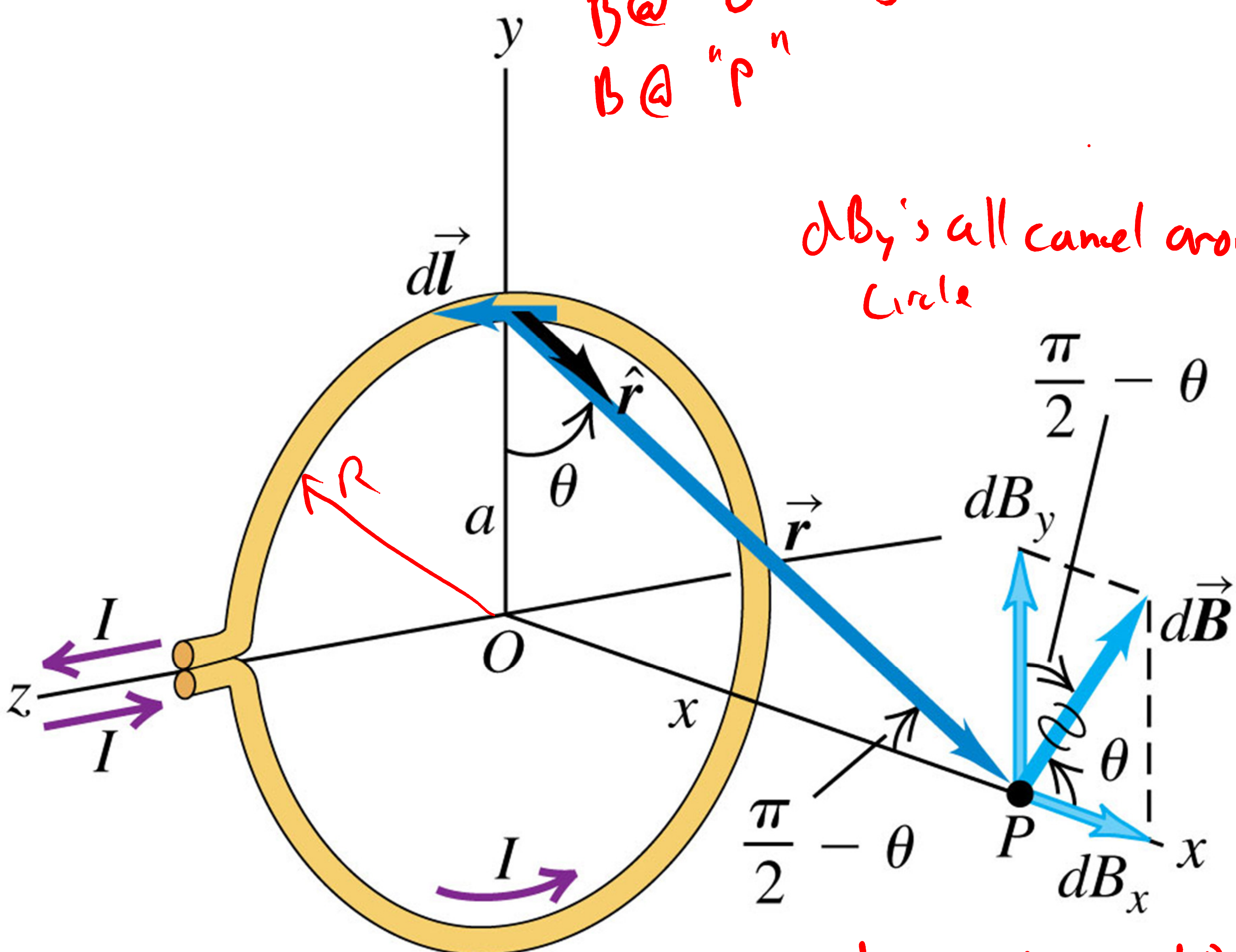
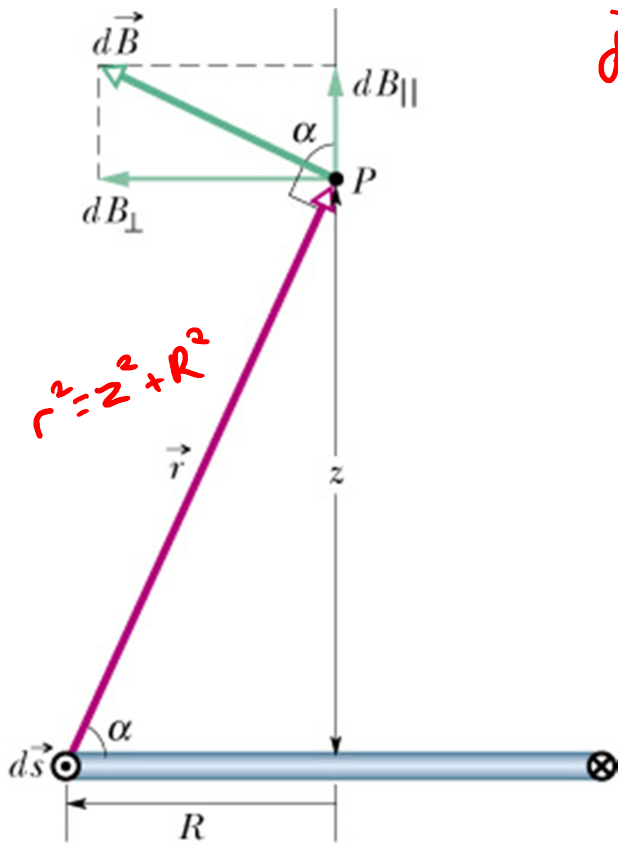
B@ "O"   "S" = 2πR
B@ "P"

dBy's all cancel around circle



need to add up dBx's



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

cross product gets you the $d\vec{B}$ shown.

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{I\, ds}{r^2}$$

we want $dB_{\parallel} = dB \cos\alpha$

use big triangle to find $\cos\alpha$

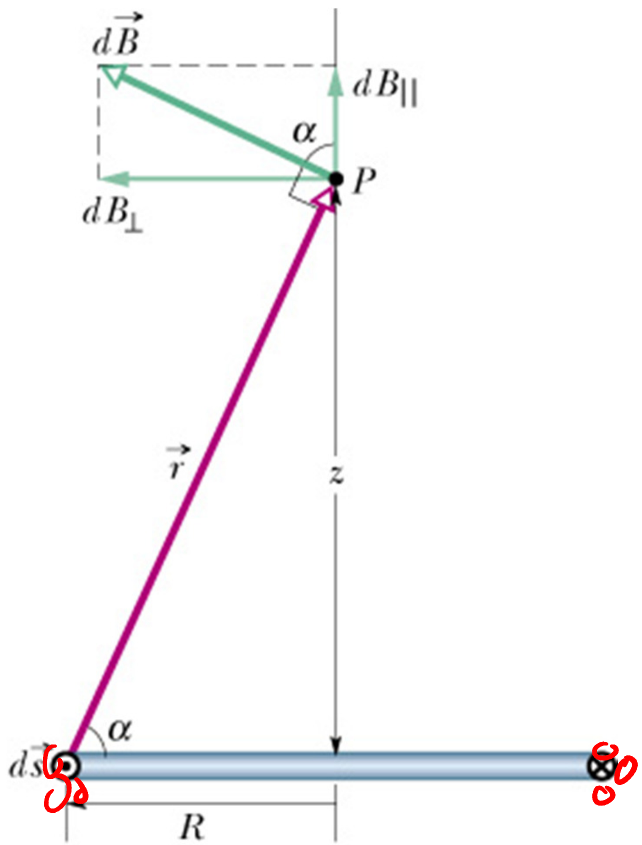
so $\cos\alpha = \frac{R}{r}$

$$dB_{\parallel} = \frac{\mu_0}{4\pi} \frac{I\, ds}{r^2} \frac{R}{r}$$

$$\int dB_{\parallel} = B_{\parallel} = \int \frac{\mu_0}{4\pi} \frac{I\, ds\, R}{(z^2+R^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi} \frac{I R}{(z^2+R^2)^{3/2}} \int ds \quad (\int ds = 2\pi R)$$

