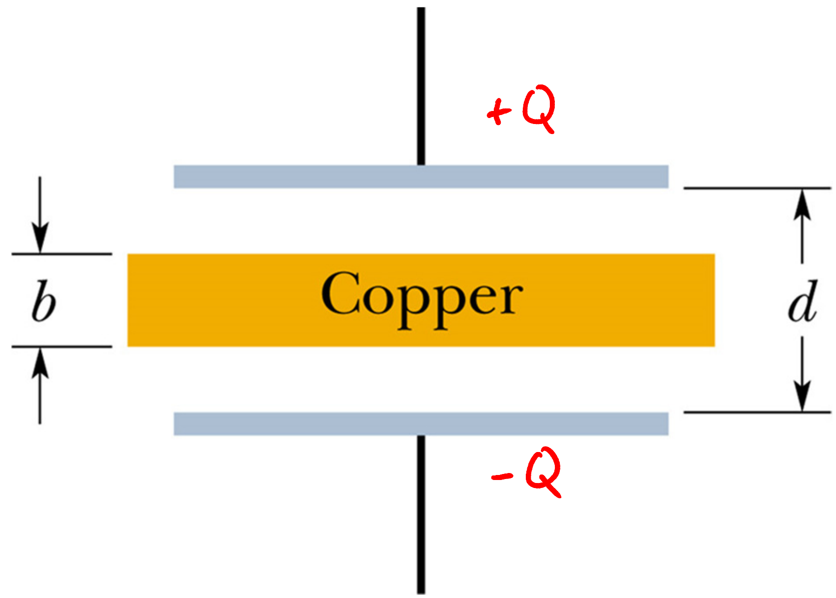
$b \to \infty$

(outer shell gone)

$$C = 4\pi\varepsilon_0 \frac{a}{1-0}$$

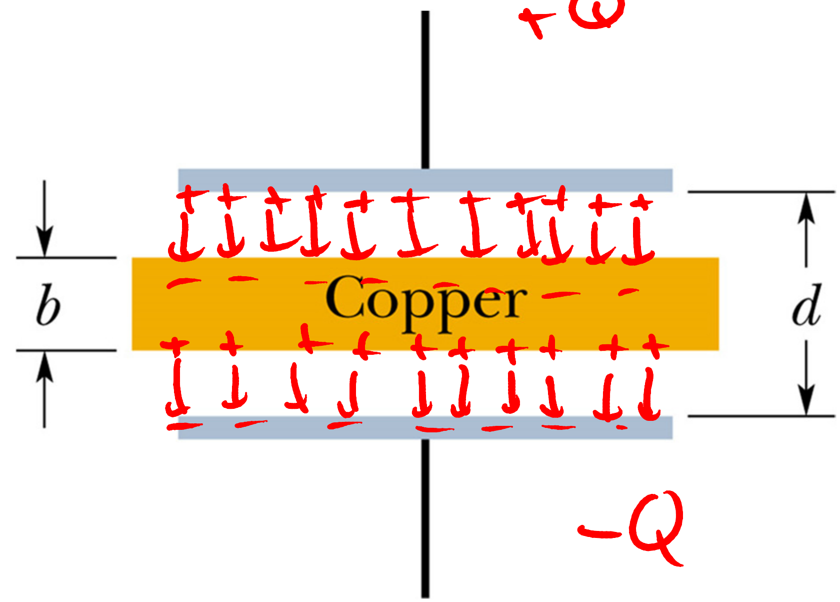
$$= 4\pi\varepsilon_0 a$$

Stick a copper slab into this capacitor
$d=5.0$ mm, $b=2.0$ mm, $A=2.4$ cm^2
What's C of the whole thing?



$b = 2\text{mm}$ $\quad d = 5\text{mm}$ $\quad A = 2.40\text{ cm}^2$

What's C of this new thing?

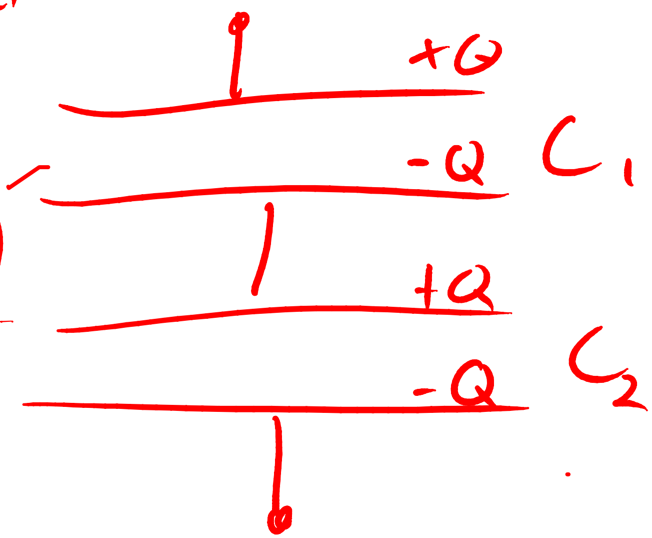


$$C_0 = \frac{\varepsilon_0 A}{d}$$

"new" cap $\quad d' = \frac{d-b}{2} = 1.5\text{mm}$

$$\text{new } C' = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

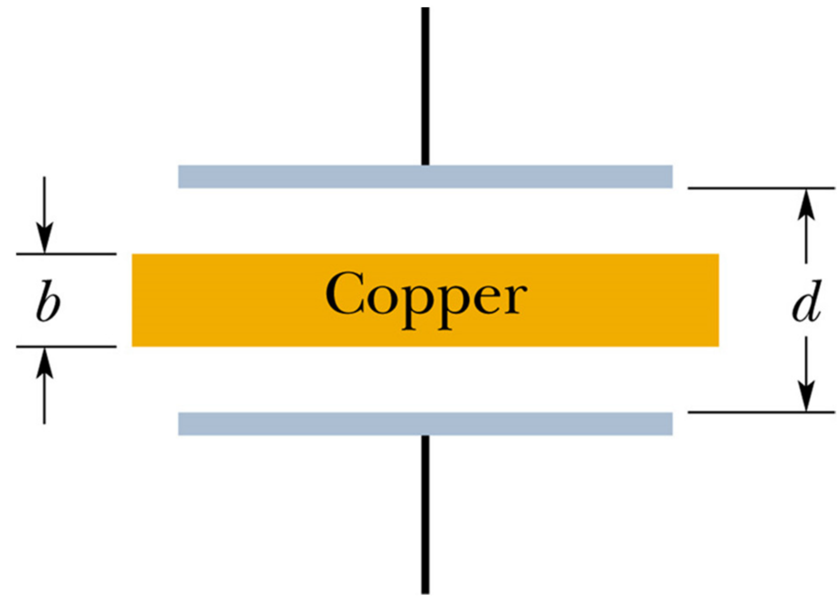
$$= \left(\frac{1}{\varepsilon_0 A/d'} + \frac{1}{\varepsilon_0 A/d'}\right)^{-1} = \left(\frac{2d'}{\varepsilon_0 A}\right)^{-1}$$



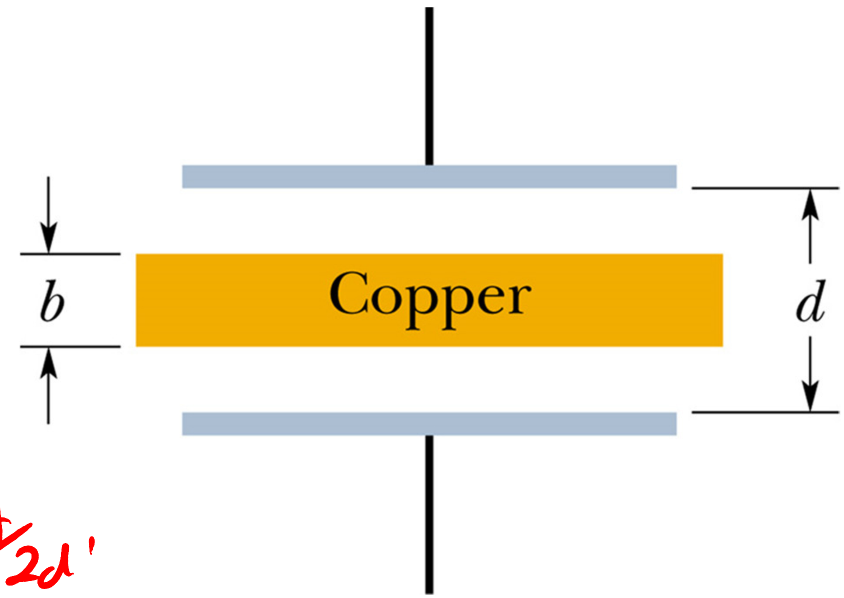
$$C' = \frac{\varepsilon_0 A}{2d'} = \frac{(8.85 \times 10^{-12})(2.40 \times 10^{-4}\text{m}^2)}{2(1.5 \times 10^{-3}\text{m})}$$

$$= 0.708 \text{ pF}$$

What's ratio of energy stored before and after the copper is put in? (*let Q=3.4μC if you want, but you don't need to know it*)



If $Q = 3.4\,\mu C$, what's ratio of energy stored before + after?



