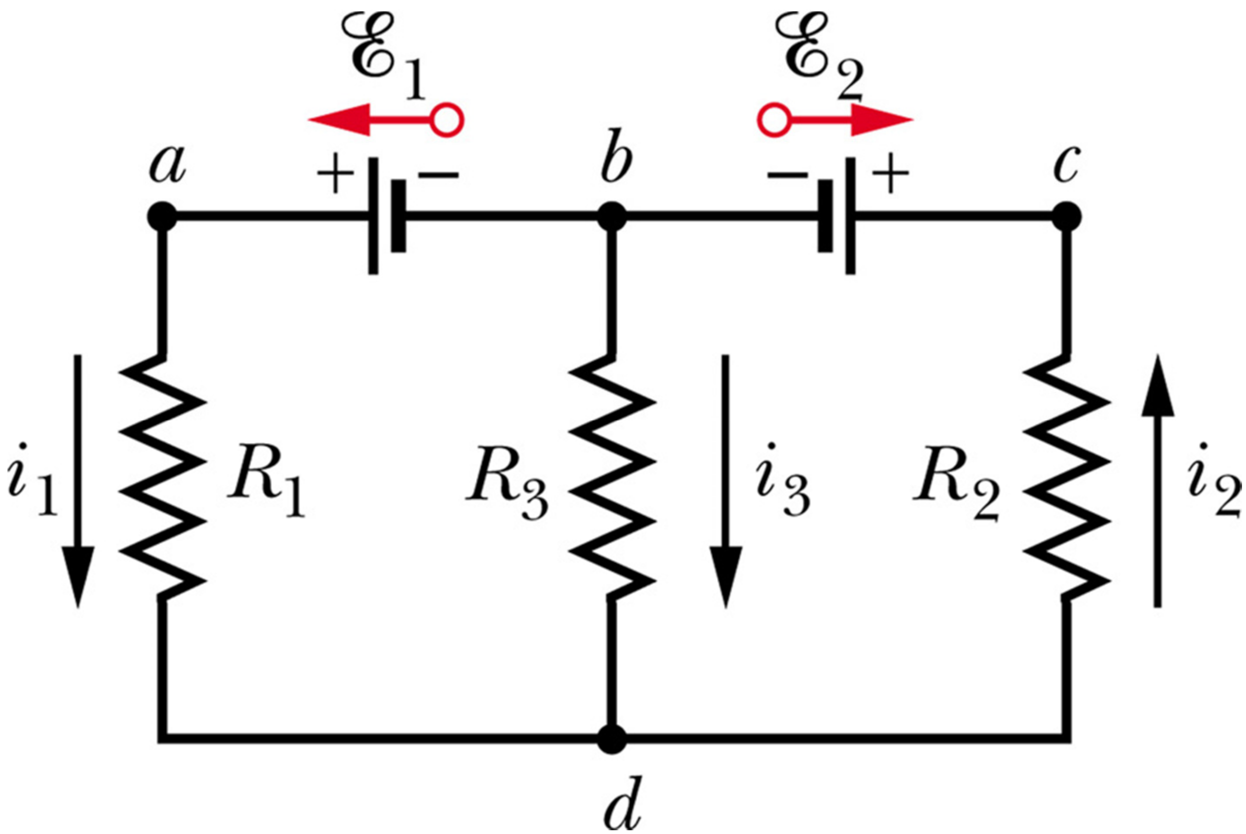
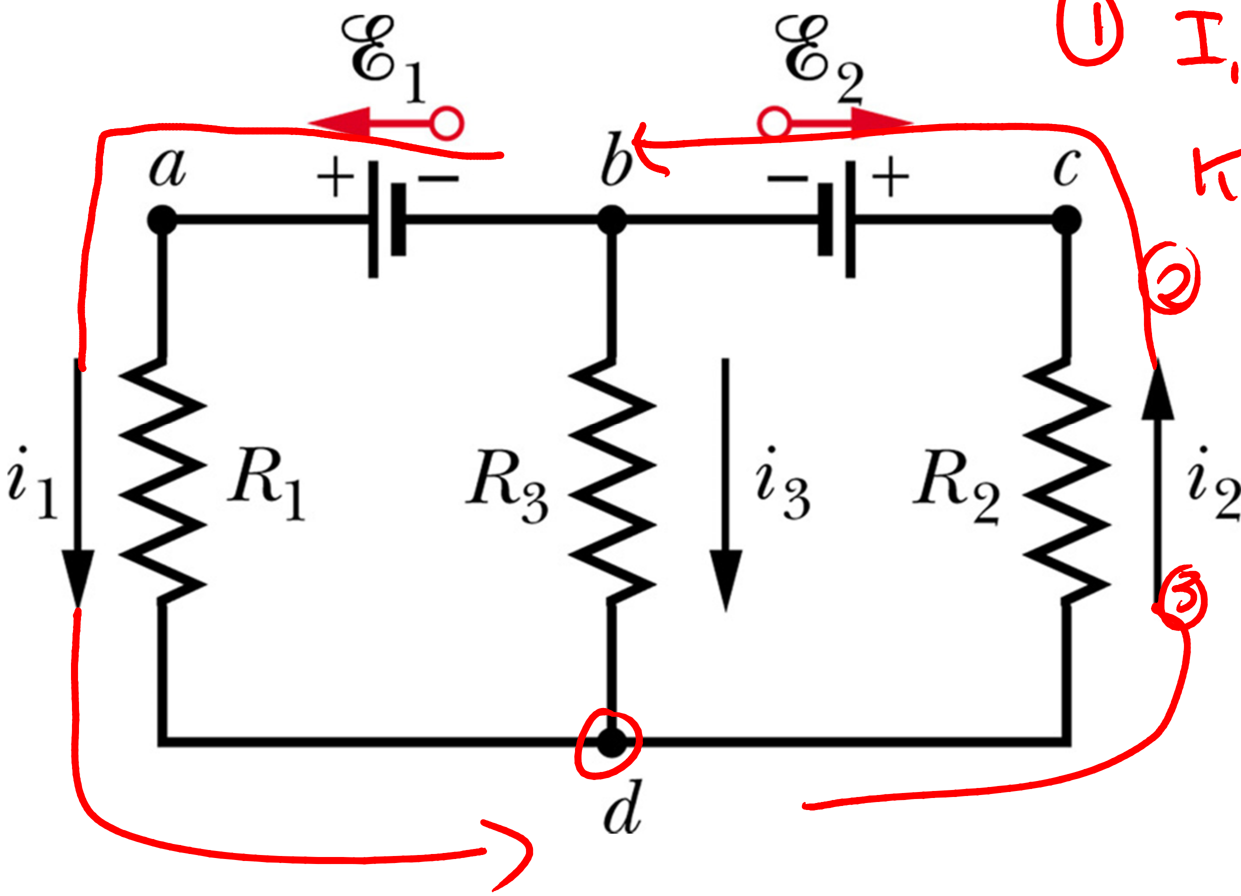


$\mathscr{E}_1$
$\mathscr{E}_2$
a
b
c
d
+
−
−
+
i_1
R_1
R_3
i_3
R_2
i_2



Kirchoff's Rules: Loop Junc.

① $I_1 + I_3 = I_2$

(badb)

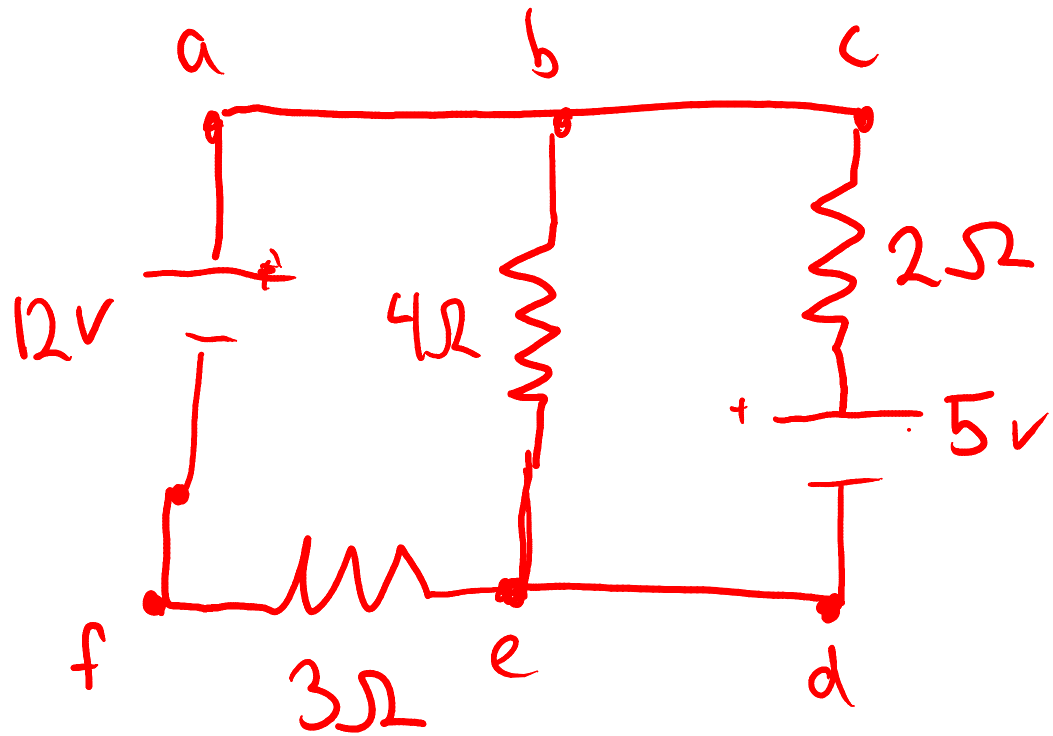
KLR: CCW over left loop (L)