$$B_{\parallel} = \frac{\mu_0}{2} \frac{R^2 I}{(R^2+z^2)^{3/2}}$$



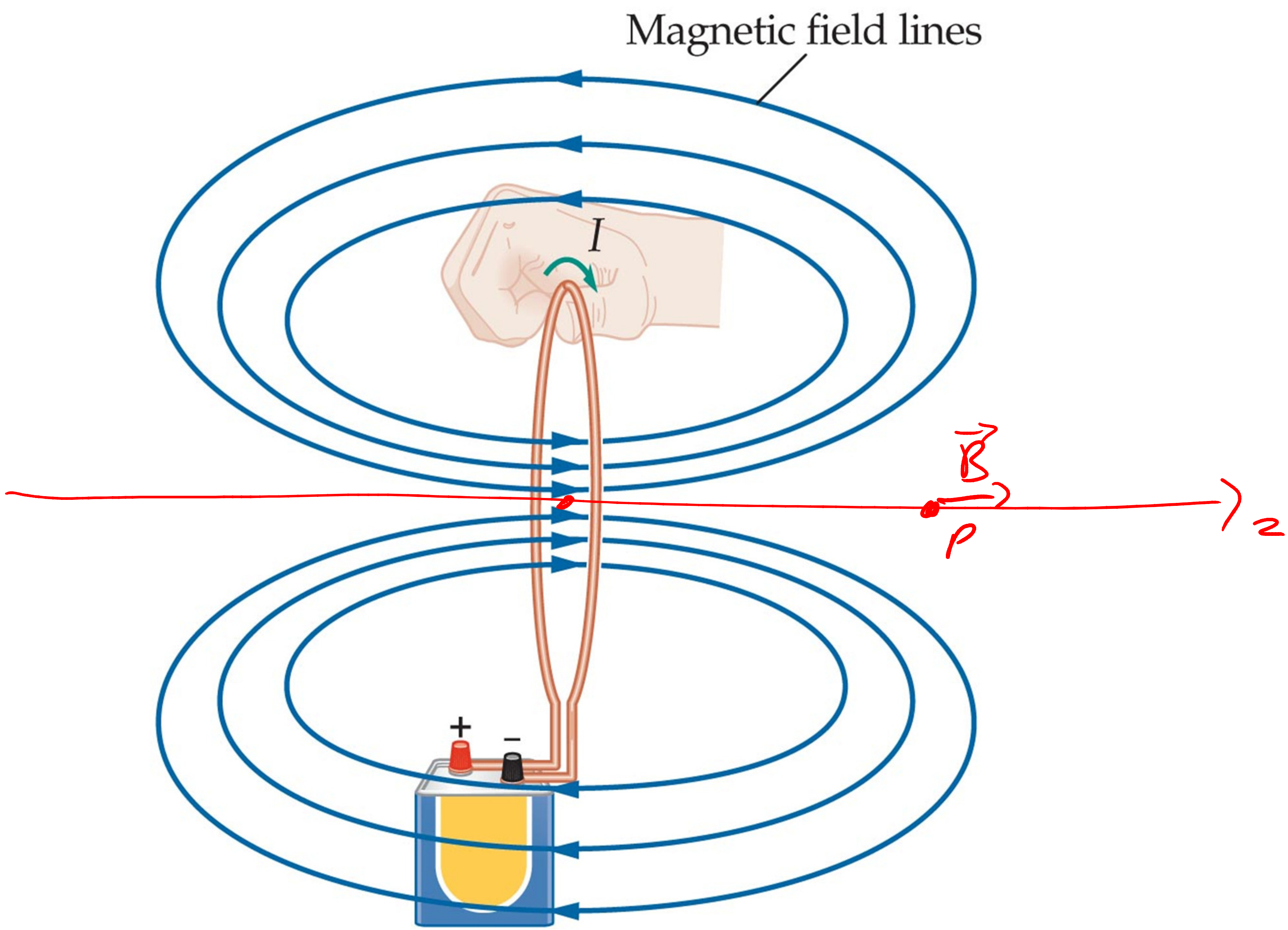
What if it was a coil?

N loops

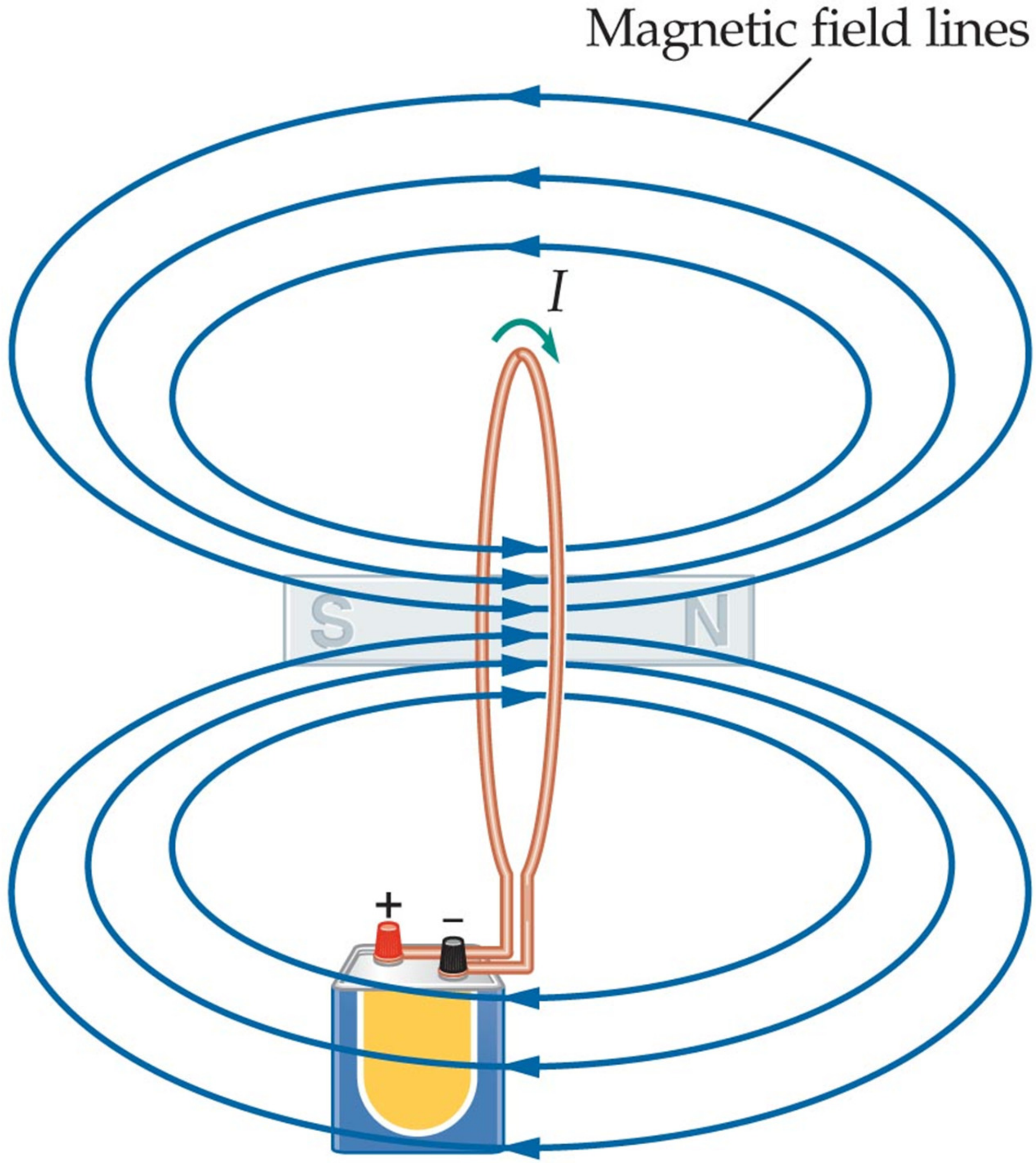
$$B_{||} = \frac{\mu_0}{2} \frac{R^2 I \quad N}{(R^2+z^2)^{3/2}}$$

What is B @ z = 0?