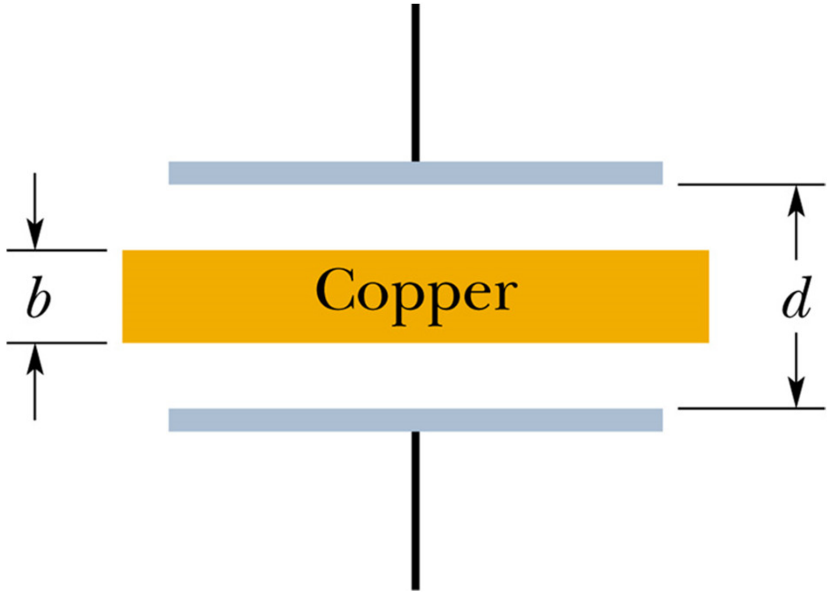
$$U = \frac{Q^2}{2C} \qquad U' = \frac{Q^2}{2C'}$$

$$\frac{U}{U'} = \frac{Q^2/2C}{Q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/2d'}{\varepsilon_0 A/d}$$

$$U/U' = \frac{d}{2d'} = \frac{5.00\text{ mm}}{2(1.5\text{ mm})} = 1.67$$

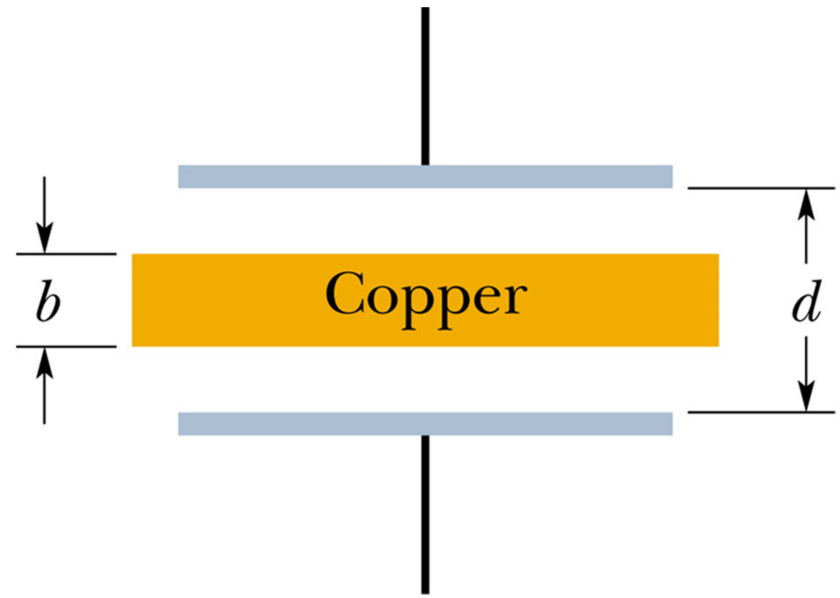
U = after $\qquad U'$ = before

As you put the copper in, is it being pulled in or do you have to shove it?

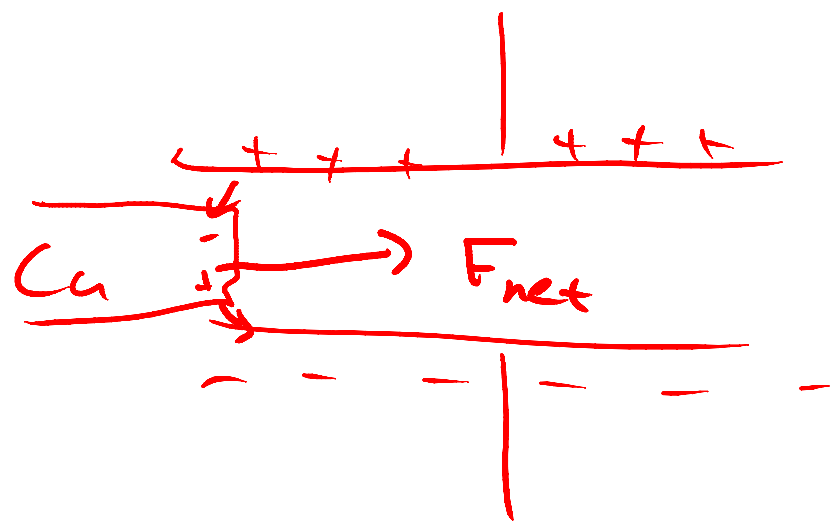


Is Cu being pulled in,

Or do you have to push?



adding Cu, we have $\frac{1}{1.67}$ less U: so it "falls" in



Dielectrics

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

ε_0: permittivity of free space
↳ vacuum

$\varepsilon = K\varepsilon_0$

K dielectric constant $K \geq 1$

air:

$K = 1.00059$

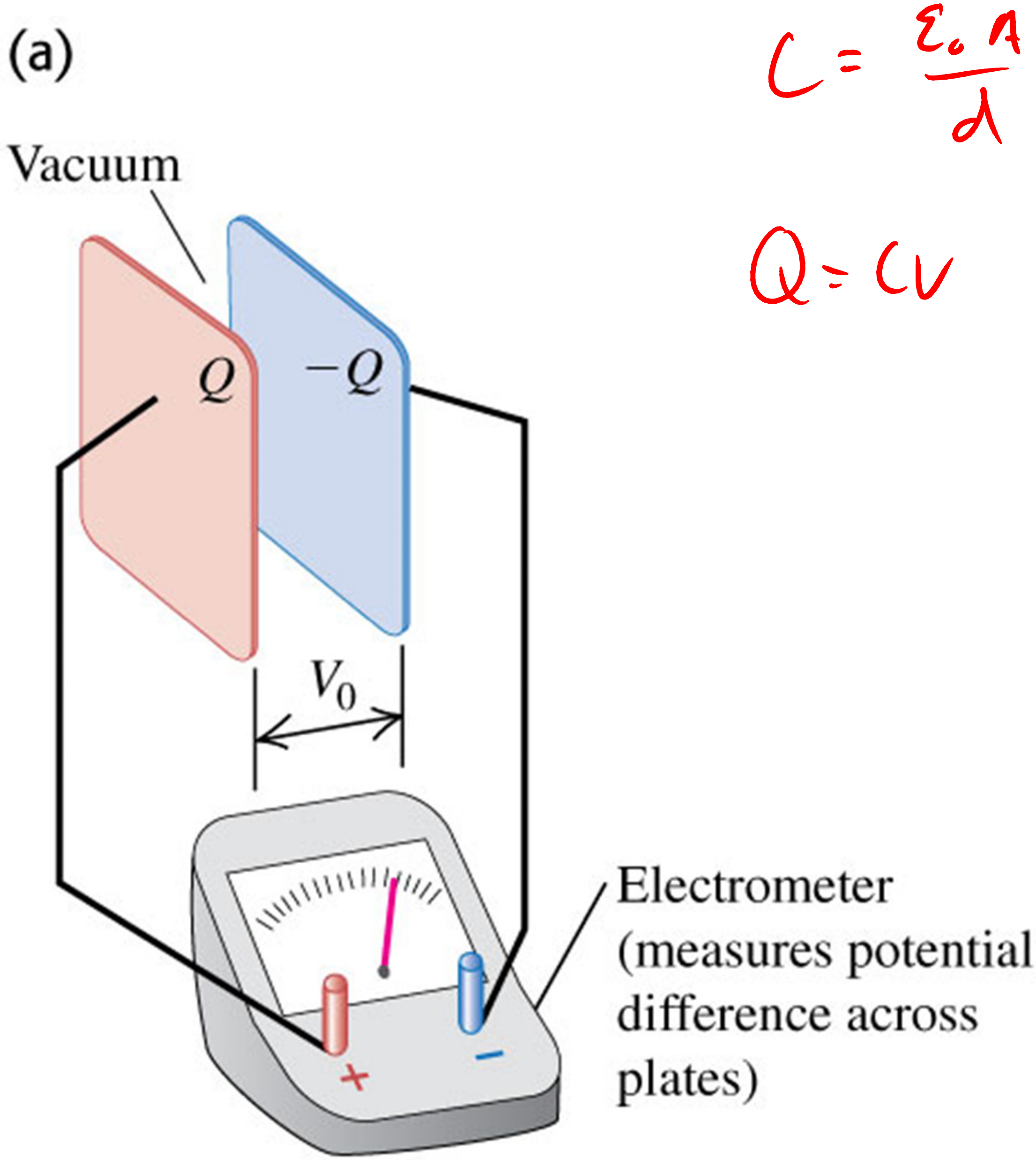
$K_{teflon} = 2.1$

$K_{water} = 80.4$

anywhere we've had ε_0
replace w/ $K\varepsilon_0$

example: $\vec{E} = \frac{\sigma}{\varepsilon_0}$ replace $\varepsilon_0 \rightarrow \varepsilon_0' = K\varepsilon_0$