② $+\mathcal{E}_1 - I_1R_1 + I_3R_3 = 0$

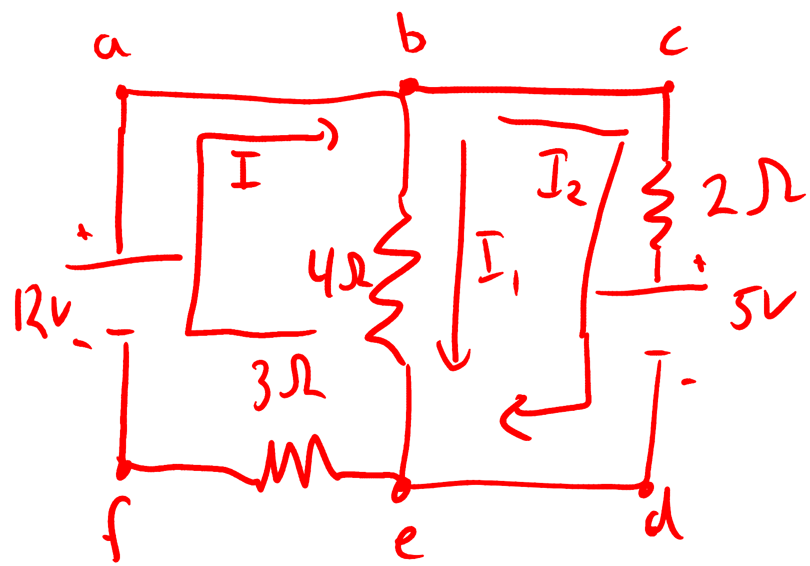
(bdcb)

KL: CCW over right loop (L)

③ $-I_3R_3 - I_2R_2 - \mathcal{E}_2 = 0$



a
b
c
12V
+
4Ω
2Ω
+
5V
f
3Ω
e
d



KJR: @ b $I = I_1 + I_2$ ①

KLR: around CW [fabcdef]

$+12V - I_2 2\Omega - 5V - I\,3\Omega = 0$ ②

CW [fabef]

$+12V - I_1 4\Omega - I(3\Omega) = 0$ ③

Subst ① into ②,③

② $12V - 2\Omega I_2 - 5V - 3\Omega(I_1 + I_2) = 0 \;=\; 7V - 3\Omega I_1 - 5\Omega I_2 = 0$

③ $12V - 4\Omega I_1 - 3\Omega(I_1 + I_2) = 0 \;=\; 12V - 7\Omega I_1 - 3\Omega I_2 = 0$

divide all by 1Ω

② $7A - 3I_1 - 5I_2 = 0$

③ $12A - 7I_1 - 3I_2 = 0$

Solve ③ for $I_2 = \dfrac{12A - 7I_1}{3} = 4A - \frac{7}{3} I_1$

Subst into 2

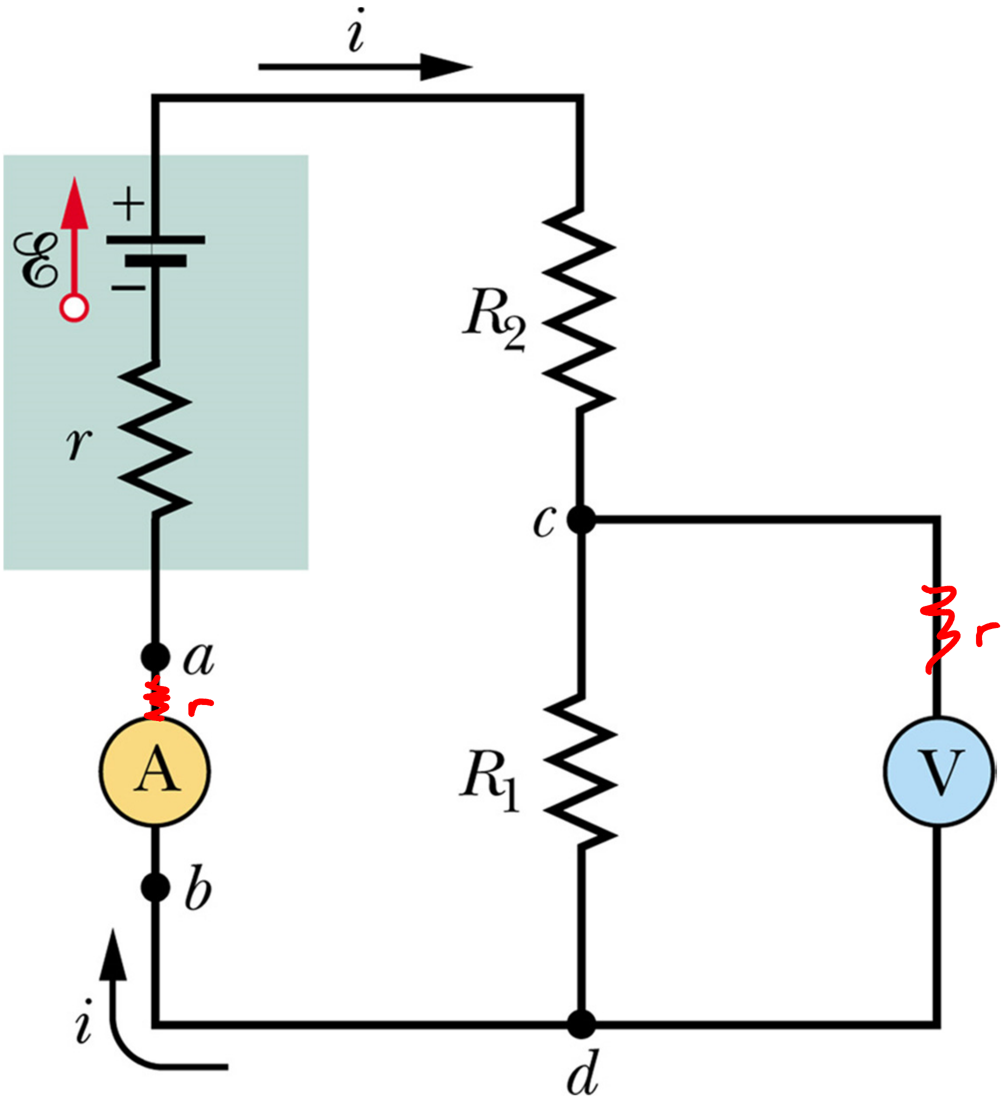
$0 = 7A - 3I_1 - 5\left(4A - \frac{7}{3}I_1\right)$