$$B_{||} = \frac{\mu_0 N I}{2R}$$

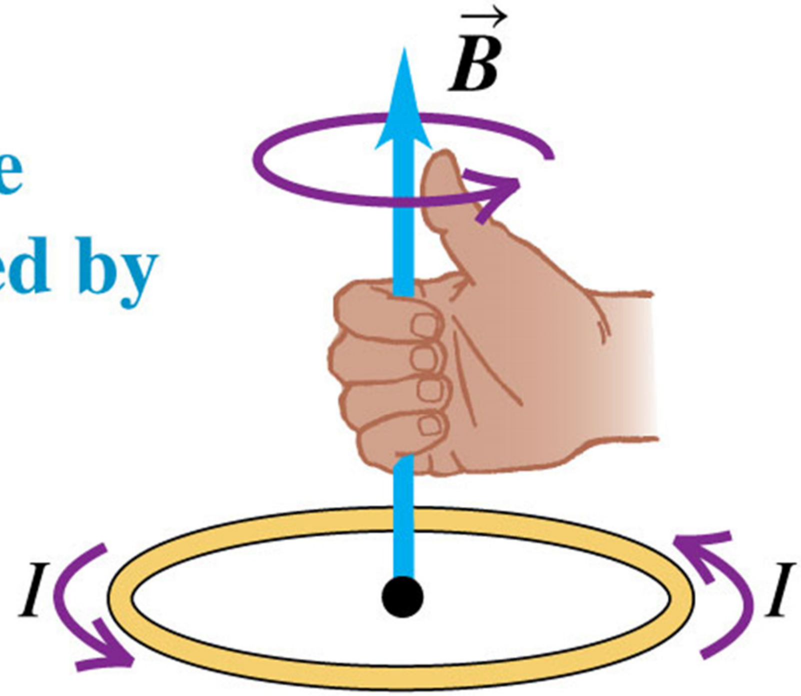


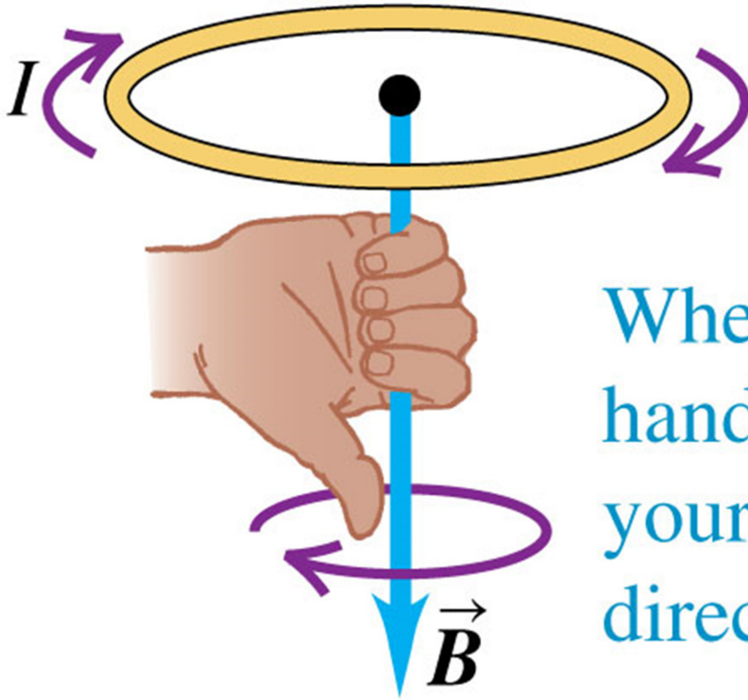
(a) Magnetic field of a current loop



(b) Magnetic field of bar magnet is similar

Right-hand rule for the magnetic field produced by a current in a loop:





When the fingers of your right hand curl in the direction of I, your right thumb points in the direction of $\vec{B}$.

The result for the ring of current looks like a dipole if the ring is really small

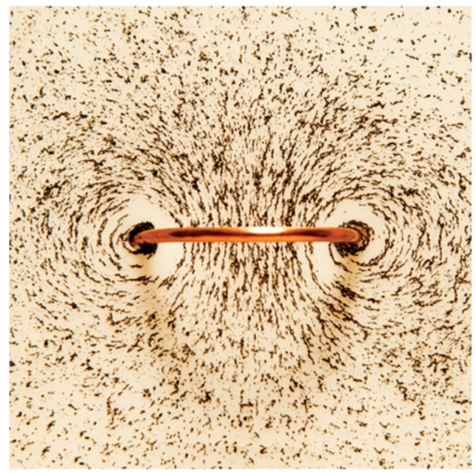
On axis:

$$\vec{B} = \frac{\mu_0 I R^2}{2\left(R^2 + y^2\right)^{3/2}}\hat{j}$$

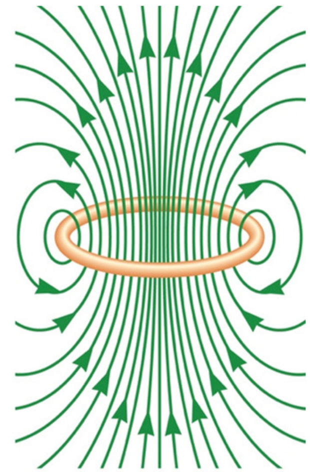
Special Case

For $R << y$,

$$\vec{B} \approx \frac{\mu_0 I R^2}{2\left(y^2\right)^{3/2}}\hat{j} = \frac{\mu_0 I R^2}{2y^3}\hat{j}$$



A.



B.

Define the magnetic dipole moment

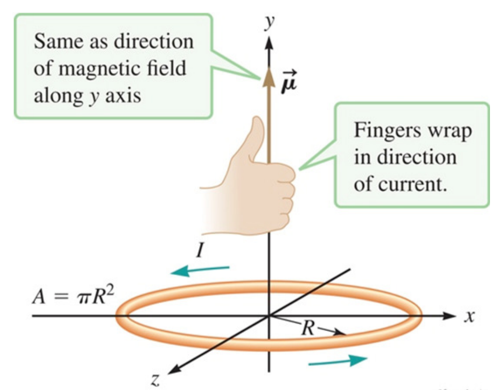
Then

$$\vec{B} \approx \left(\frac{\mu_0}{2\pi}\right)\frac{\vec{\mu}}{y^3}$$

$$\left|\vec{\mu}\right| \equiv IA$$

$$\vec{\mu} = I\pi R^2\hat{j}$$

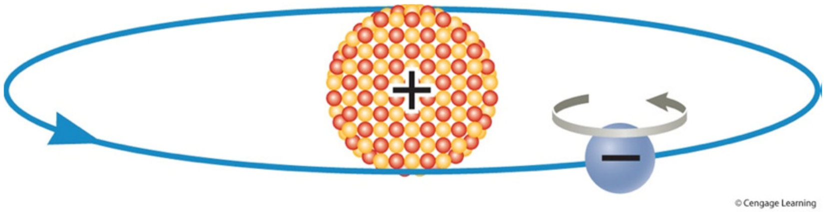
Right hand rule for direction



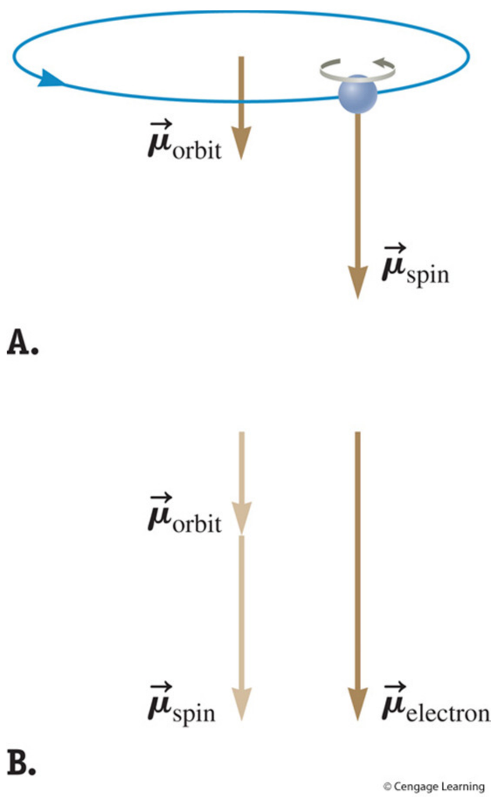
Similar to the electric dipole field,

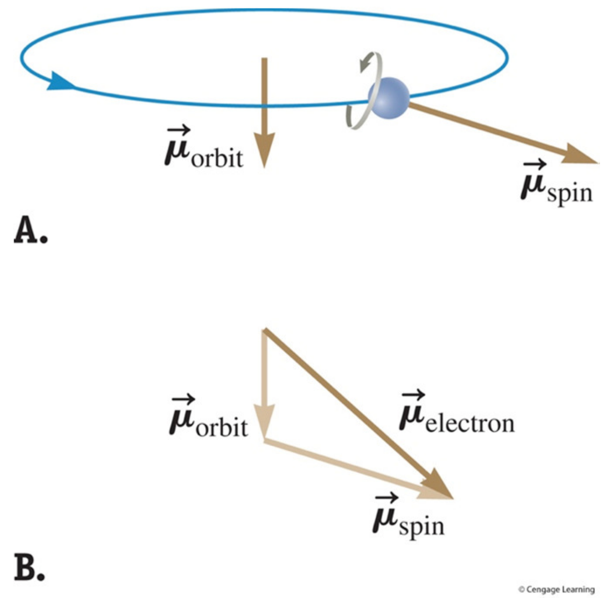
$$\vec{E} \approx \frac{2k\vec{p}}{y^3} = \left(\frac{1}{2\pi\varepsilon_0}\right)\frac{k\vec{p}}{y^3}$$

Dipoles are useful for modeling atoms



(Mental image, not to scale)





$\vec{E}$: a "force field" made by charge

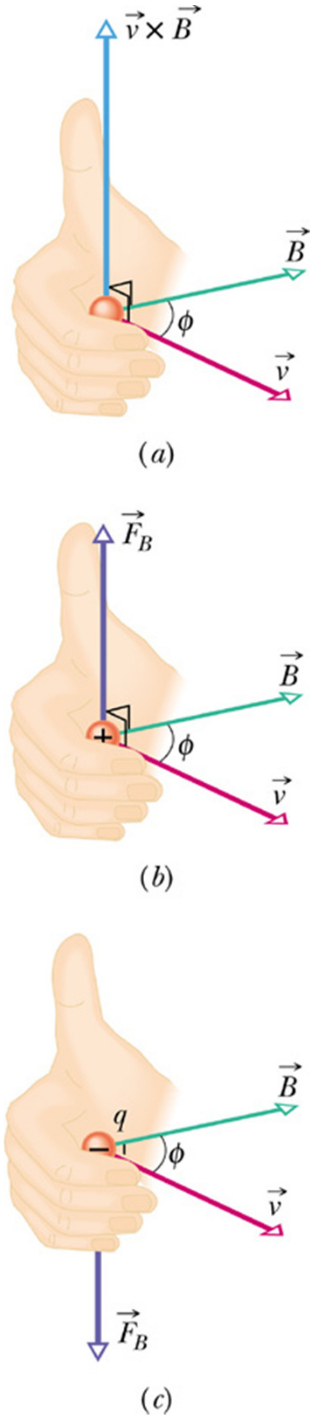
a map of the force some other charge would feel $\vec{F}_E = q\vec{E}$