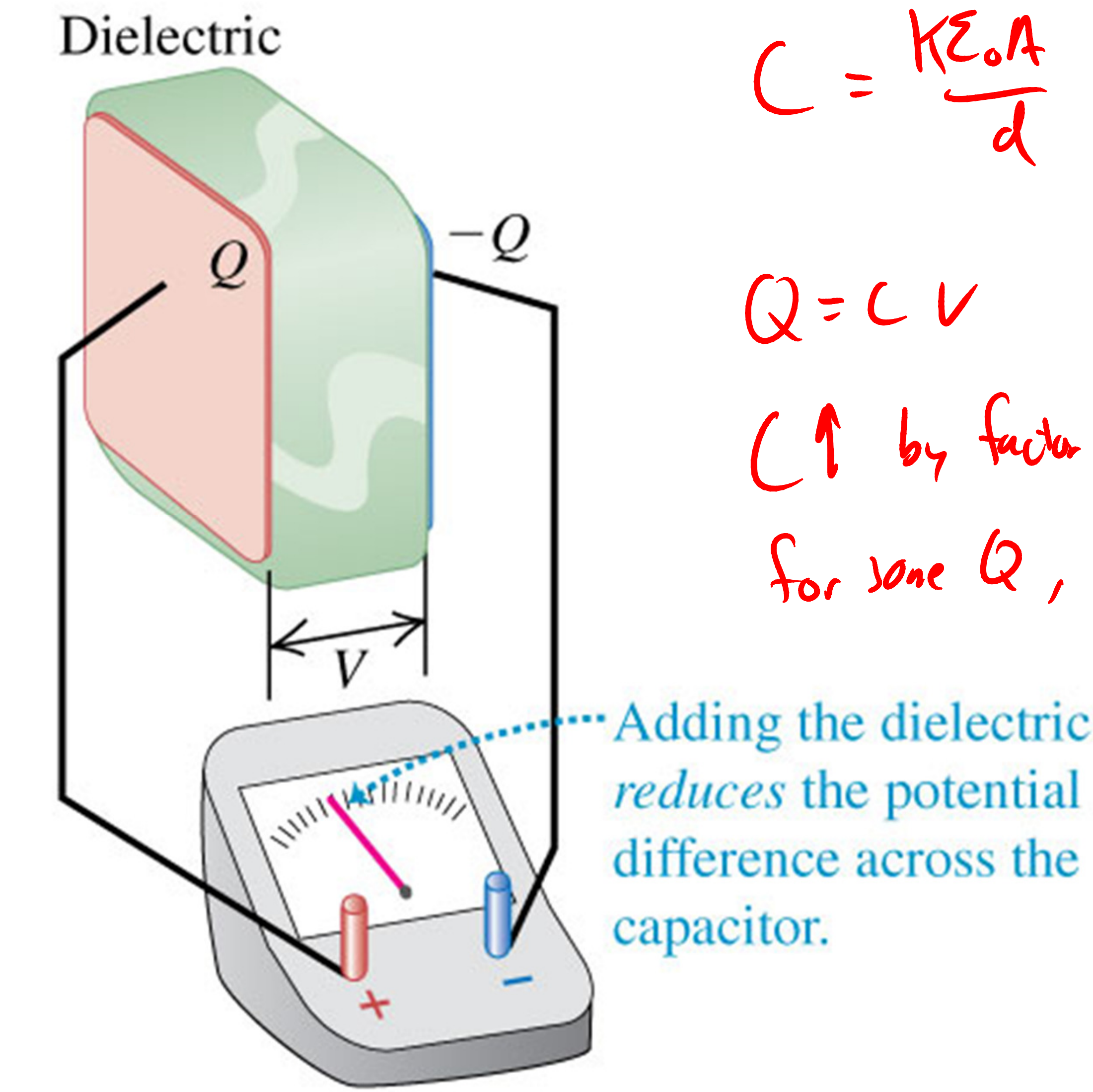
$$\vec{E} = \frac{\sigma}{K\varepsilon_0}$$



$$C = \frac{\varepsilon_0 A}{d}$$

$$Q = CV$$

(b)



$K \qquad \varepsilon = K\varepsilon_0$

$$C = \frac{K\varepsilon_0 A}{d}$$

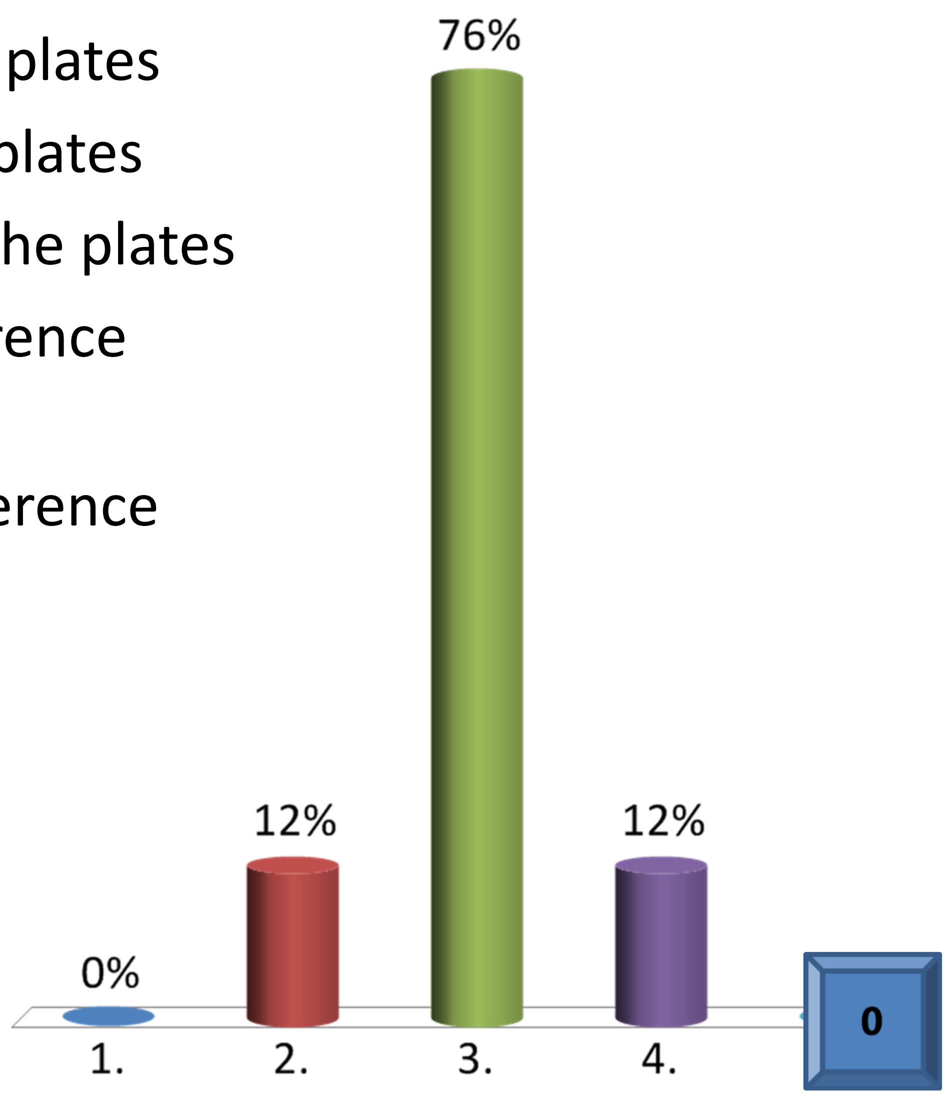
$Q = CV$

$C\uparrow$ by factor of K

for same Q, $V\downarrow$ by K

Which one of the following changes will necessarily increase the capacitance of a capacitor?

1. decreasing the charge on the plates
2. increasing the charge on the plates
3. placing a dielectric between the plates
4. increasing the potential difference between the plates
5. decreasing the potential difference between the plates



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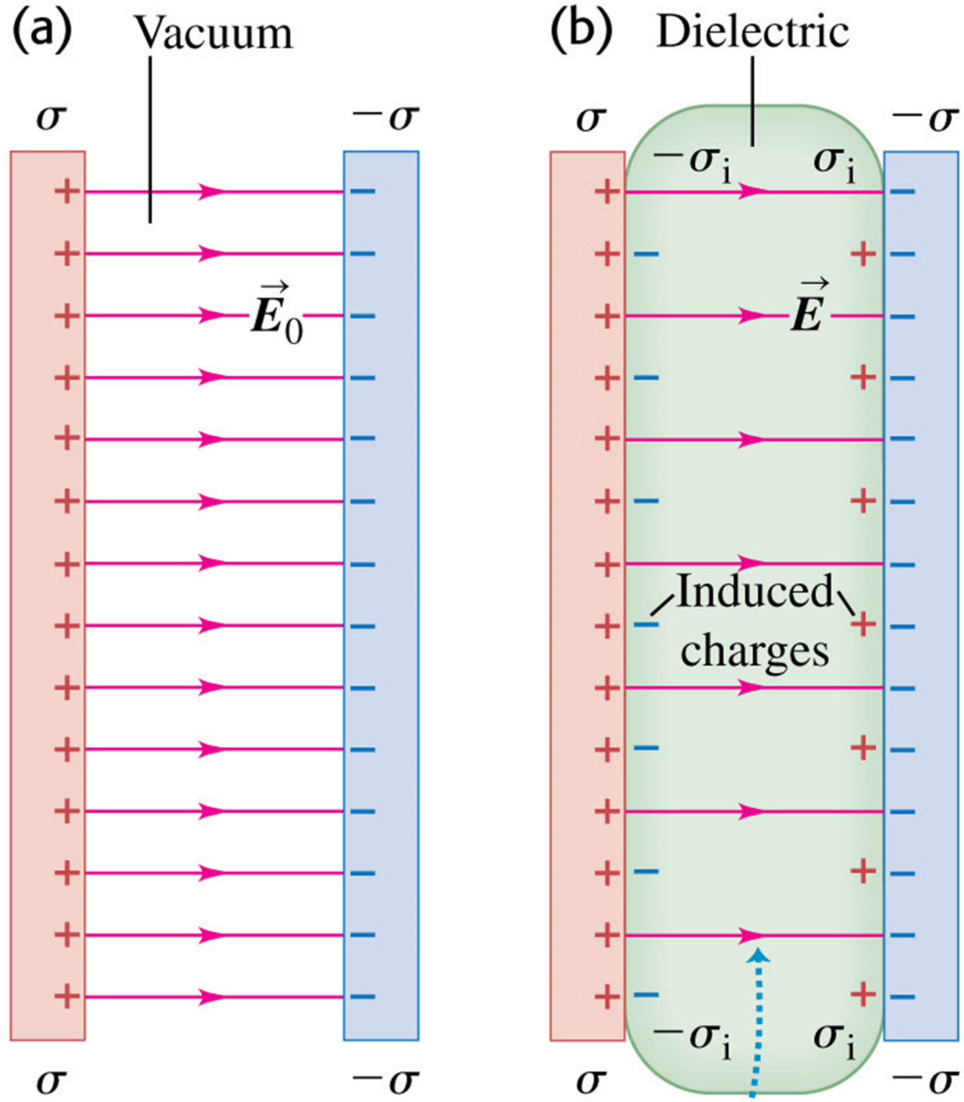
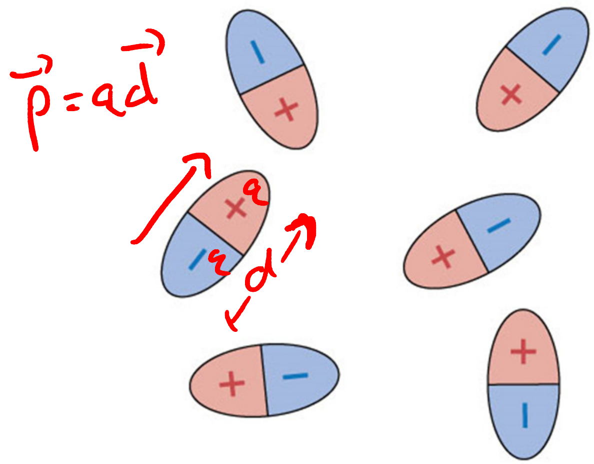
Do Capacitor & Battery Handout