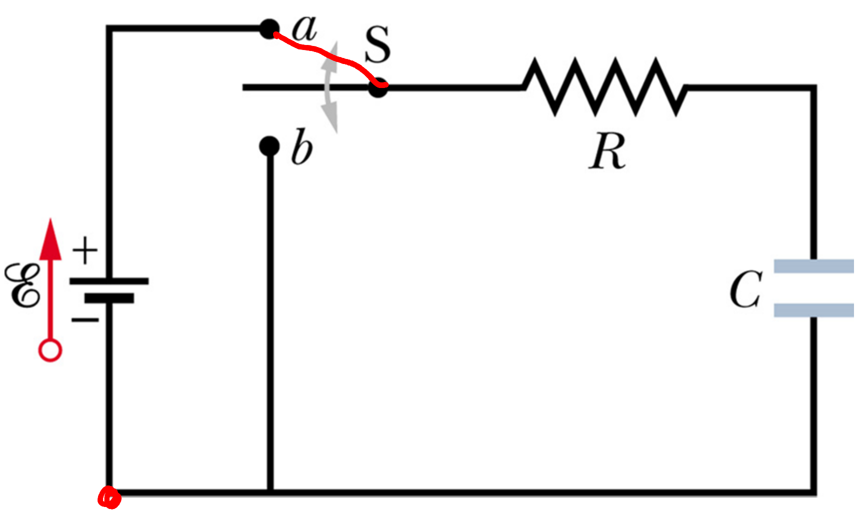
solve for $I_1 = 1.5A$

so $I_2 = \frac{12A - 7(1.5A)}{3} = 0.5A$

$I = I_1 + I_2 = 2A$

Ideal vs. real meters





$q = 0 \quad @ \; t = 0$

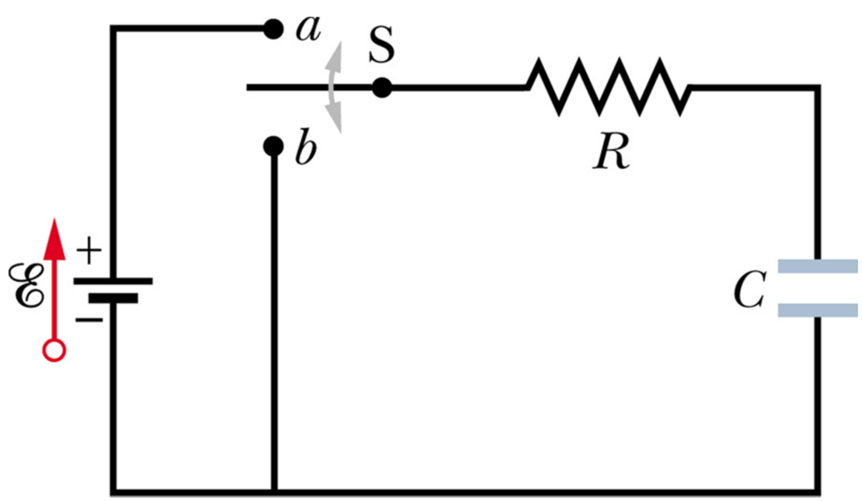
$V_c = q/C$

KLR: $+\mathcal{E} - IR - q/C = 0$

$$\mathcal{E} - \frac{dq}{dt}R - q/C = 0$$

$$R\frac{dq}{dt} + q/C = \mathcal{E}$$

$$\frac{dq}{dt} + \frac{q}{RC} = \mathcal{E}/R$$



$$\frac{dq}{dt} + \frac{q}{RC} = \mathcal{E}/R$$

Soln: $q(t) = q_{Final} + Ke^{-\alpha t}$

q_{Final}: @ $t=\infty$ $V_c = \mathcal{E}$

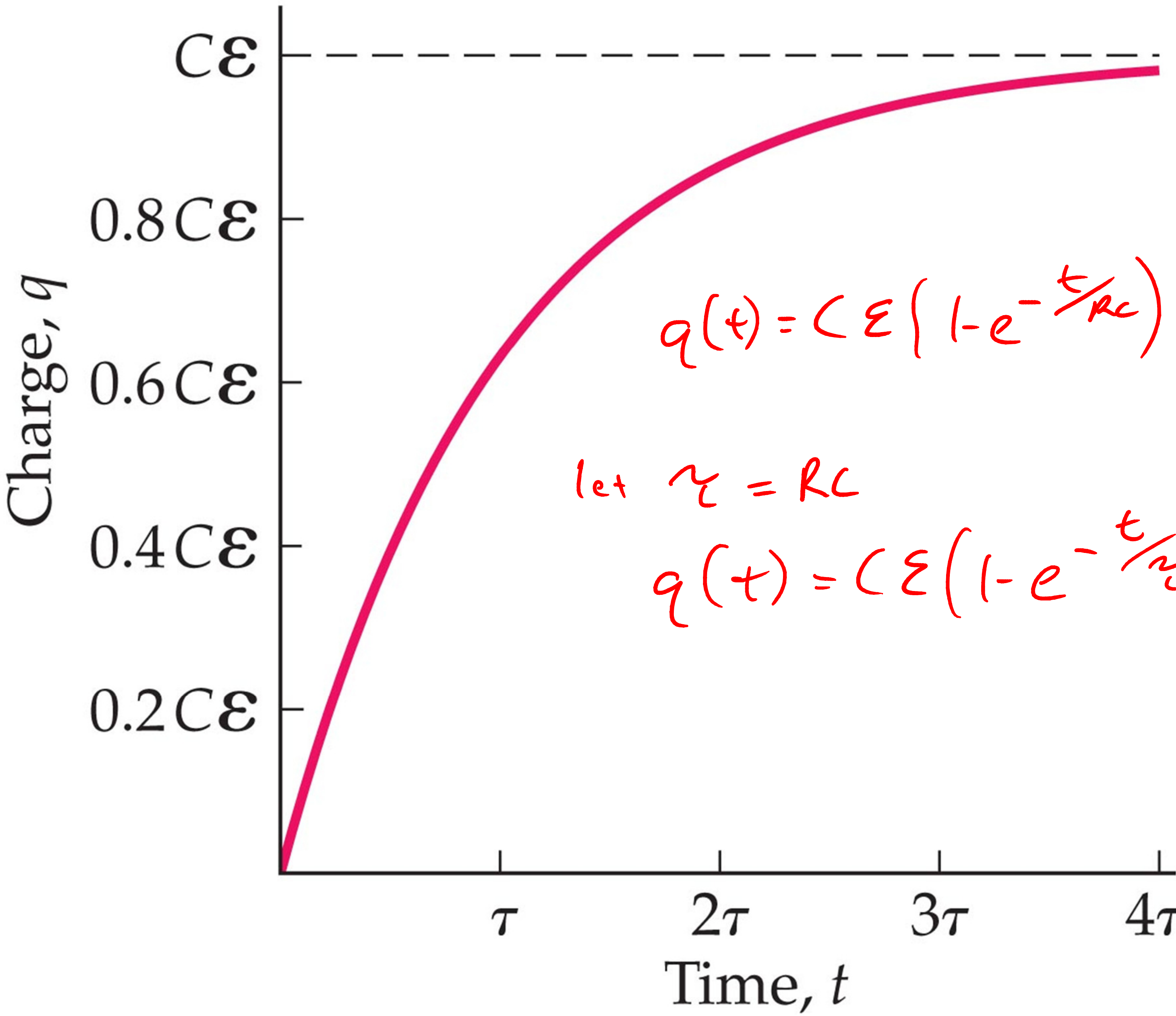
$$q(\infty) = q_{final} + \cancel{e^{-\infty}}^{0}$$

$$0 + \frac{q_f}{RC} = \mathcal{E}/R \Rightarrow q_f = C\mathcal{E}$$

@ $t=0$, $q=0$ $\quad q(0) = 0 = q_f + K\cancel{e^{-0}}^{1}$ so $K = -C\mathcal{E}$

$$q(t) = C\mathcal{E} - C\mathcal{E}e^{-t/RC}$$

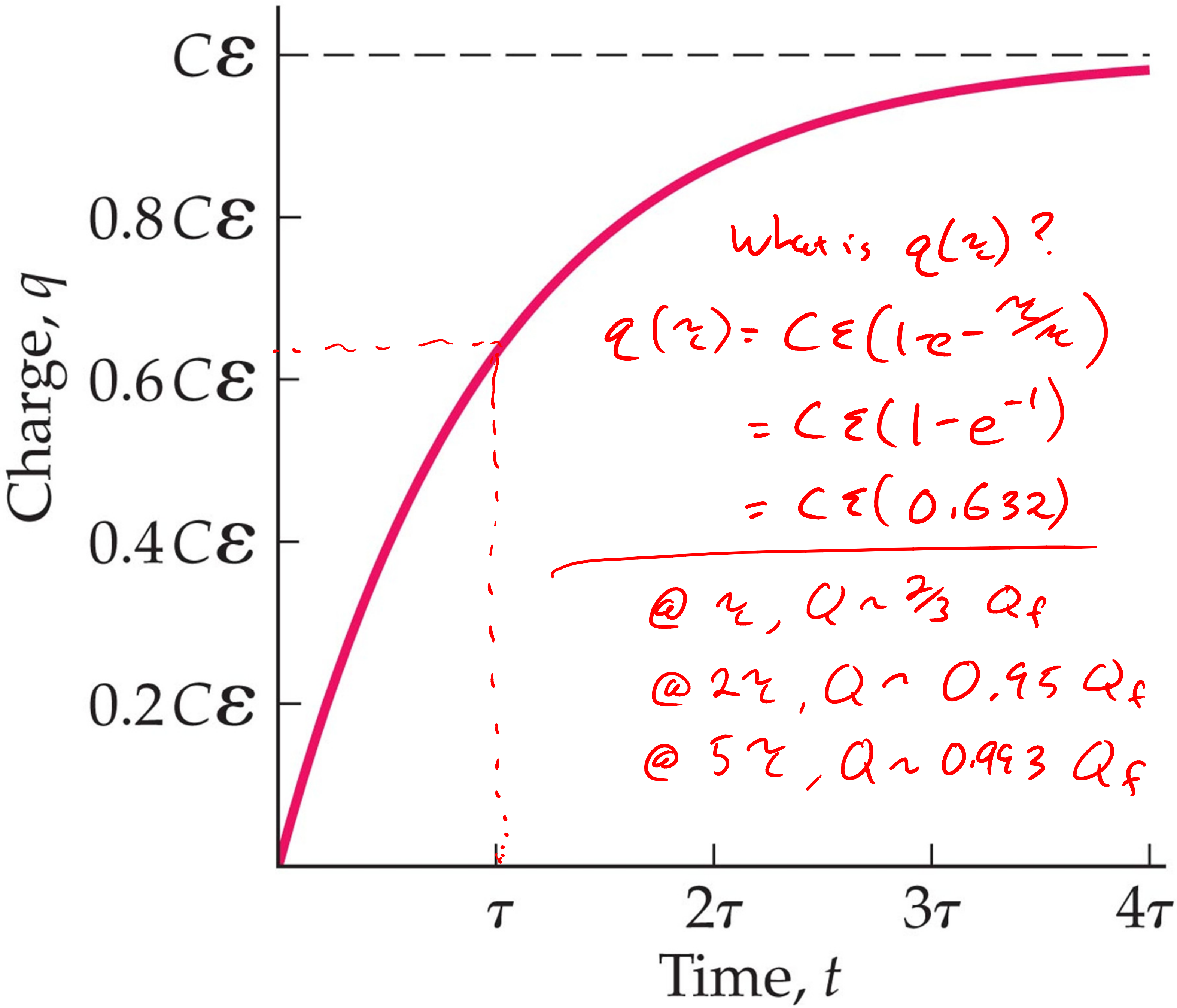
$$\boxed{q(t) = C\mathcal{E}(1 - e^{-t/RC})}$$



$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

let $\tau = RC$

$$q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$



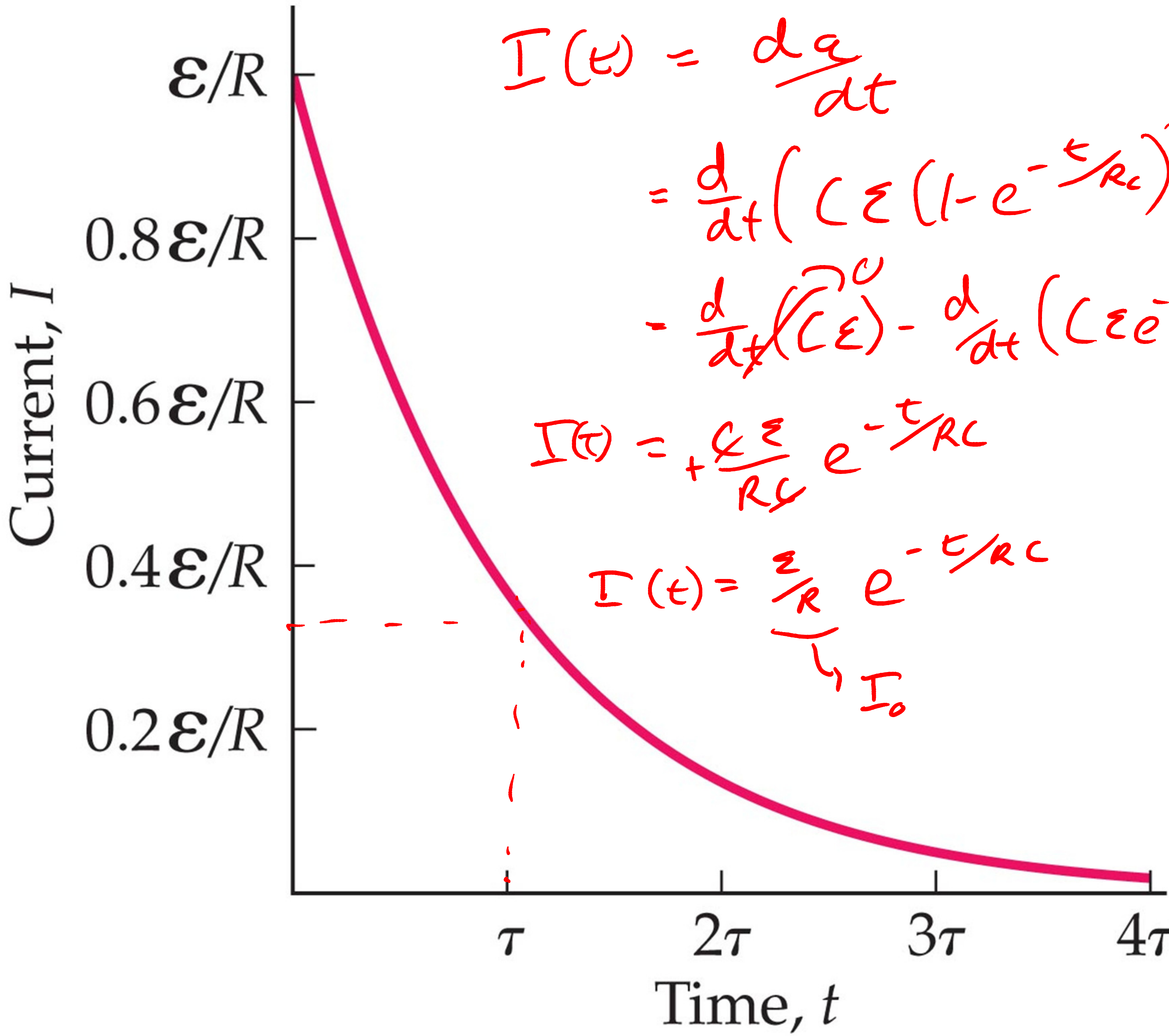
What is $q(\tau)$?