$\vec{B}$: another force field made by moving charges (we have never found a stationary "magnetic charge")

charges moving through $\vec{B}$ feel a force

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

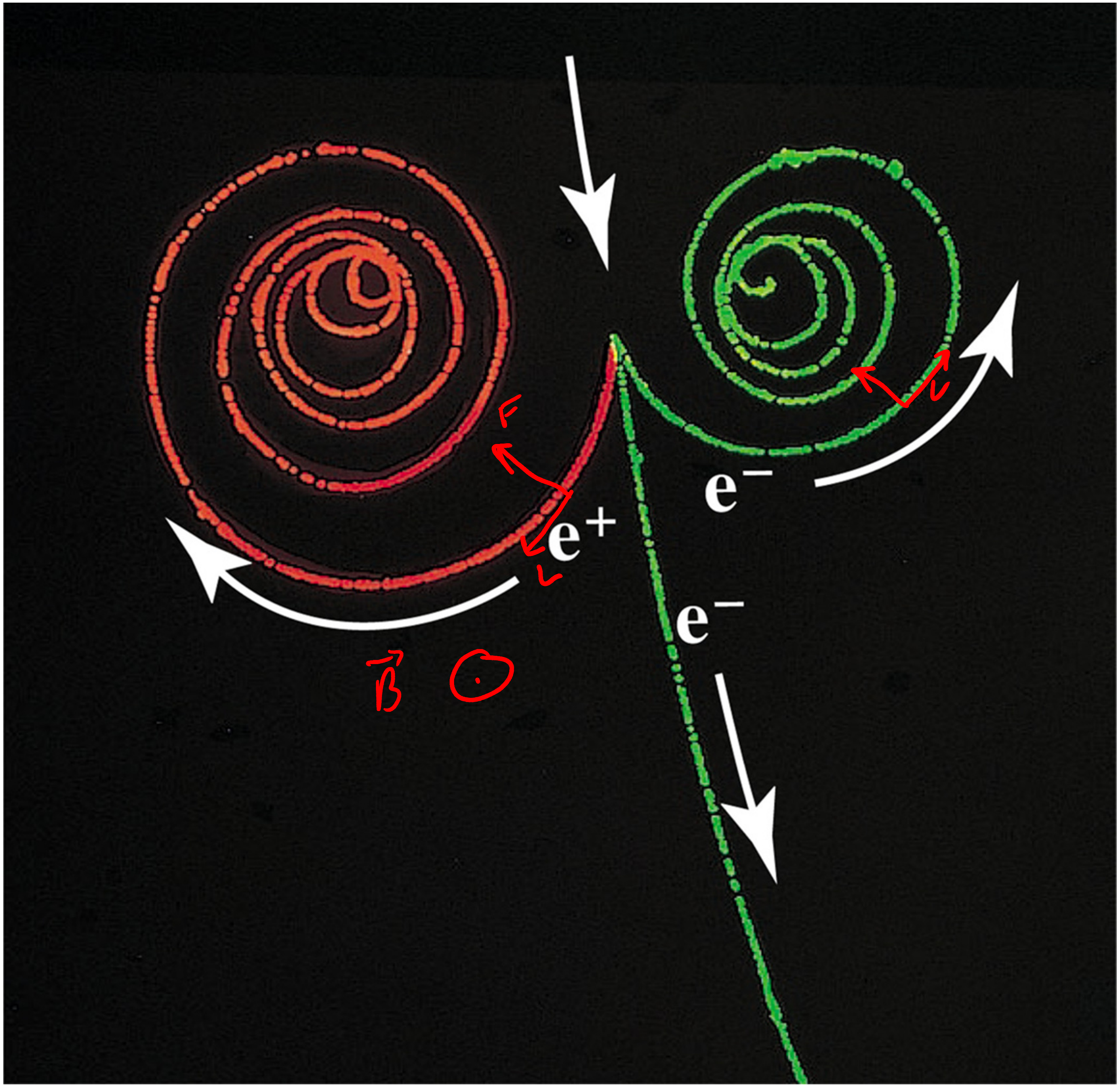
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