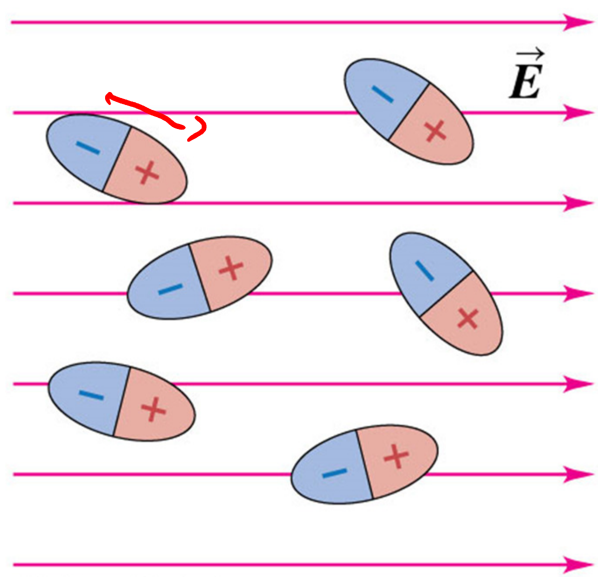
For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

(a)

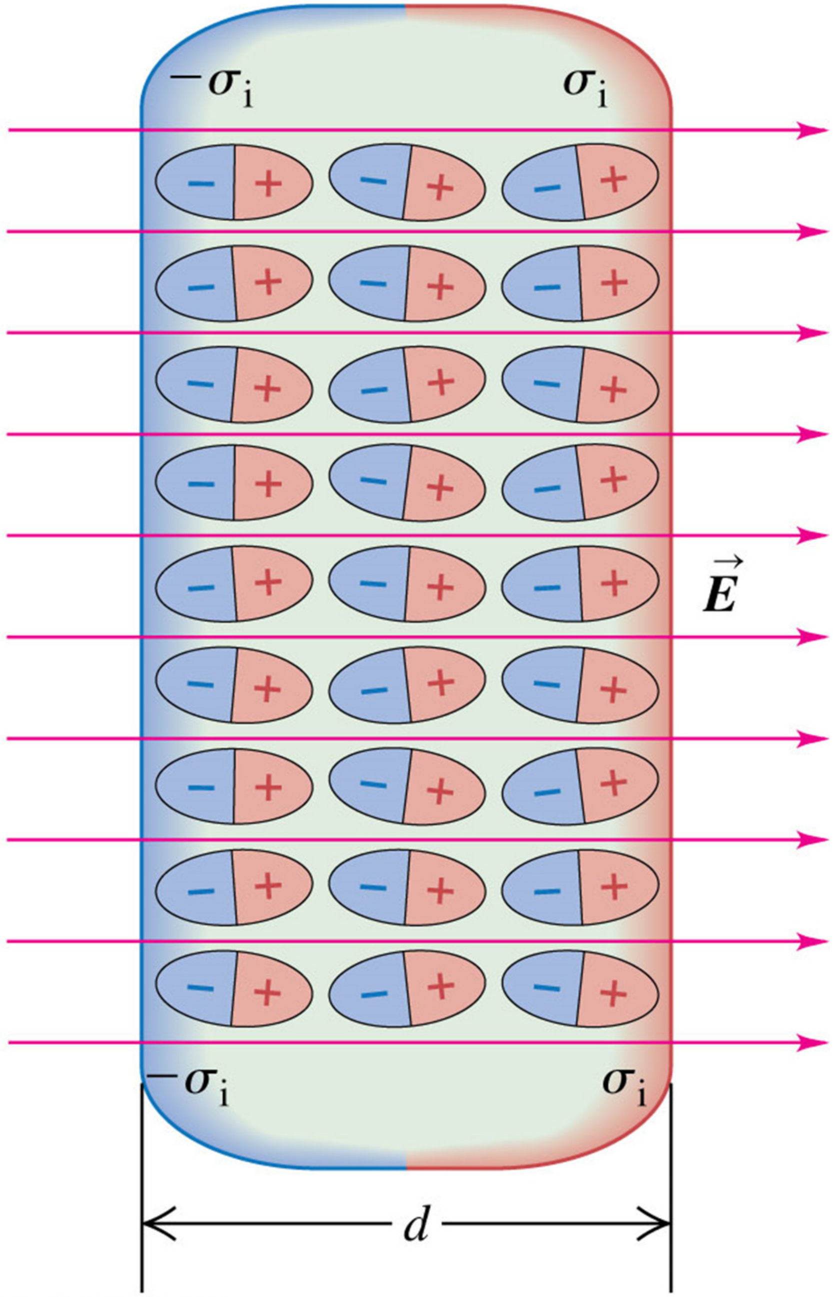


In the absence of an electric field, polar molecules orient randomly.

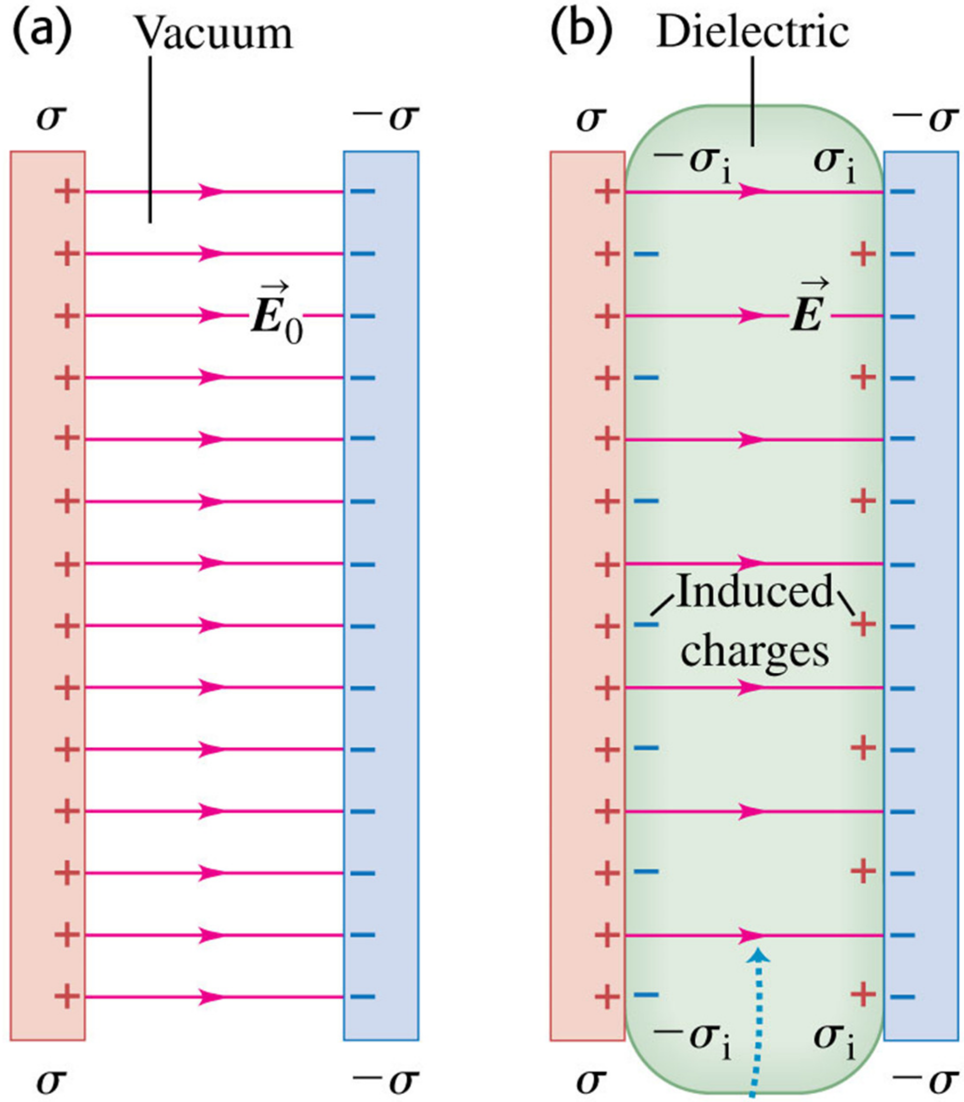
(b)



When an electric field is applied, the molecules tend to align with it.

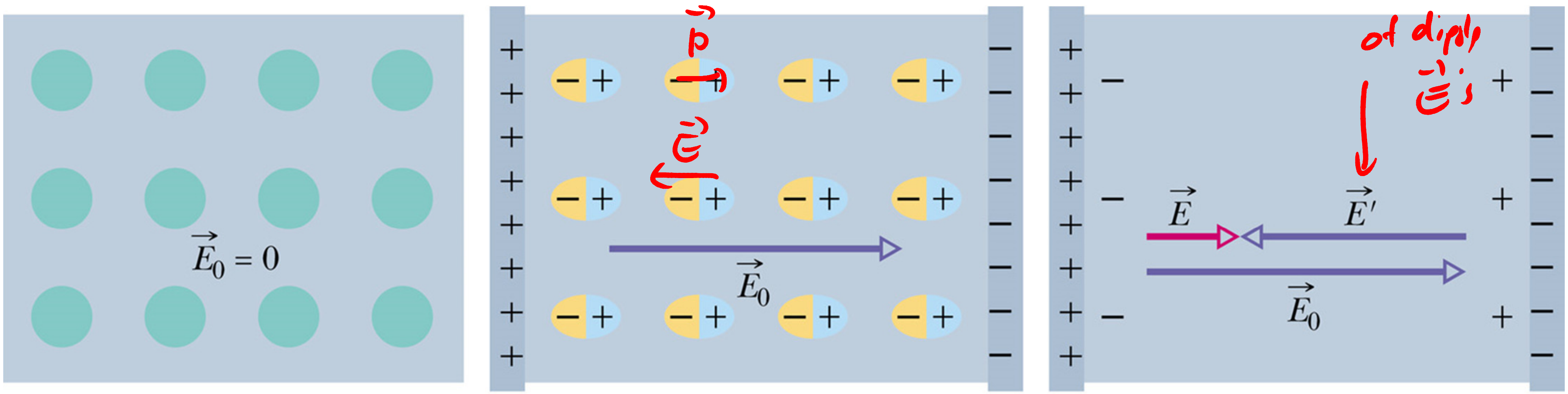


$-\sigma_i$
σ_i
$\vec{E}$
$-\sigma_i$
σ_i
d



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

$\vec{E}'$ from sum of dipole $\vec{E}$'s



(a) (b) (c)