$$q(\tau) = C\mathcal{E}(1 - e^{-\tau/\tau})$$
$$= C\mathcal{E}(1 - e^{-1})$$
$$= C\mathcal{E}(0.632)$$

@ τ, $Q \sim 2/3\ Q_f$

@ 2τ, $Q \sim 0.95\ Q_f$

@ 5τ, $Q \sim 0.993\ Q_f$



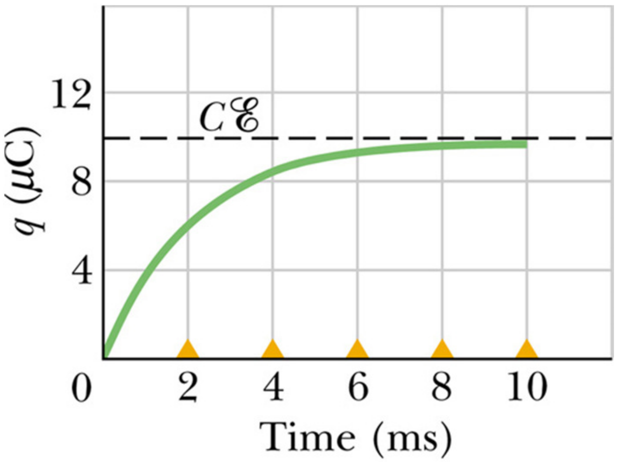
$$I(t) = \frac{dq}{dt}$$

$$= \frac{d}{dt}\left(C\varepsilon\left(1 - e^{-t/RC}\right)\right)$$

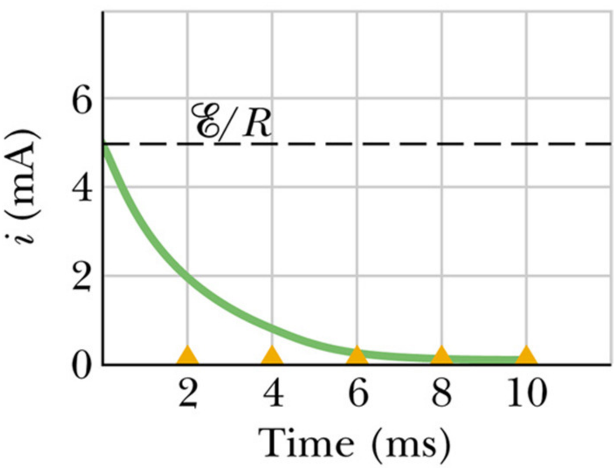
$$= \underbrace{\frac{d}{dt}(C\varepsilon)}_{0} - \frac{d}{dt}\left(C\varepsilon e^{-t/R}\right)$$

$$I(t) = +\frac{\cancel{C}\varepsilon}{R\cancel{C}} e^{-t/RC}$$

$$I(t) = \underbrace{\frac{\varepsilon}{R}}_{I_0} e^{-t/RC}$$

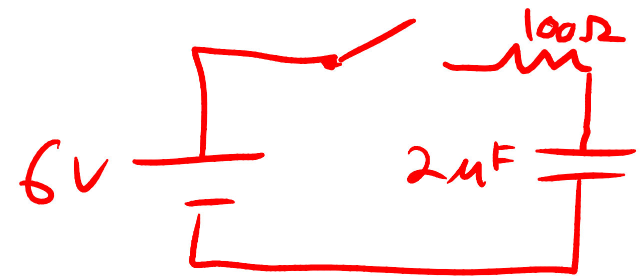

12
8
4
0
$C\mathscr{E}$
q (μC)
2
4
6
8
10
Time (ms)

(a)


6
4
2
0
$\mathscr{E}/R$
i (mA)
2
4
6
8
10
Time (ms)

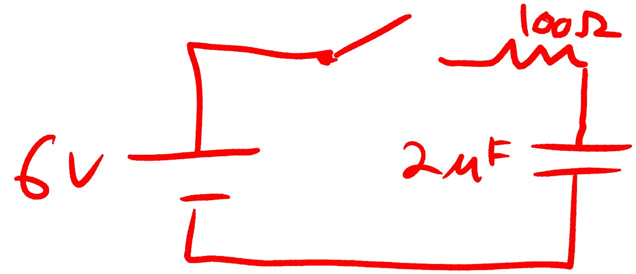
(b)

Charging example:



a) What is initial current?

Charging example:

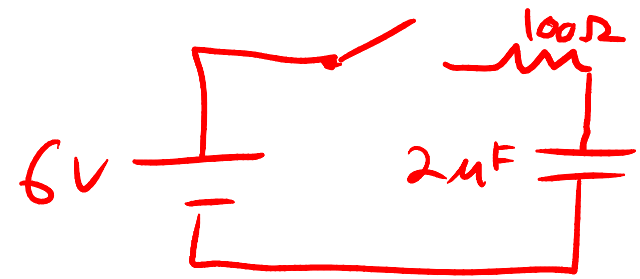


a) What is initial current?

$$I(t) = \mathcal{E}/R \, e^{-t/\tau}$$

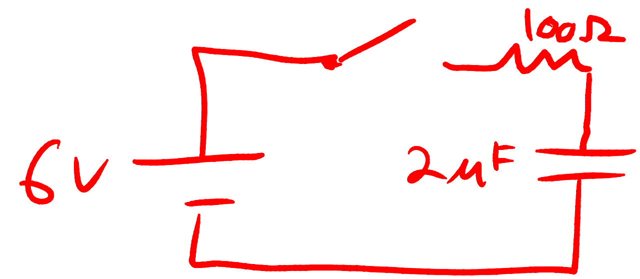
$$I(0) = \mathcal{E}/R \, \cancel{e^{-0}}^{\,1} = \mathcal{E}/R = \frac{6\,V}{100\,\Omega} = 0.06\,A$$

Charging example:



b) What is final charge?

Charging example:



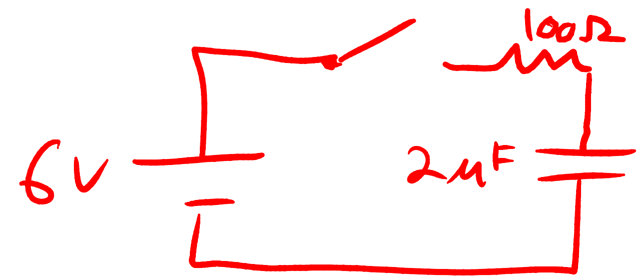
b) What is final charge?

$$q(t) = C\varepsilon\left(1 - e^{t/\tau}\right)$$

$$q(\infty) = C\varepsilon\left(1 - \underbrace{\cancel{e^{-\infty}}}_{\to 0}\right) = C\varepsilon = (2\times10^{-6}\,F)(6V)$$

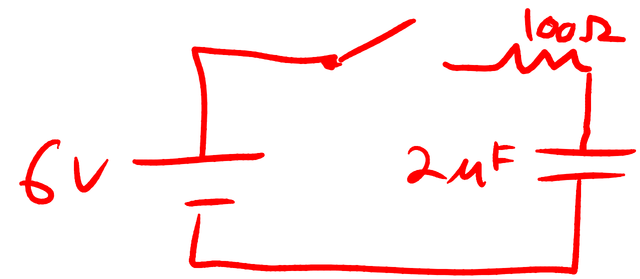
$$= 12\,\mu C$$

Charging example:



c) How long to get to 90% of full charge?