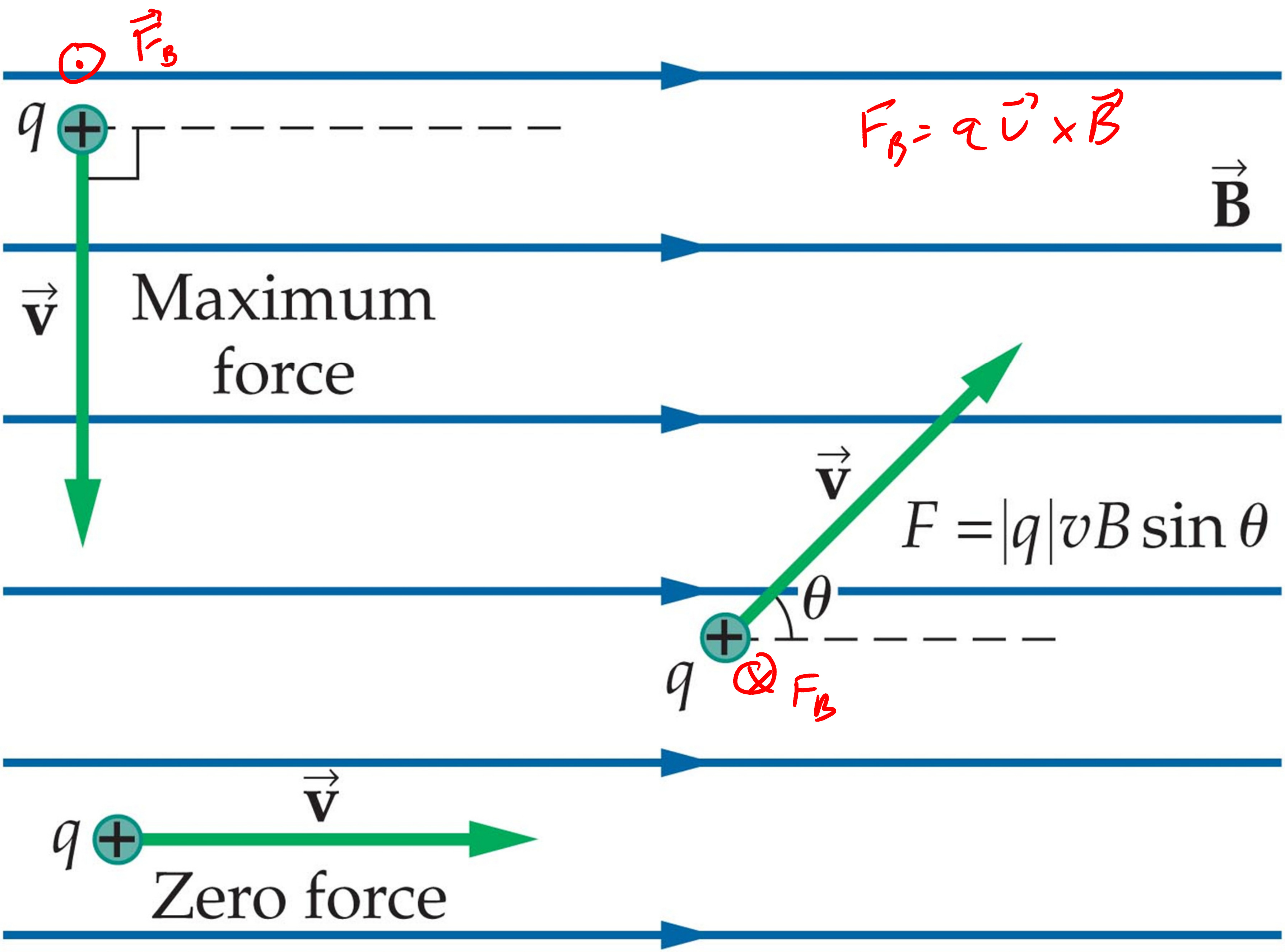


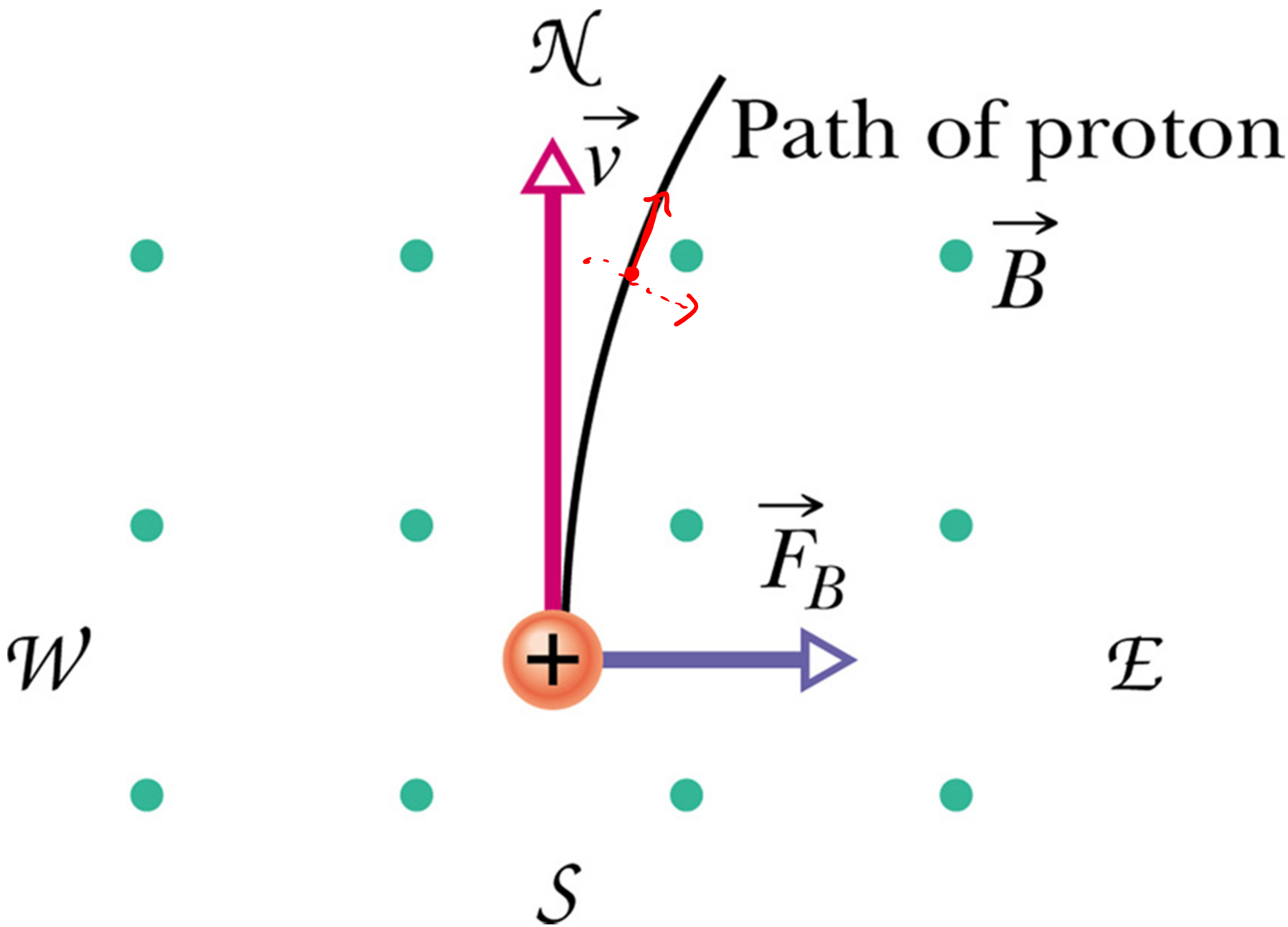
$$\vec{F}_B = q\vec{v} \times \vec{B} \quad \text{(Lorentz Force)}$$

$\vec{F}$ always $\perp$ to both $\vec{v}$, $\vec{B}$

$$|\vec{F}_B| = q|\vec{v}||\vec{B}| \sin\theta$$

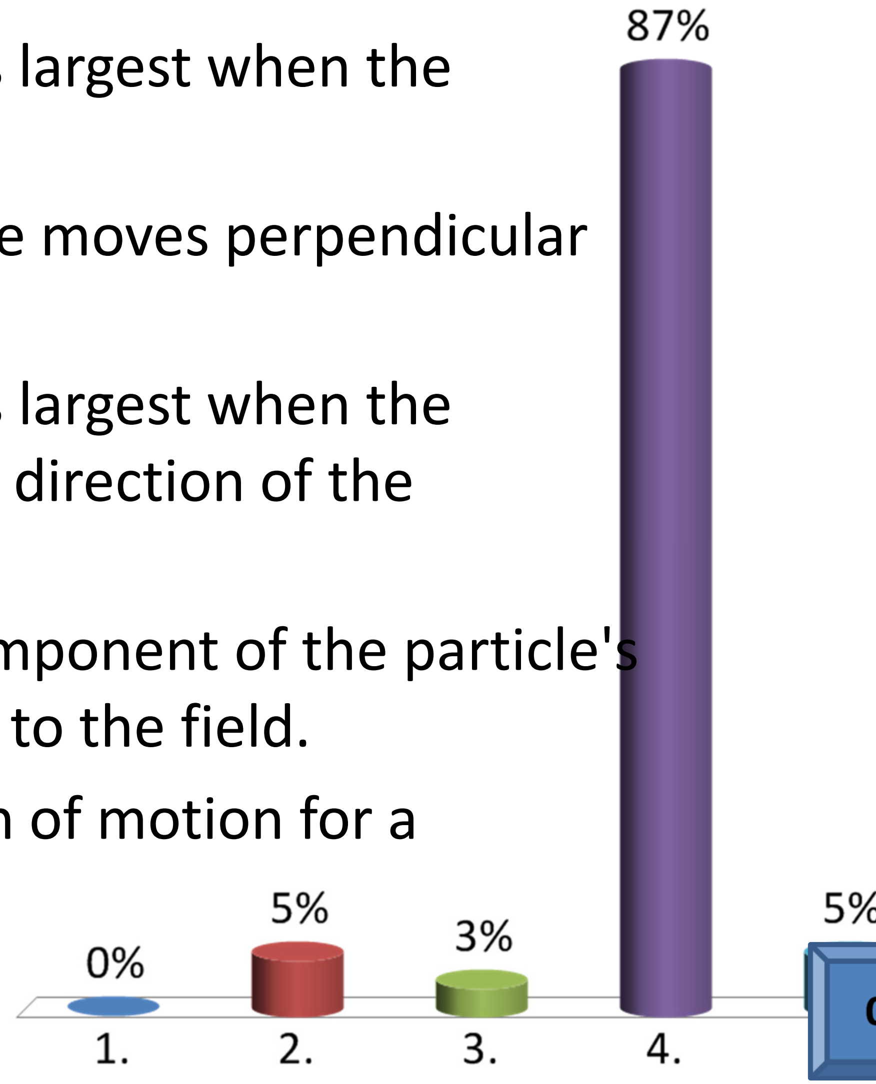

e+
e−
e−
F
v
B


$\vec{F}_B$