Less $\vec{E}$ by a factor K

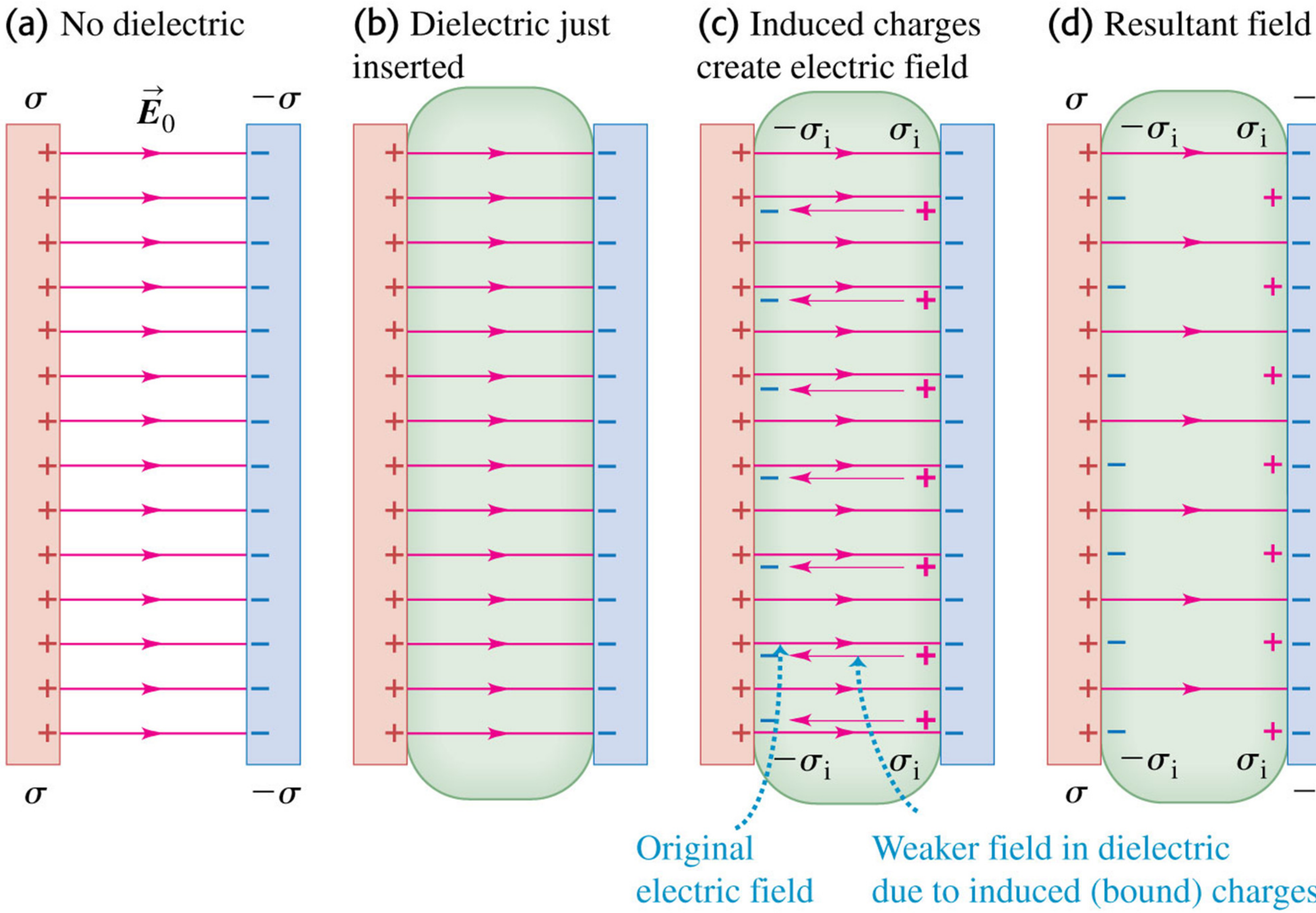
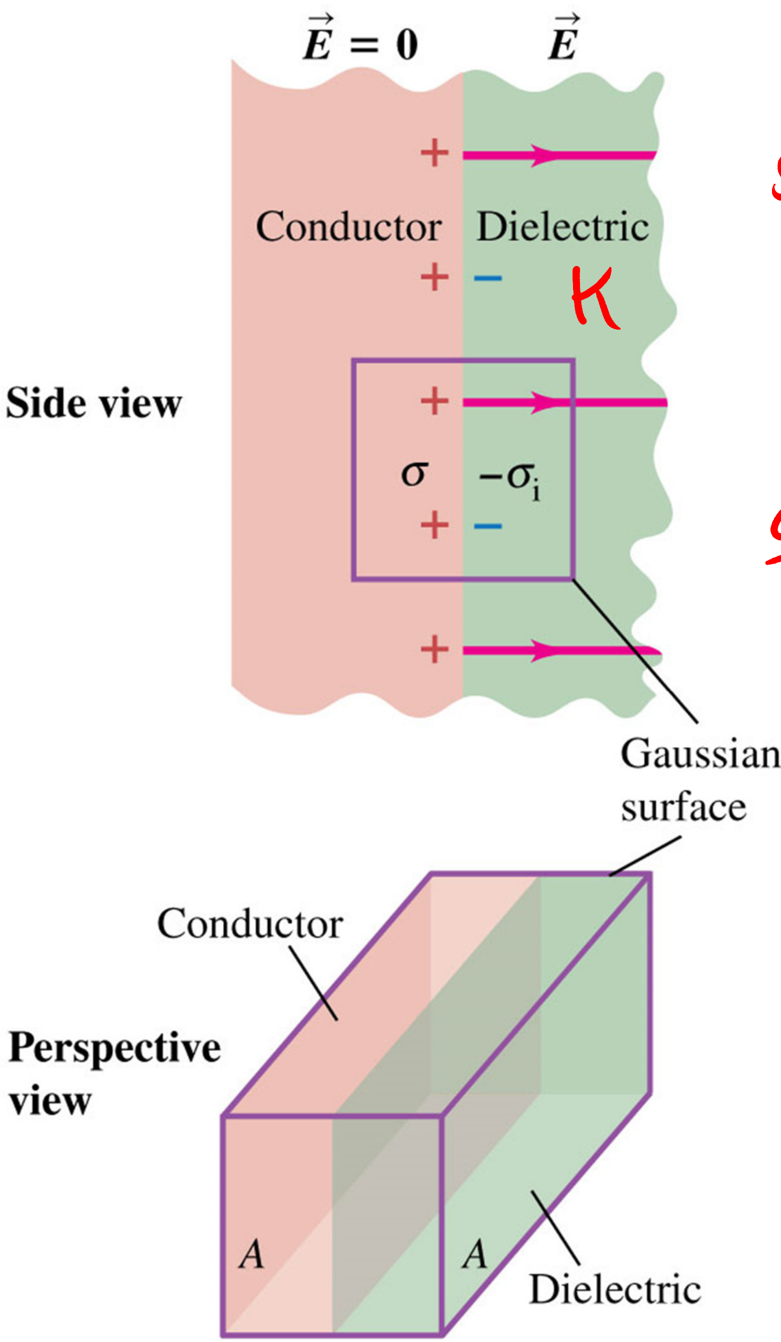

(a) No dielectric
σ
$\vec{E}_0$
$-\sigma$
σ
$-\sigma$
(b) Dielectric just inserted
(c) Induced charges create electric field
$-\sigma_i$
σ_i
$-\sigma_i$
σ_i
Original electric field
Weaker field in dielectric due to induced (bound) charges
(d) Resultant field
σ
$-\sigma_i$
σ_i
$-\sigma$
σ
$-\sigma_i$
σ_i
$-\sigma$

Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Dielectric Constant, K	Dielectric Strength, E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

Air 1.0005 3×10^6



$\varepsilon = K\varepsilon_0$ $\quad \Phi_E = \dfrac{q_{enc}}{\varepsilon_0}$

Skip to $EA = \Phi_E = \dfrac{1}{\varepsilon_0}(\sigma A - \sigma_i A)$

$= \dfrac{A}{\varepsilon_0}(\sigma - \sigma_i)$

$\dfrac{\sigma}{K} = \sigma - \sigma_i \quad \left(\sigma_i = \sigma\left(1 - \dfrac{1}{K}\right)\right)$