Charging example:



c) How long to get to 90% of full charge?

$$q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$Q_F = C\mathcal{E}$$

$$0.9\,Q_F = 0.9(\cancel{C\mathcal{E}}) = \cancel{C\mathcal{E}}\left(1 - e^{-t/\tau}\right)$$

$$0.9 = 1 - e^{-t/\tau}$$

$$-0.1 = -e^{-t/\tau}$$

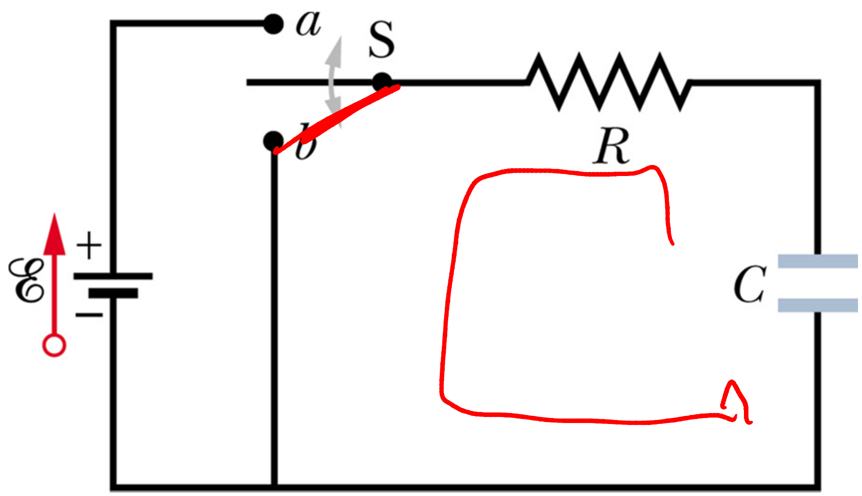
$$0.1 = e^{-t/\tau}$$

$$\ln(0.1) = -t/\tau$$

$$t = \tau \ln(0.1) = RC \ln(0.1)$$

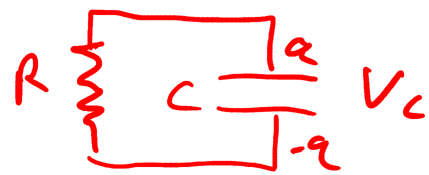
$$= \ln(0.1)(100\,\Omega)(2\times10^{-6}\,F) = 460\,\mu s$$

Discharging a capacitor



Charged up to q, has V_c

move switch to b



$I = V_c / R \qquad q = C V_c$

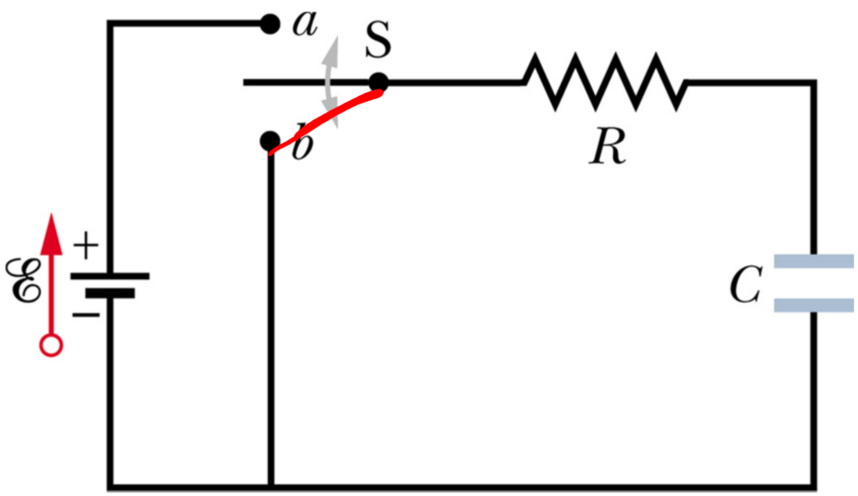
Loop rule: $-IR - q/C = 0$

$$R \frac{dq}{dt} + q/C = 0$$

$$q(t) = q_0 e^{-t/RC}$$

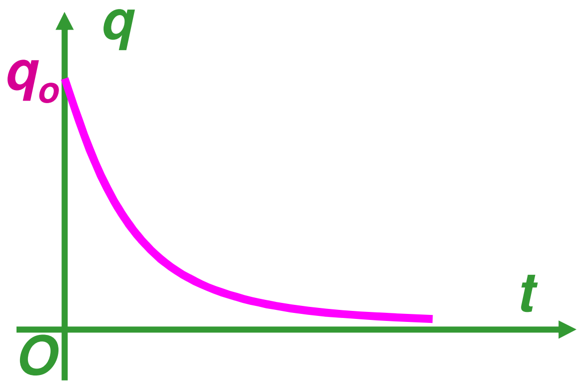
$q_0 = C V_0$ = initial charge

Current?



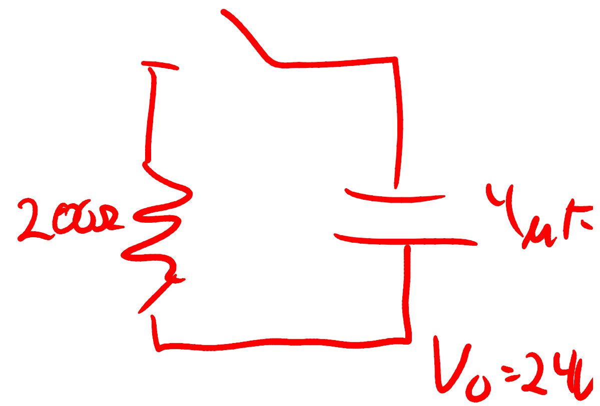
$$I = \frac{dq}{dt} = \frac{d}{dt}\left(q_0 e^{-t/RC}\right)$$

$$I(t) = \underbrace{-\frac{q_0}{RC}}_{\hookrightarrow I_0} e^{-t/RC}$$

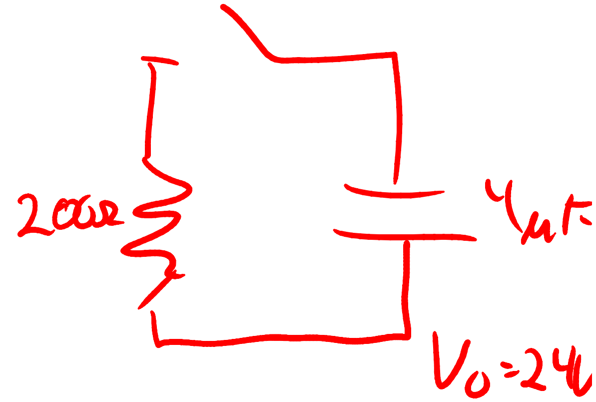

q
q_o
t
O

Discharging example:

a) What is initial charge?



Discharging example:

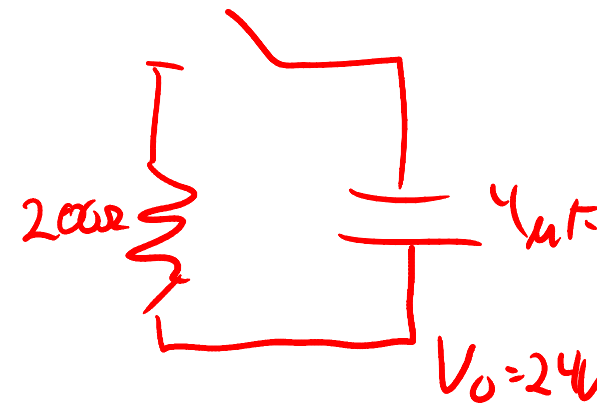


a) What is initial charge?

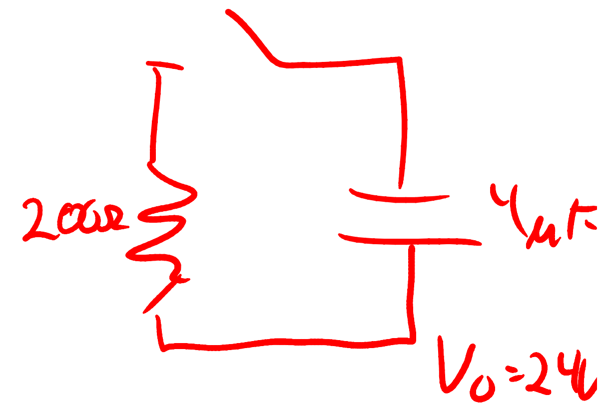
$$Q = CV = (4 \times 10^{-6}\,F)(24\,V)$$
$$= 96\,\mu C$$

Discharging example:

b) What is initial current?



Discharging example:

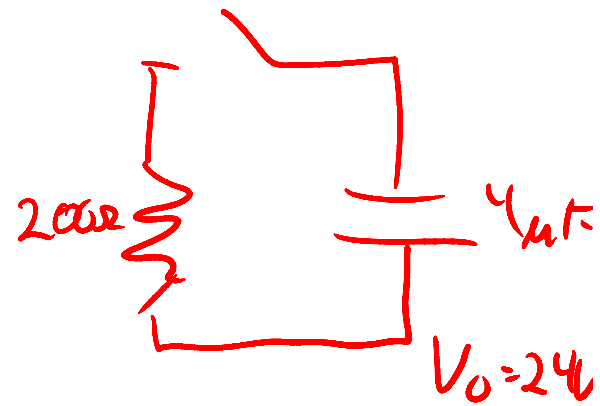


b) What is initial current?

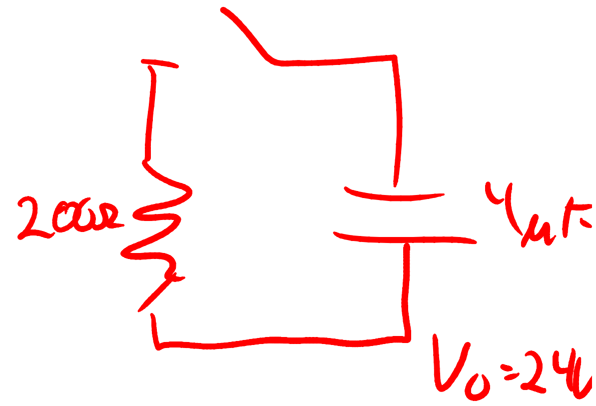
$$I_0 = V_0/R = \frac{24V}{200\Omega} = 0.12A$$

Discharging example:

c) What is time constant?