$F_B = q\vec{v} \times \vec{B}$
$\vec{B}$
q
$\vec{v}$
Maximum force
$\vec{v}$
$F = |q| vB \sin\theta$
θ
q
F_B
$\vec{v}$
q
Zero force


N
$\vec{v}$
Path of proton
$\vec{B}$
$\vec{F}_B$
W
E
S

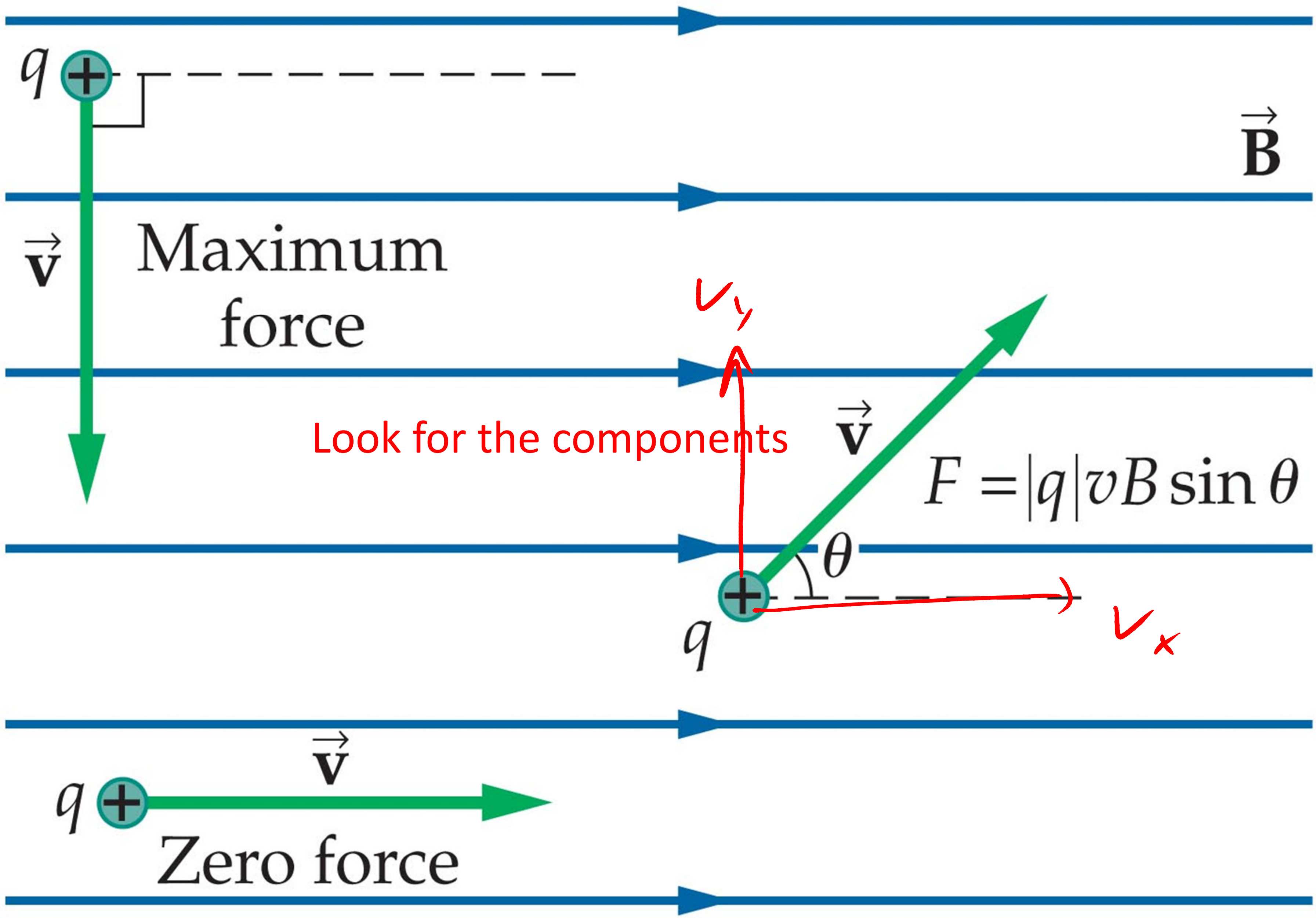
Which one of the following statements concerning the magnetic force on a charged particle in a magnetic field is true?