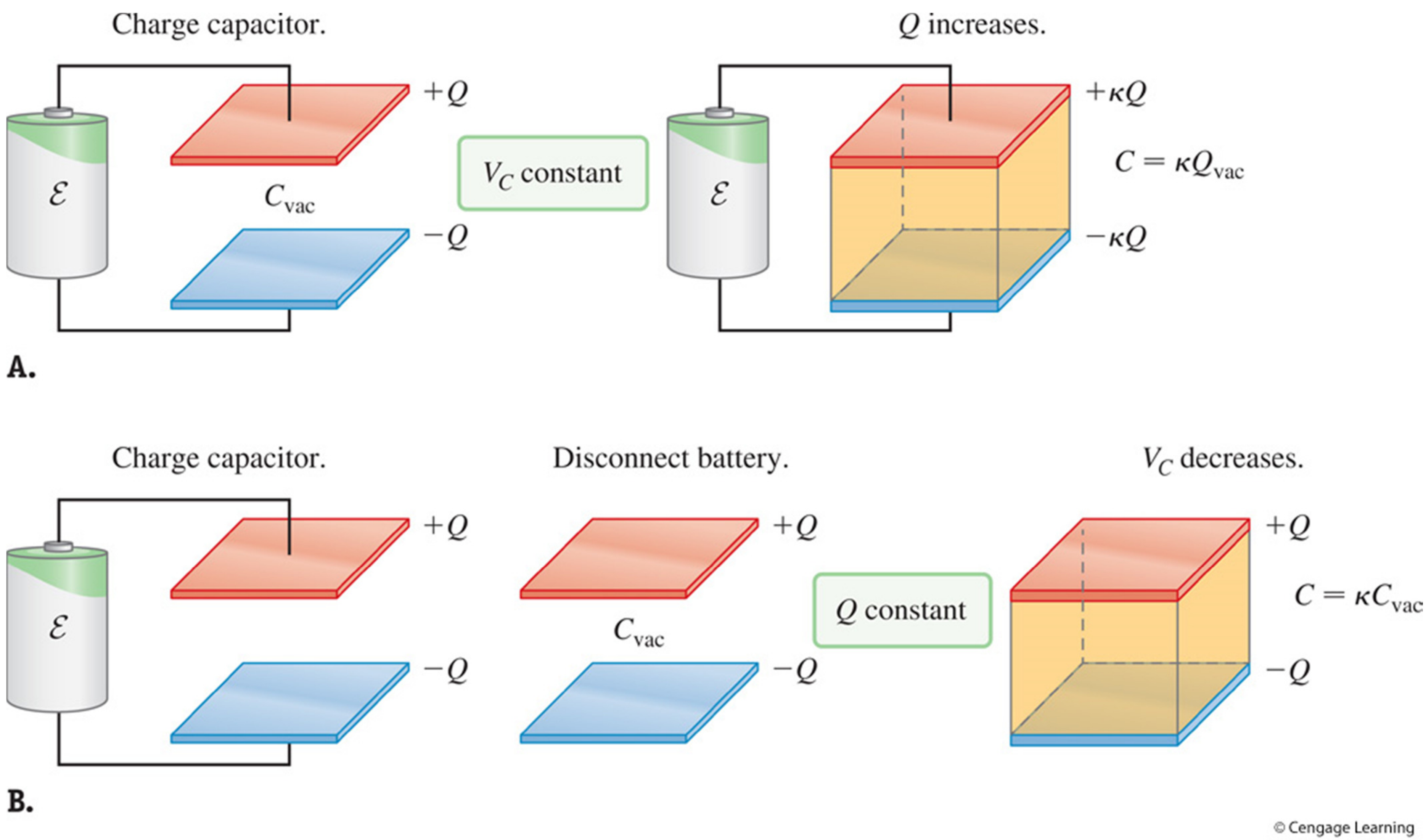
$EA = \dfrac{\sigma A}{K\varepsilon_0}$ back to $\vec{E}$ near σ

$\oint K\vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\varepsilon_0}$

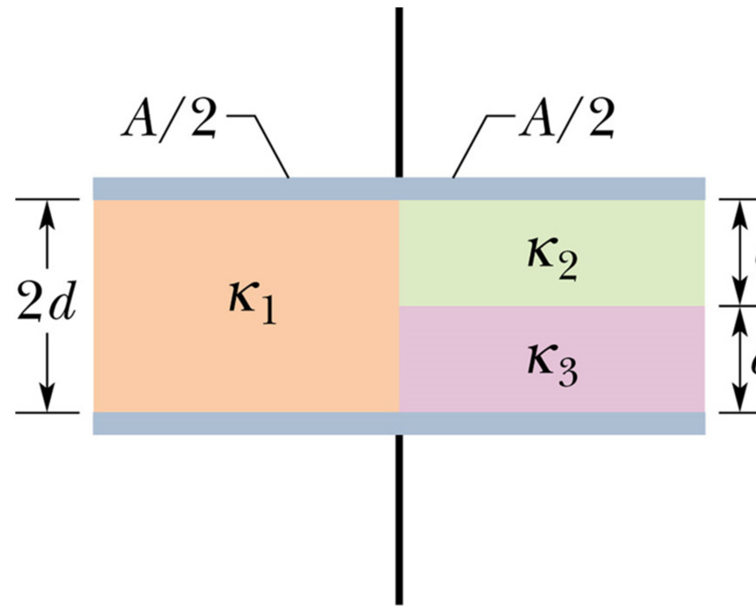
$K\vec{E} = \vec{D}$

$\vec{D}$ = electric displacement

Fig.27.31


Charge capacitor.
Q increases.
+Q
−Q
$\mathcal{E}$
C_{vac}
V_C constant
+κQ
−κQ
$C = \kappa Q_{vac}$
A.
Charge capacitor.
Disconnect battery.
V_C decreases.
+Q
−Q
C_{vac}
Q constant
$C = \kappa C_{vac}$
B.
© Cengage Learning

$A = 10.5\ cm^2$, $2d = 7.17$ mm,
$\kappa_1 = 21.0$, $\kappa_2 = 42.0$, $\kappa_3 = 58.0$
What's C of the whole thing?

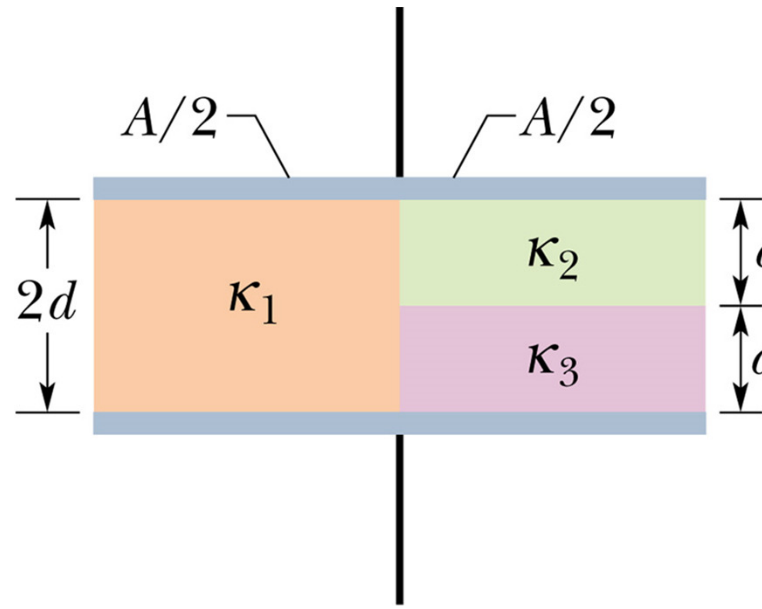


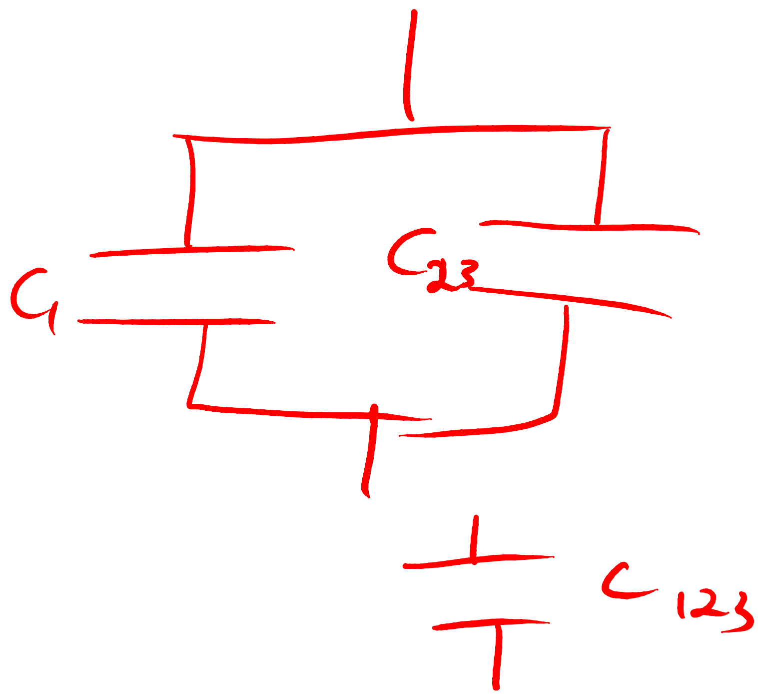
$A = 10.5\ cm^2$ $2d = 7.17\ mm$ $\kappa_1 = 21.0$ $\kappa_2 = 42.0$ $\kappa = 58.0$

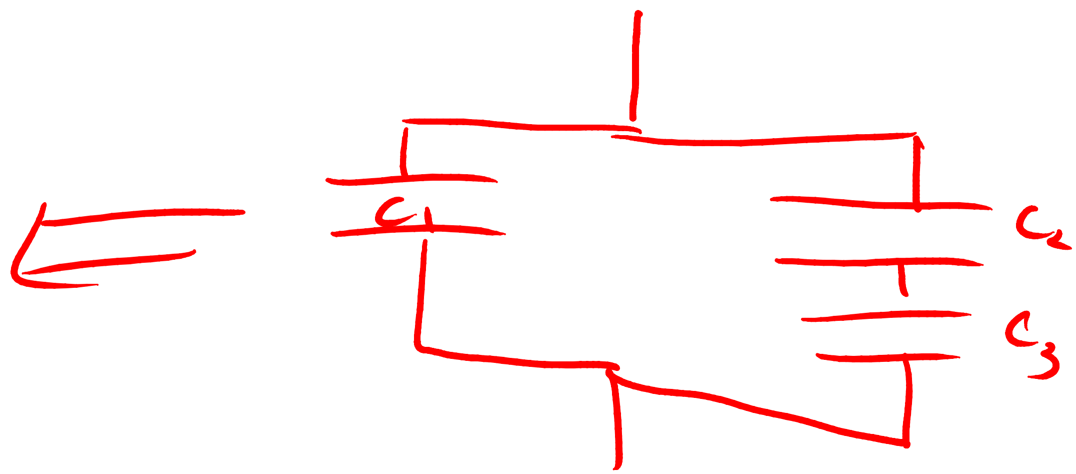
$= 3.56\ mm$

what is C?