Discharging example:



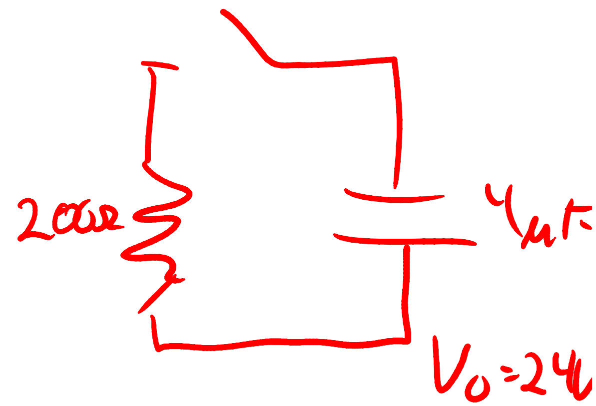
c) What is time constant?

$$\tau = RC = (200\,\Omega)(4\,\mu F)$$

$$= 0.8\ ms$$

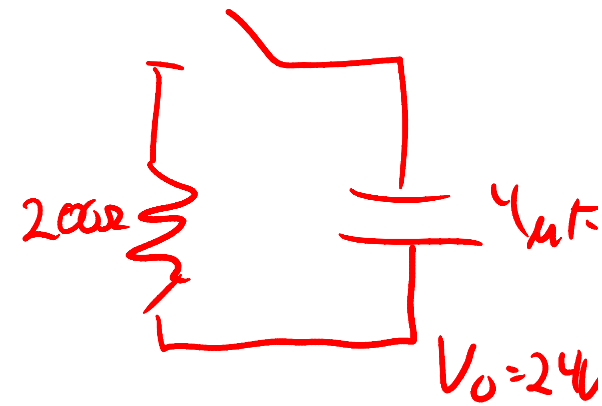
Discharging example:

d) What is charge at t=4ms?



Discharging example:

d) What is charge at t=4ms?



$$q(4ms) = Q_0 e^{-\frac{4ms}{0.8ms}}$$

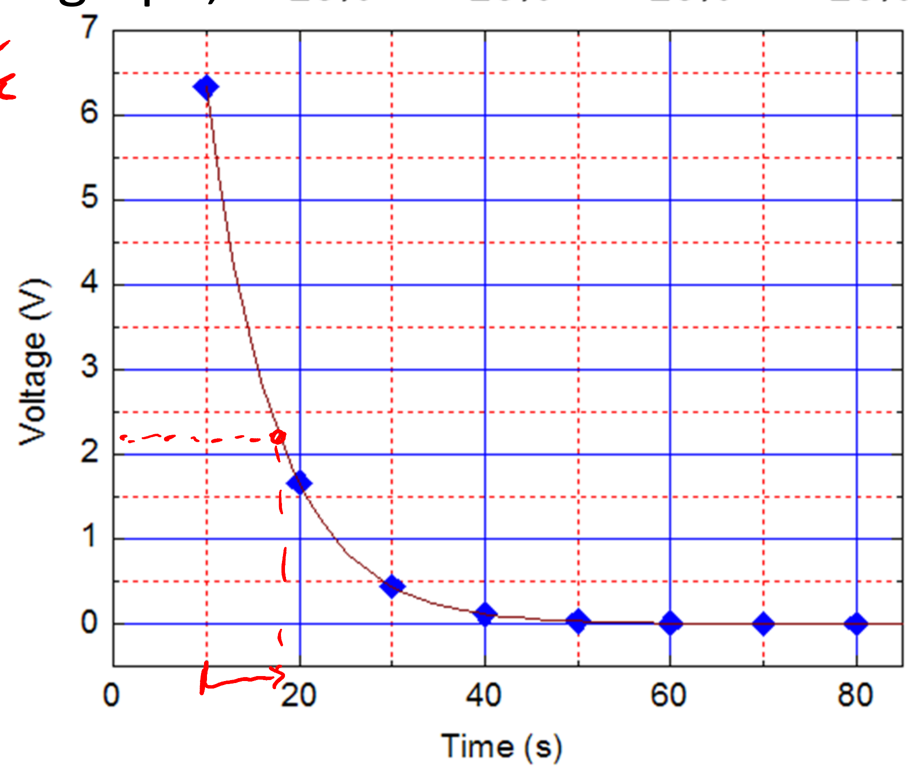
$$= 96\,\mu C\, e^{-5}$$

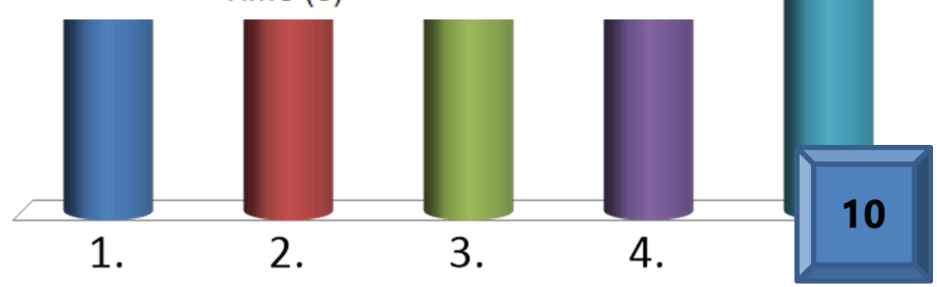
$$= 0.647\,\mu C$$

Voltage across an unknown capacitor in an RC circuit, every ten seconds after a switch in the circuit that allows the capacitor to discharge is closed. The capacitor was initially fully charged. Using the graph, estimate the time constant.

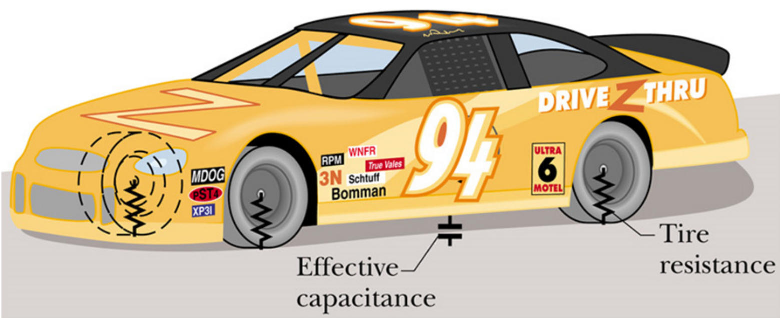
1. 7.5s
2. 15s
3. 30s
4. 45s
5. 60s

$Q(t) = Q_0 e^{-t/\tau}$





Response Counter

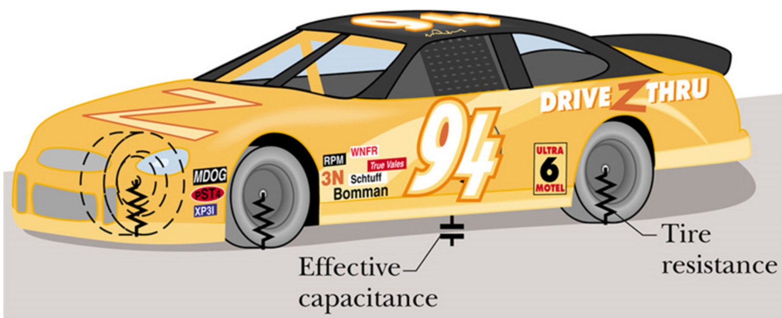


(*a*)

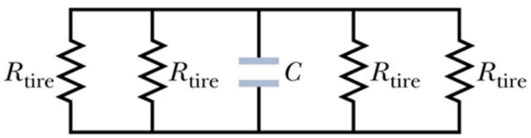
$C_{car/ground} = 500\,pF \quad V_0 = 30\,kV$

$R_{Tire} = 100\,G\Omega \quad U_{fire} = 50\,mJ$

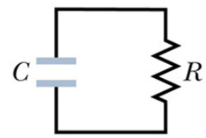
How long till you can safely refuel, till the energy stored in the static electricity won't ignite the gas?



(a)

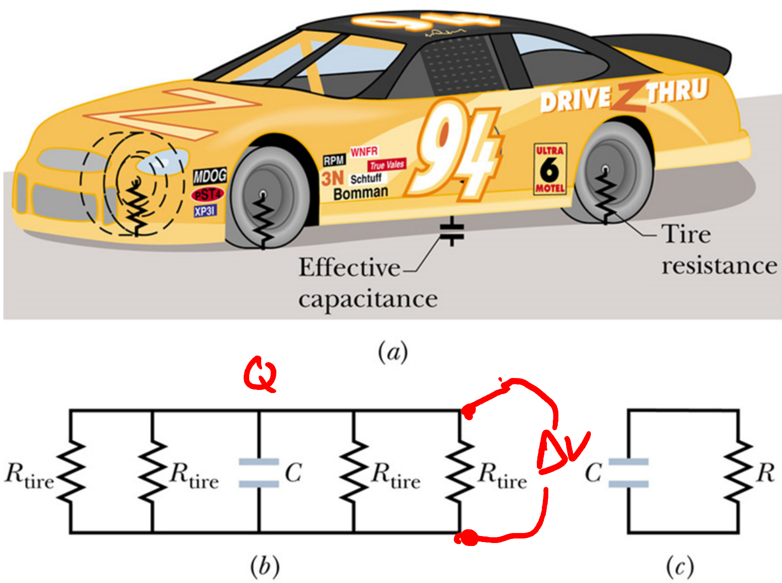


(b)



(c)

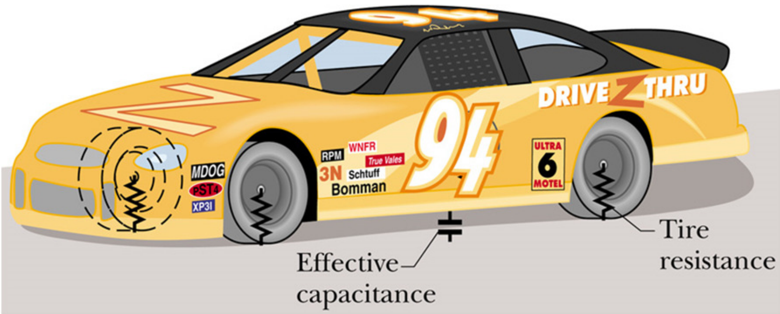
First, draw it as a circuit & simplify it



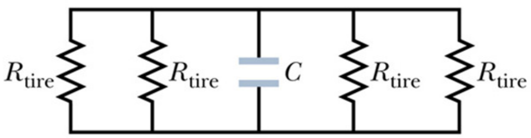
First, draw it as a circuit & simplify it

4 tires in parallel

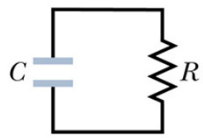
$$\frac{1}{R_{eq}} = 4\left(\frac{1}{R_{Tire}}\right) \qquad R_{eq} = 25\,G\Omega$$