1. The magnitude of the force is largest when the particle is not moving.
2. The force is zero if the particle moves perpendicular to the field.
3. The magnitude of the force is largest when the particle moves parallel to the direction of the magnetic field.
4. The force depends on the component of the particle's velocity that is perpendicular to the field.
5. The force acts in the direction of motion for a positively charged particle.



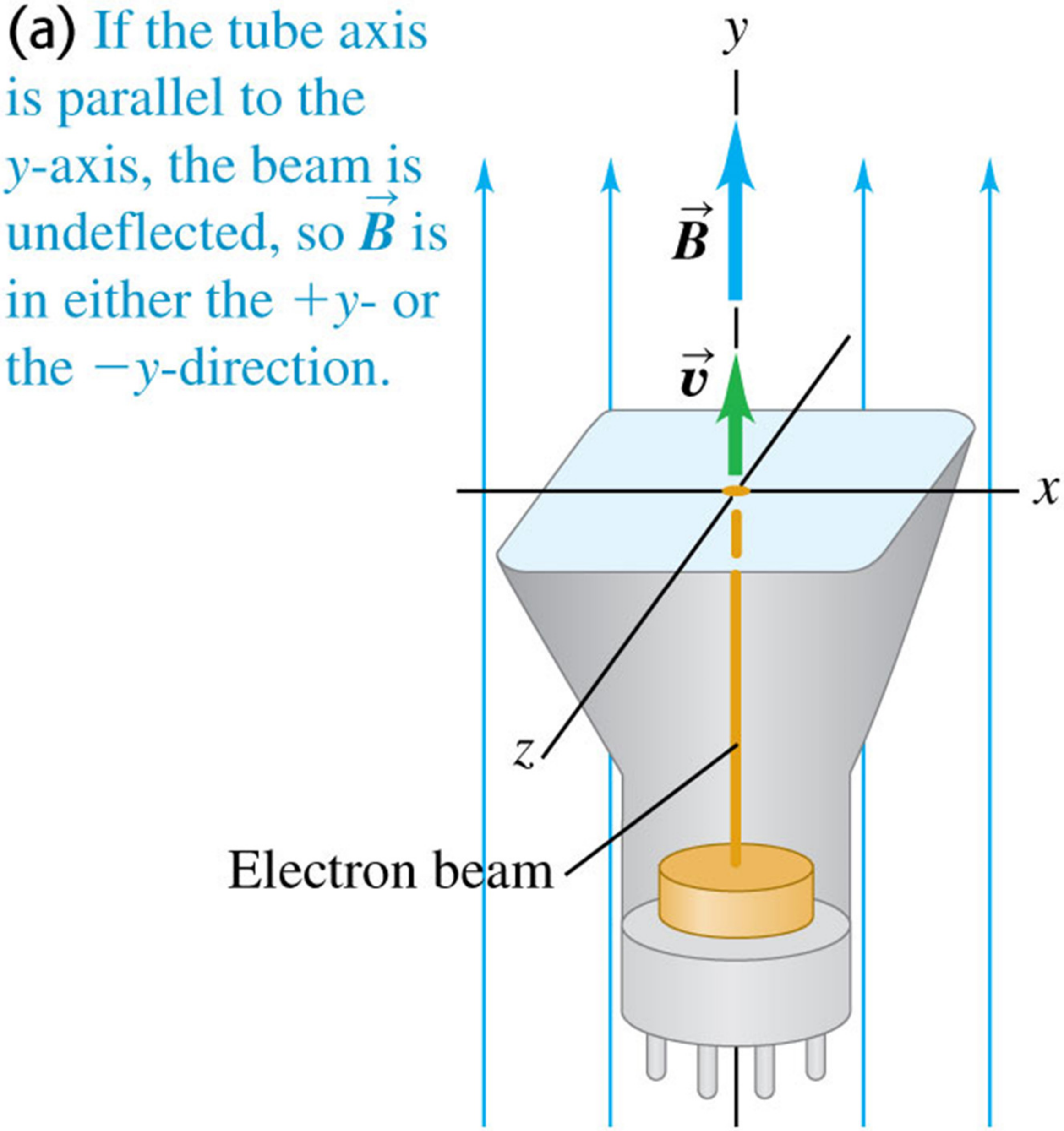
39 of 52

0

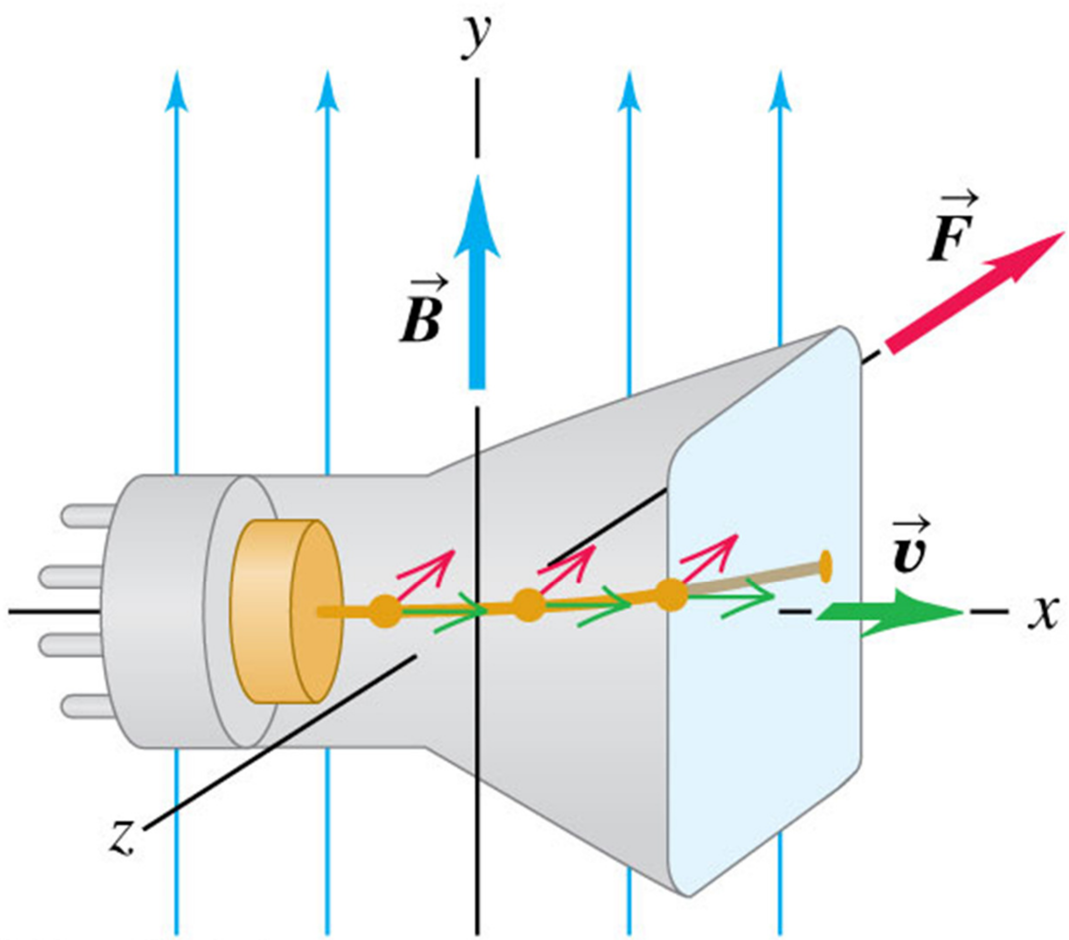


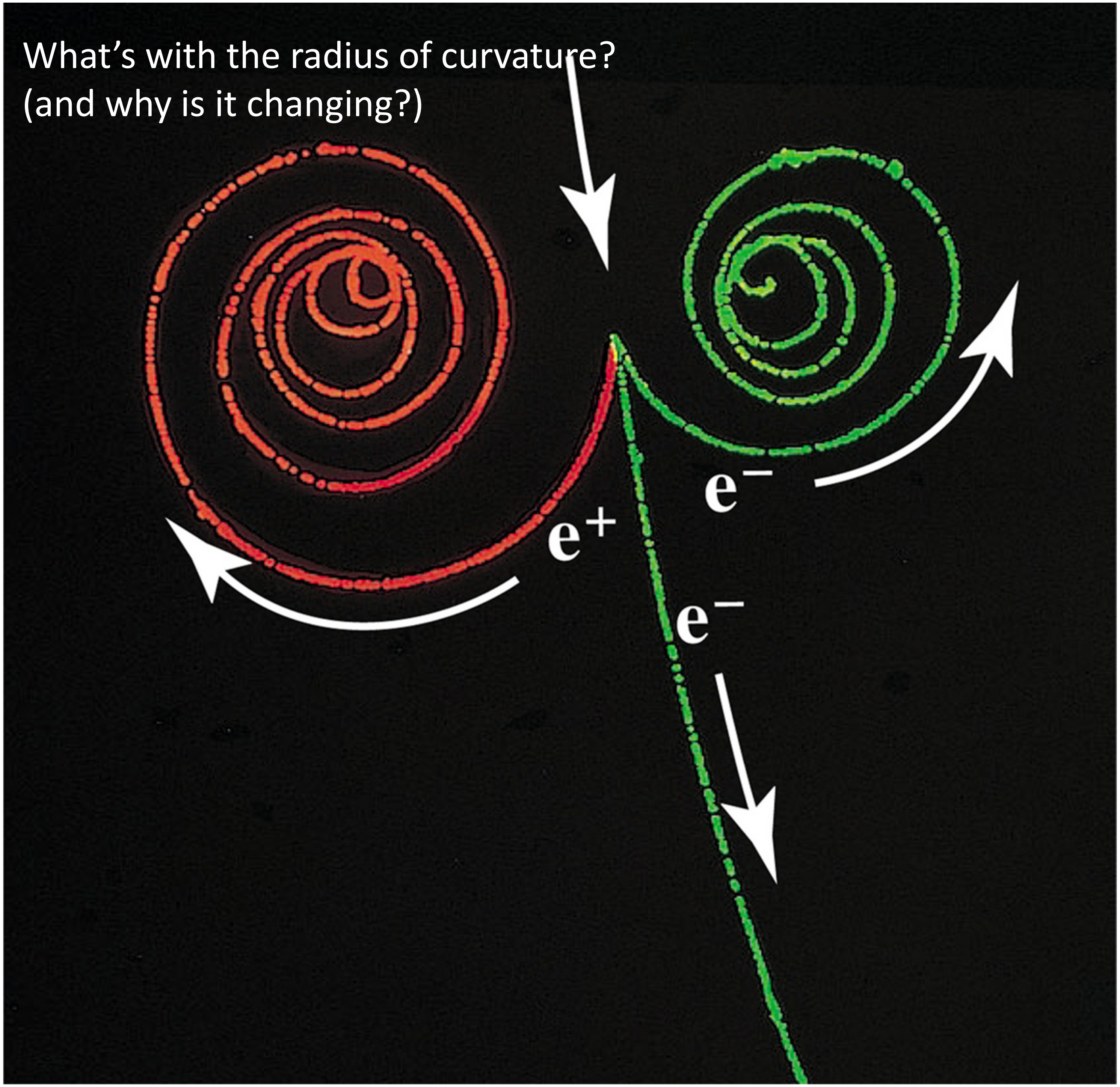
q
$\vec{B}$
$\vec{v}$
Maximum force
v_y
Look for the components
$\vec{v}$
$F = |q|vB\sin\theta$
θ
q
v_x
$\vec{v}$
q
Zero force

Force on Moving Charge worksheet


(a) If the tube axis is parallel to the y-axis, the beam is undeflected, so $\vec{B}$ is in either the $+y$- or the $-y$-direction.
y
$\vec{B}$
$\vec{v}$
x
z
Electron beam

(b) If the tube axis is parallel to the x-axis, the beam is deflected in the $-z$-direction, so $\vec{B}$ is in the $+y$-direction.