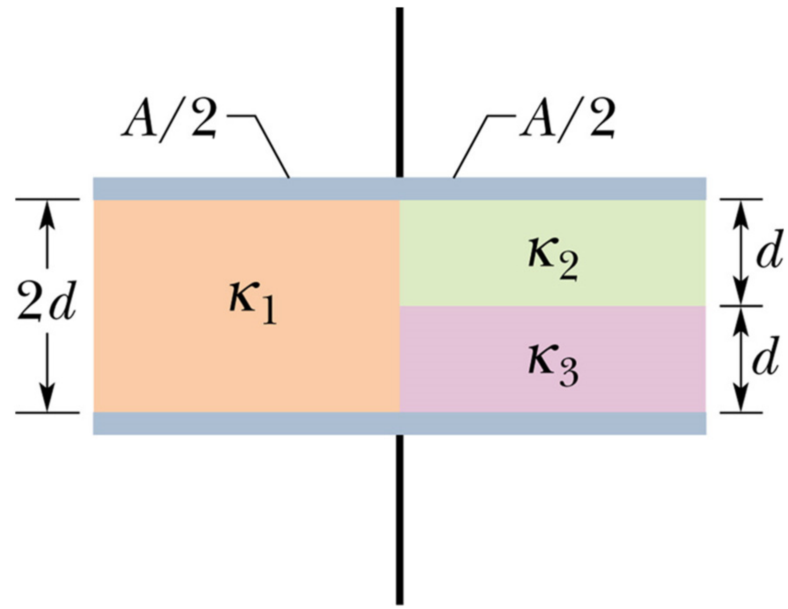
$$C = \frac{\varepsilon_0 \kappa_1 A}{d}$$







$$C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$



$$C_{TOT} = C_1 + C_{23}$$

$$C_1 = \frac{\varepsilon_0 \kappa_1 \left(\tfrac{1}{2}A\right)}{2d}$$

$$C_2 = \frac{\varepsilon_0 \kappa_2 \left(\tfrac{1}{2}A\right)}{d}$$

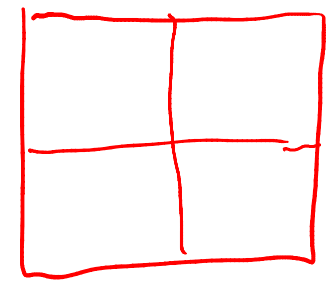
$$C_3 = \frac{\varepsilon_0 \kappa_3 \left(\tfrac{1}{2}A\right)}{d}$$

so $$C = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

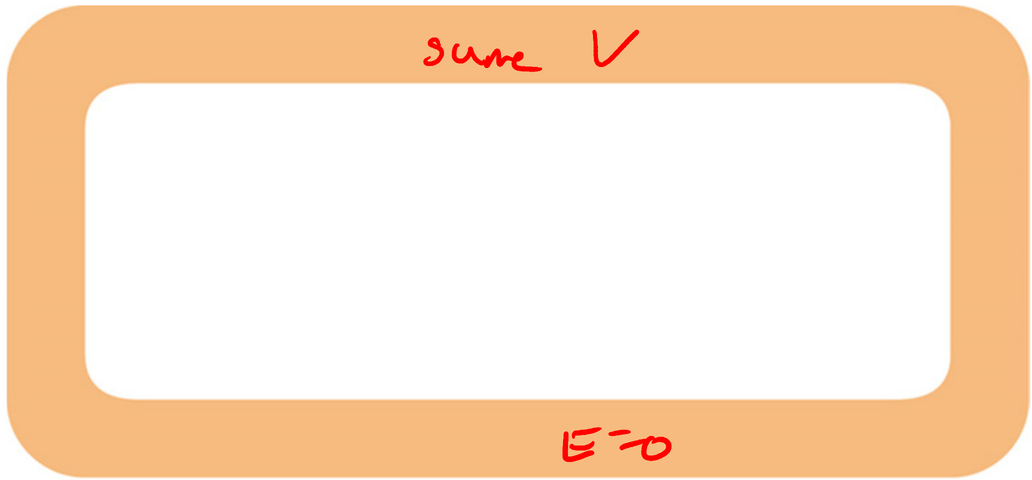
$$C = \frac{\varepsilon_0 A \kappa_1}{4d} + \frac{\left(\varepsilon_0 \frac{A}{2d}\right)^{\cancel{2}} \kappa_2 \kappa_3}{\cancel{\left(\varepsilon_0 \frac{A}{2d}\right)}\left(\kappa_2 + \kappa_3\right)} = \frac{\varepsilon_0}{2d}\left(\frac{\kappa_1}{2} + \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)$$

$$C = 4.55 \times 10^{-11}\,F = 0.455\,pF$$

Touch Screens

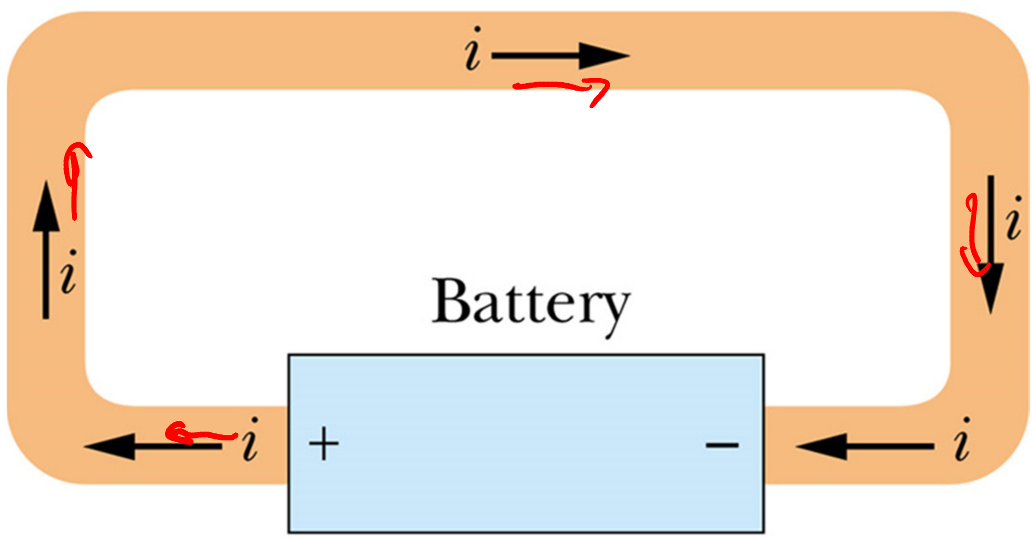


Current and Resistance



(a)

Static



(b)

not Static

Definition of Current

$I \qquad \frac{C}{s} \rightarrow \text{Amps (A)} = \frac{dq}{dt}$

Fig. 28.3

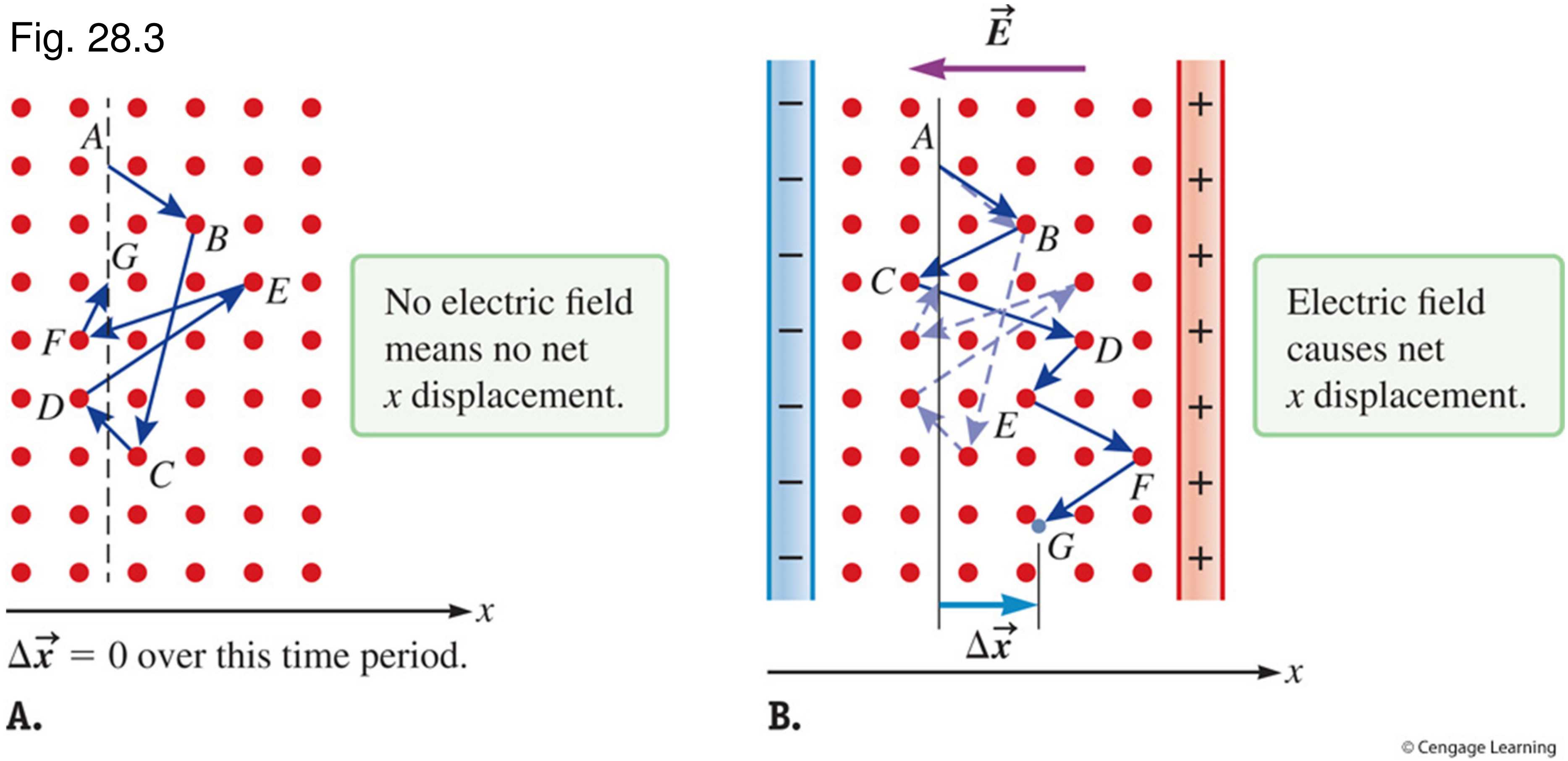
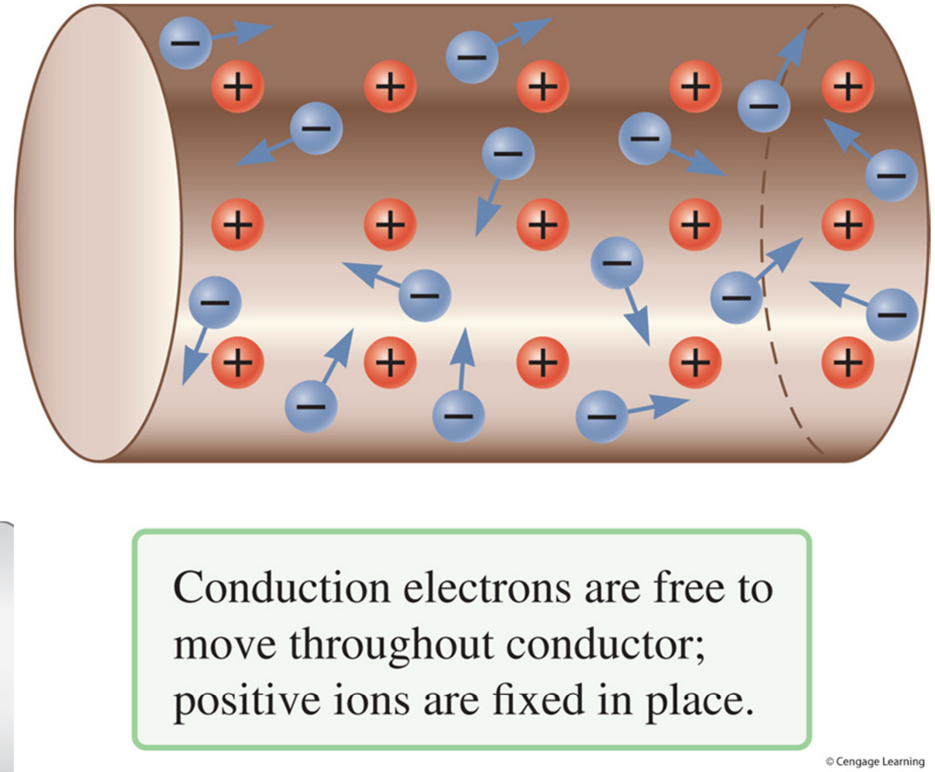