(a)

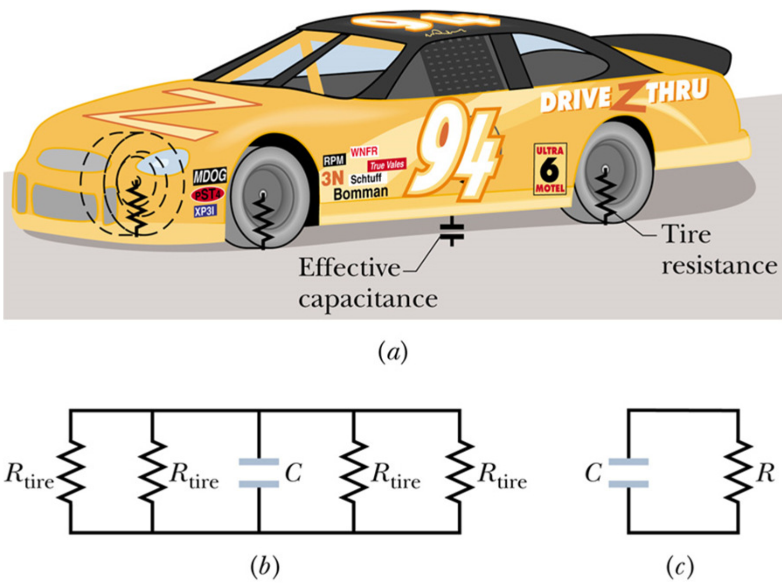


(b)



(c)

What's U_{cap}, and how does that look as a function of time?



What's U_{cap}, and how does that look as a function of time?

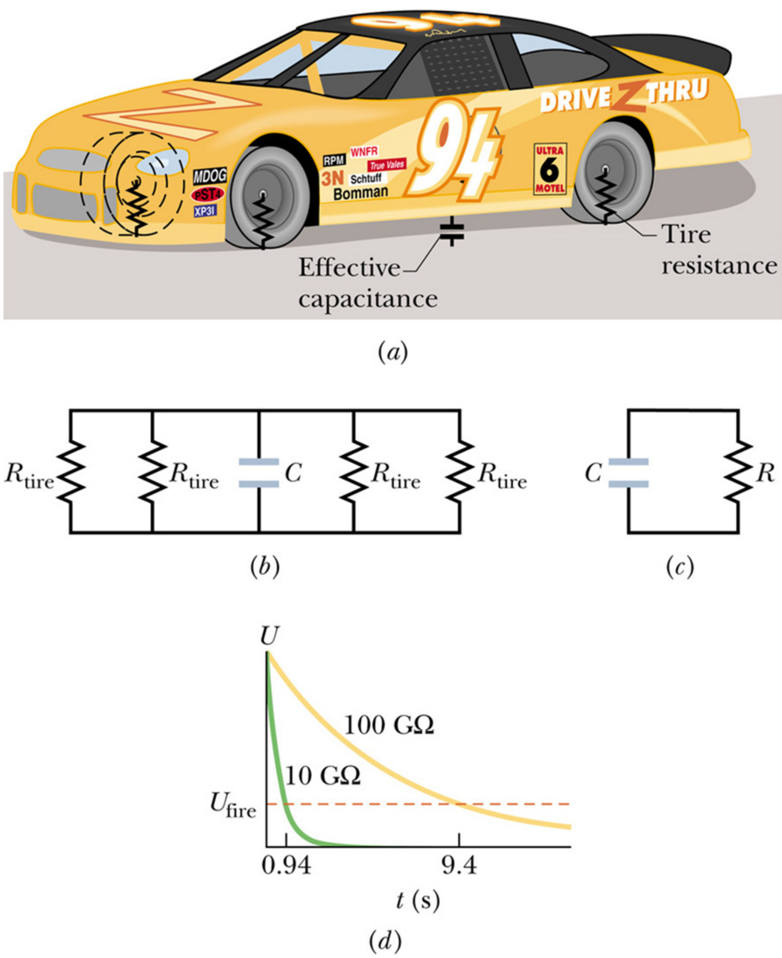
$$U = \frac{Q^2}{2C}$$

$$U(t) = \frac{Q^2(t)}{2C}$$

$$= \frac{Q_0^2}{2C} e^{-2t/RC}$$

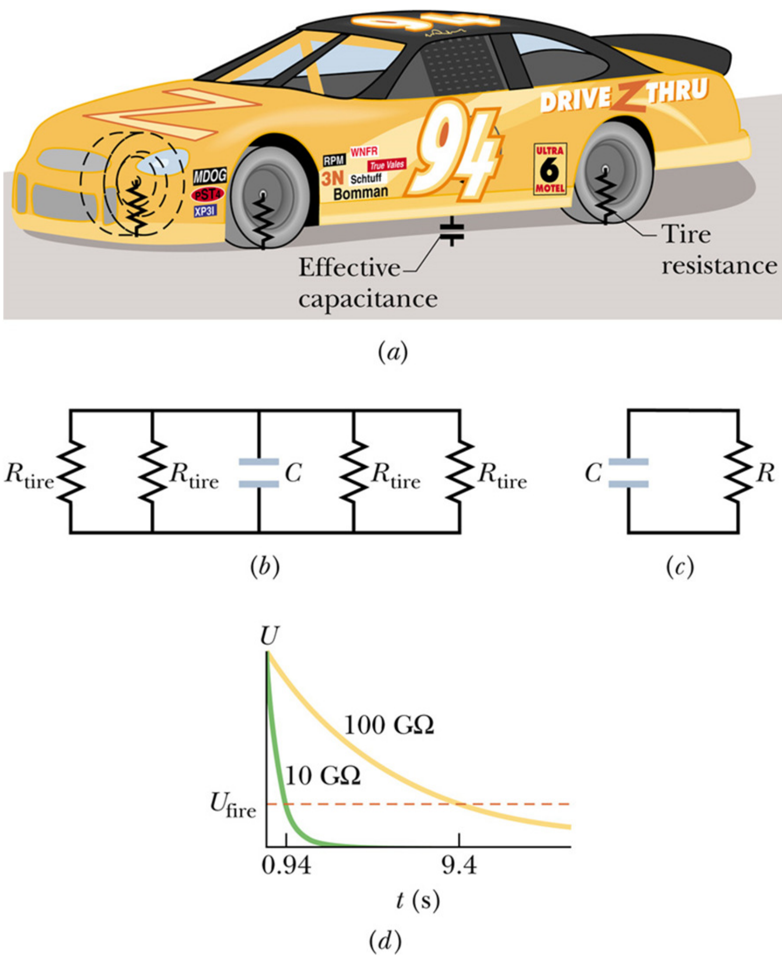
$$U(t) = \frac{(CV_0)^2}{2C} e^{-2t/RC}$$

$$= \frac{CV_0^2}{2} e^{-2t/RC}$$



Effective capacitance
Tire resistance
(a)
R_{tire}
C
(b)
C
R
(c)
U
100 GΩ
10 GΩ
U_{fire}
0.94
9.4
t (s)
(d)

Solve for t



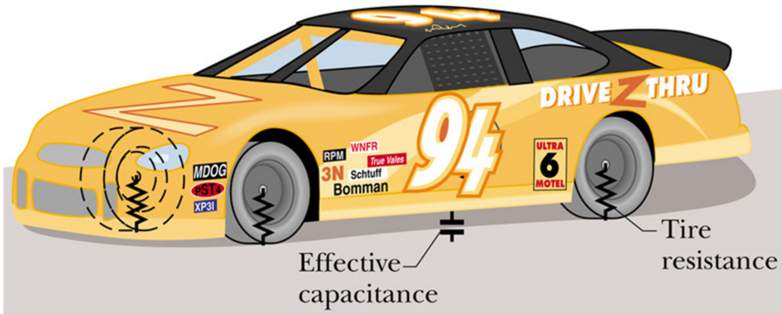
Solve for t

$$U_{exp} = \frac{CV_0^2}{2} e^{-2t/RC}$$

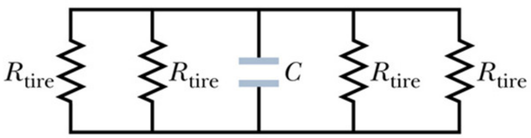
$$\ln\left(\frac{2U_{exp}}{CV_0^2}\right) = \frac{-2t}{RC}$$

$$t = -\frac{RC}{2} \ln\left(\frac{2U_{exp}}{CV_0^2}\right)$$

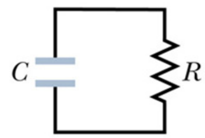
$$t = 9.4 \text{ s}$$



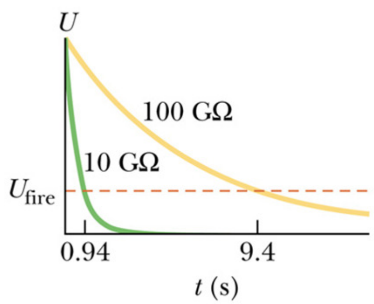
(a)



(b)

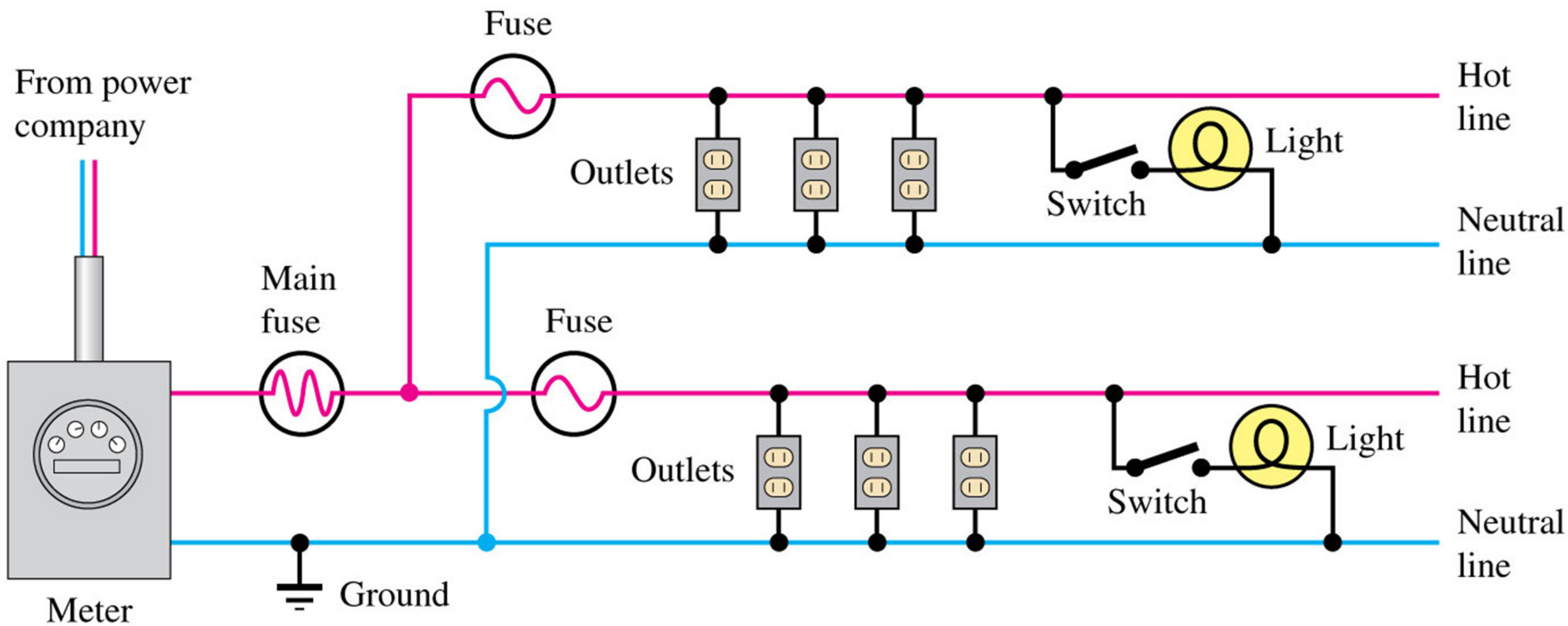


(c)



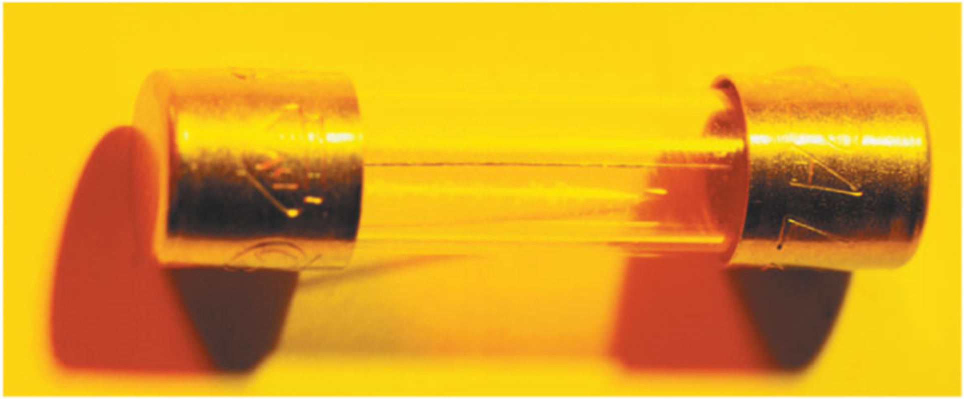
(d)

How to reduce that time, to allow for faster pit stops?

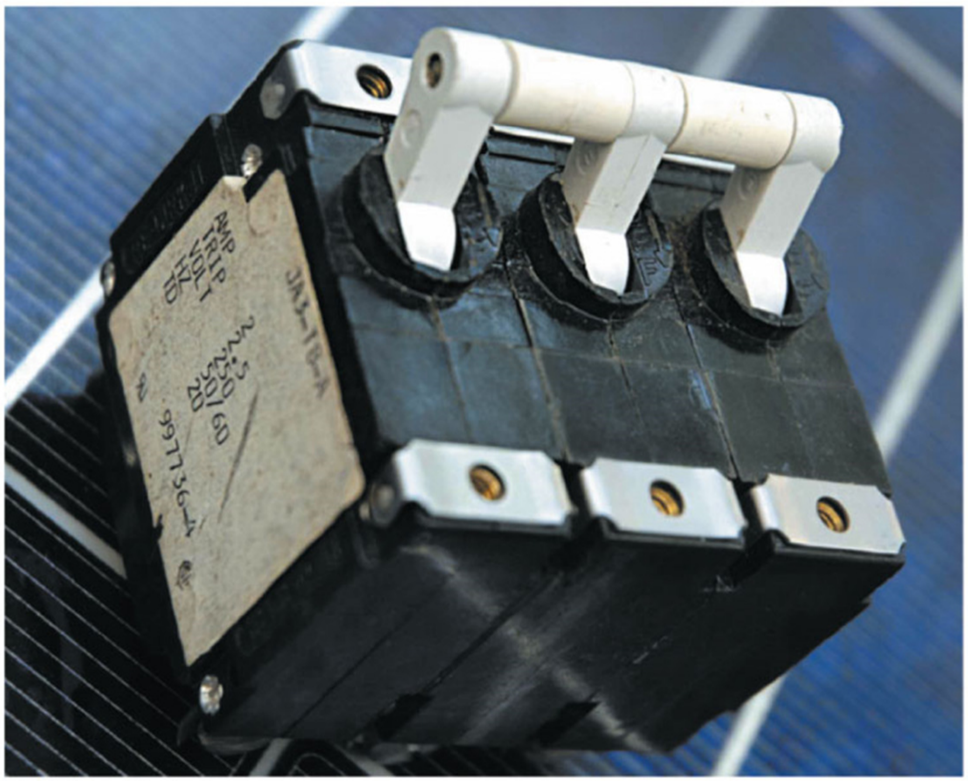


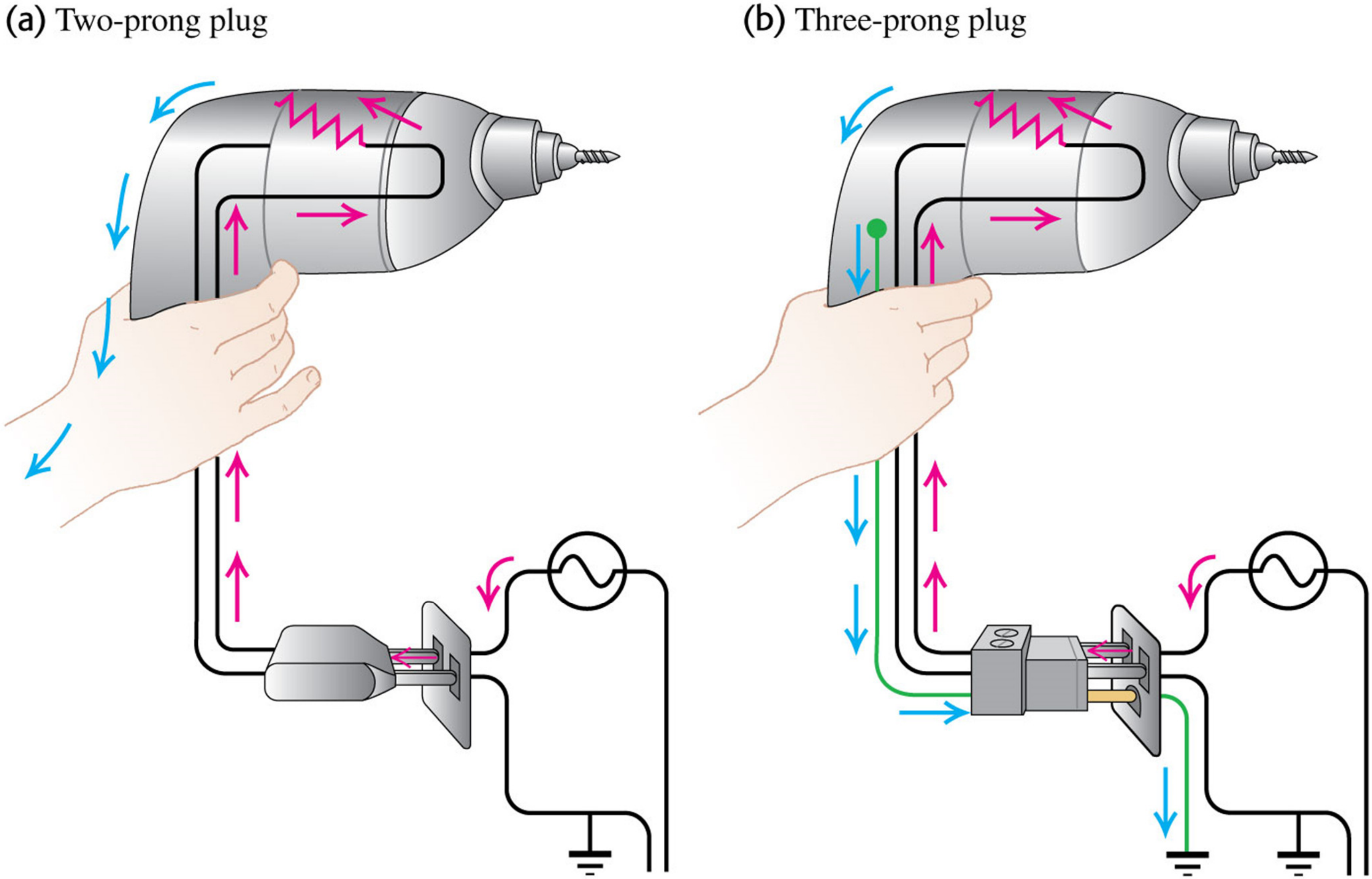
From power company
Main fuse
Fuse
Outlets
Switch
Light
Hot line
Neutral line
Fuse
Outlets
Switch
Light
Hot line
Neutral line
Meter
Ground

(a)



(b)




(a) Two-prong plug
(b) Three-prong plug