A
B
G
E
F
D
C
No electric field means no net x displacement.
x
$\Delta\vec{x} = 0$ over this time period.
A.
$\vec{E}$
A
B
C
D
E
F
G
Electric field causes net x displacement.
$\Delta\vec{x}$
x
B.
© Cengage Learning

Fig. 28.1



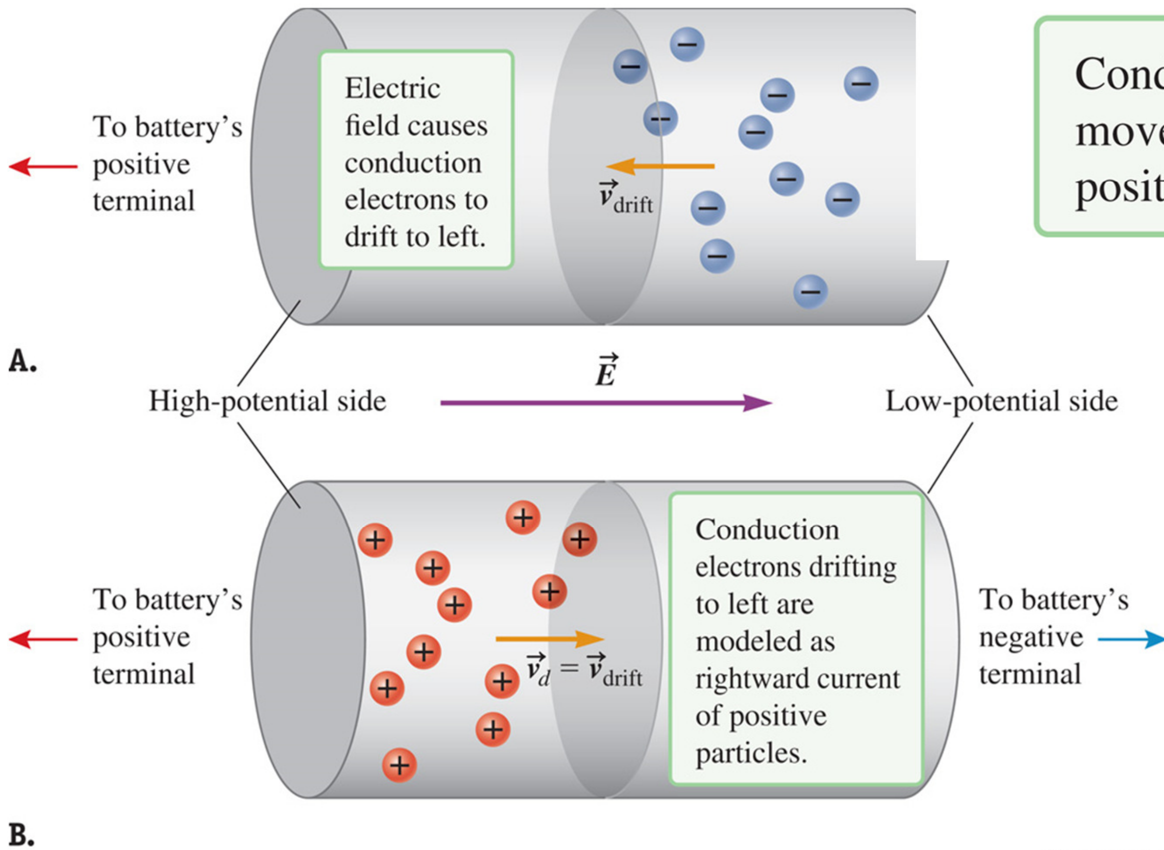
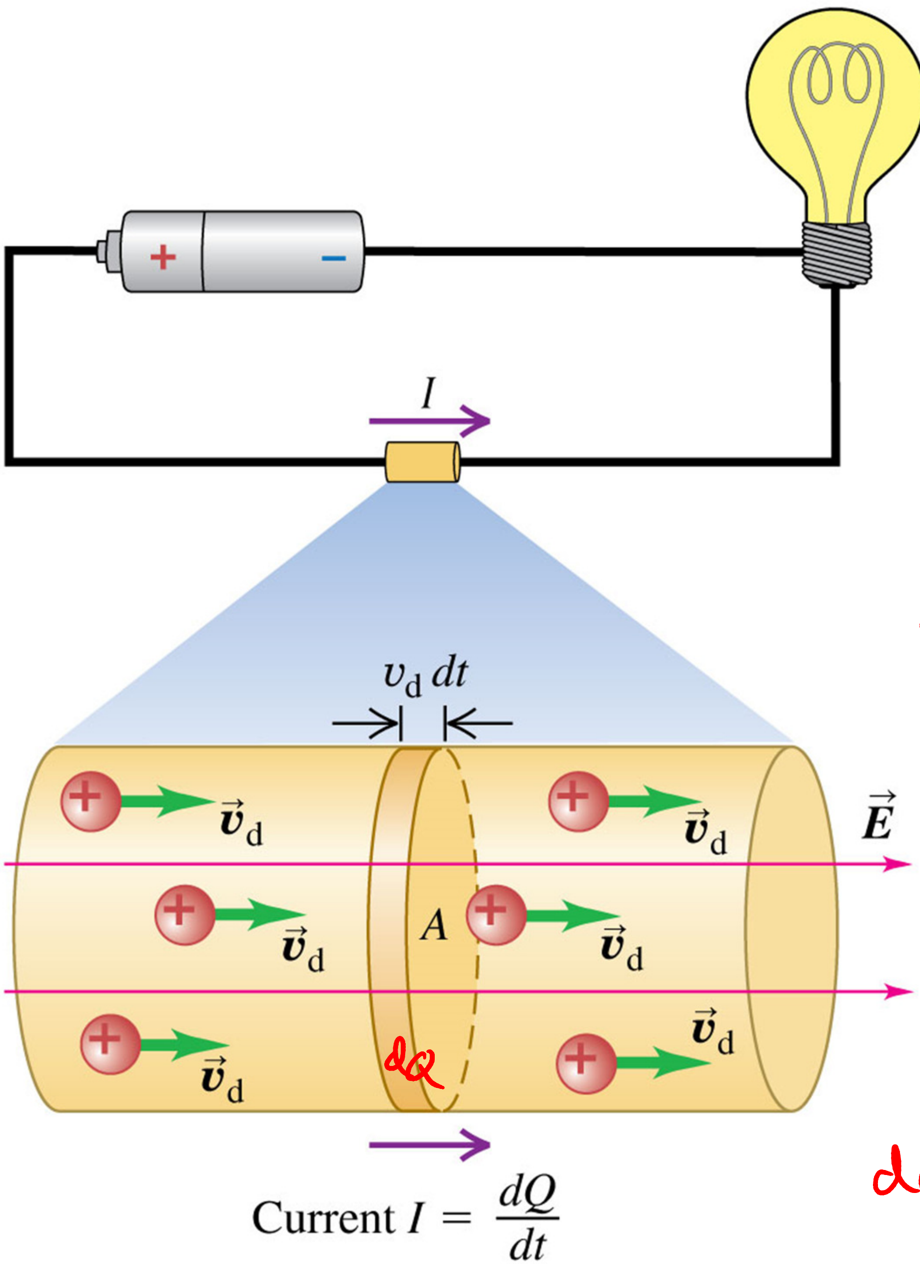


Fig. 28.6



$\vec{F} = q\vec{E} = ma \qquad \vec{a} = \frac{-e\vec{E}}{m_e}$

Over Δt, a makes $\vec{v}_d = \vec{a}\,\Delta t$

$$V_d = \frac{e\vec{E}\,\Delta t}{m_e}$$

$n \quad \frac{\text{charges}}{m^3}$

each charge moves $(v_d\,dt)$ in dt

Volume $(A\,v_d\,dt)$ contains $n\,(A\,v_d\,dt)$

$$dQ = q\,n\,v_d\,dt\,A$$

$$\frac{dQ}{dt} = q\,n\,v_d\,A = I$$

define J = current density

$$J = I/A = q\,n\,v_d$$