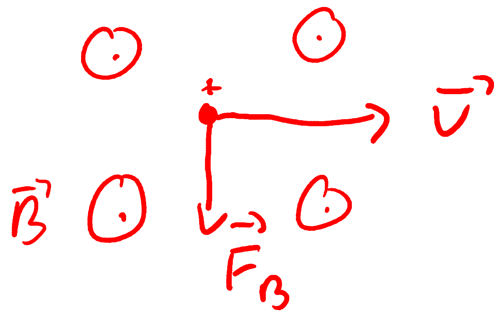



What's with the radius of curvature?
(and why is it changing?)
e⁺
e⁻
e⁻

Circular Motion worksheet

Circular Motion worksheet

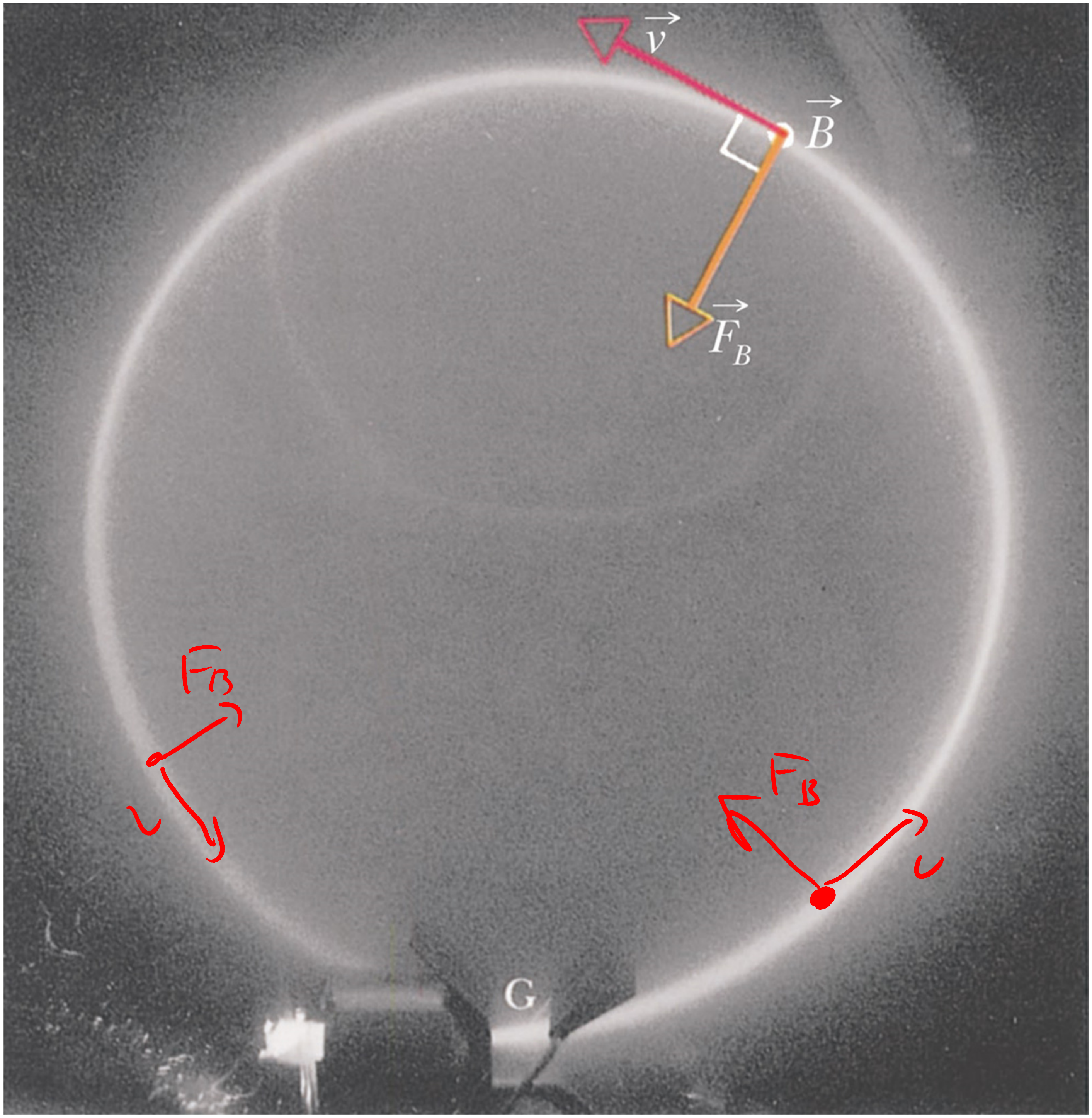
$$W = -\int \vec{F}\cdot d\vec{s}$$

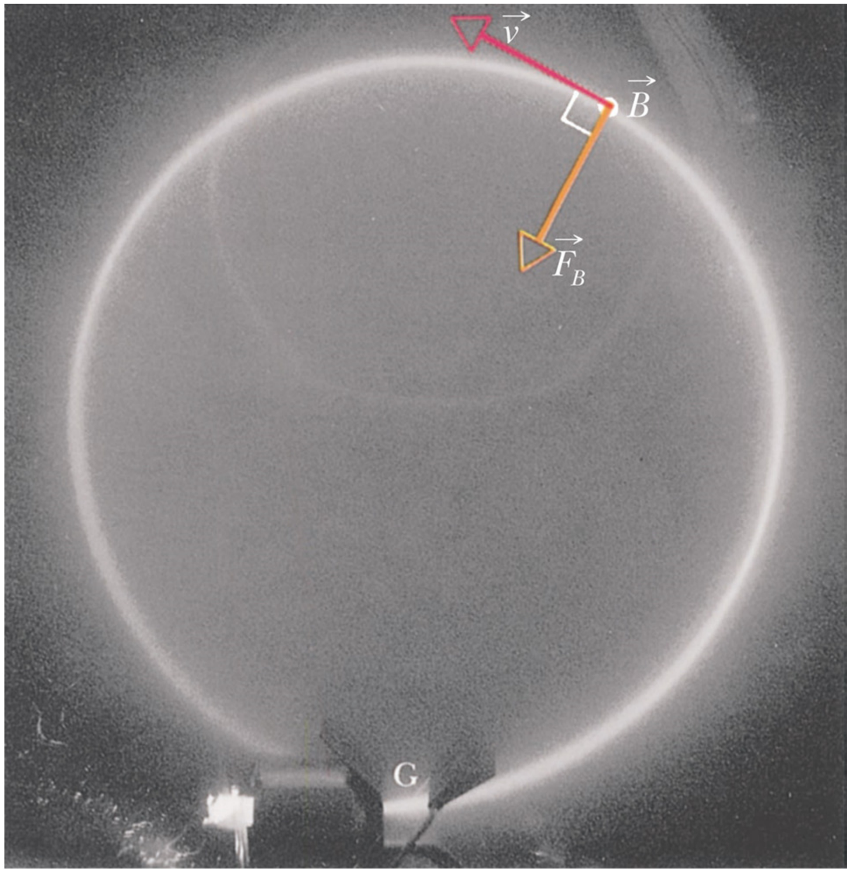


$\vec{F}$ and $d\vec{s}$ always $\perp$, so $W=0$

Thus, $|\vec{v}|$ does not change (Kinetic Energy!)

(useful fact: Centripital Force is $F = \frac{mv^2}{r}$)


$\vec{v}$
$\vec{B}$
$\vec{F}_B$
G
F_B
v
F_B
v



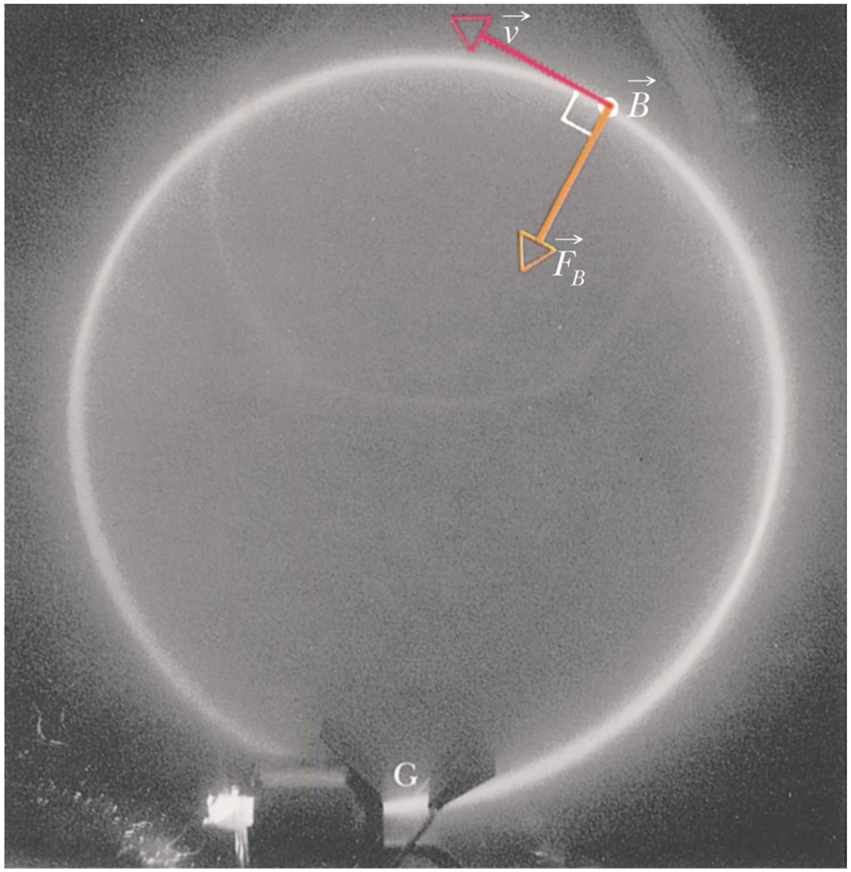
Compare F_B to centripital force to work out gyroradius

$$F_B = \frac{mv^2}{r}$$

$$qvB\sin\theta = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

gyroradius



How long does it take to orbit?

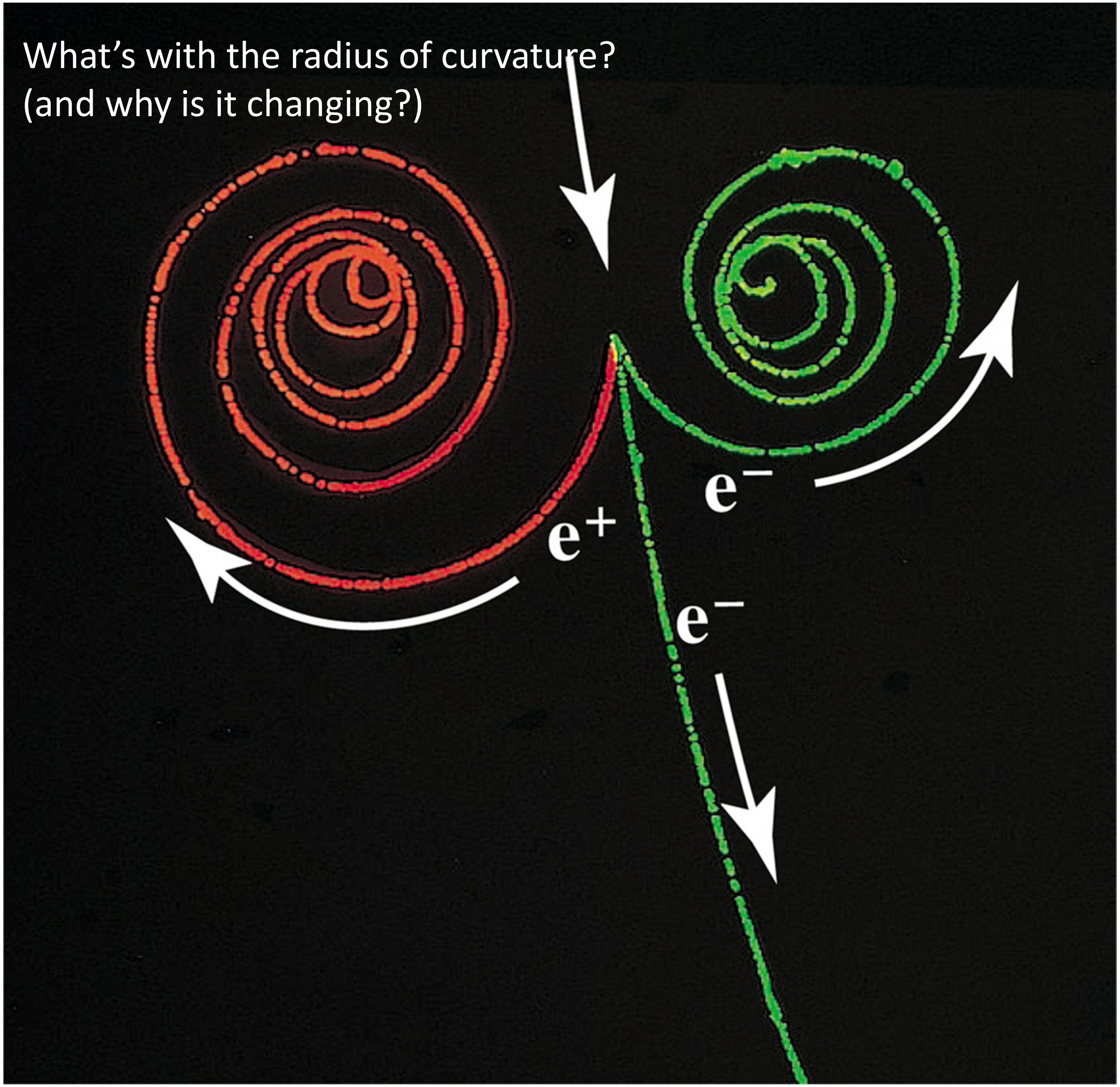
$$T = \frac{\text{distance}}{\text{speed}} = \frac{2\pi r}{v}$$

$$T = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$$

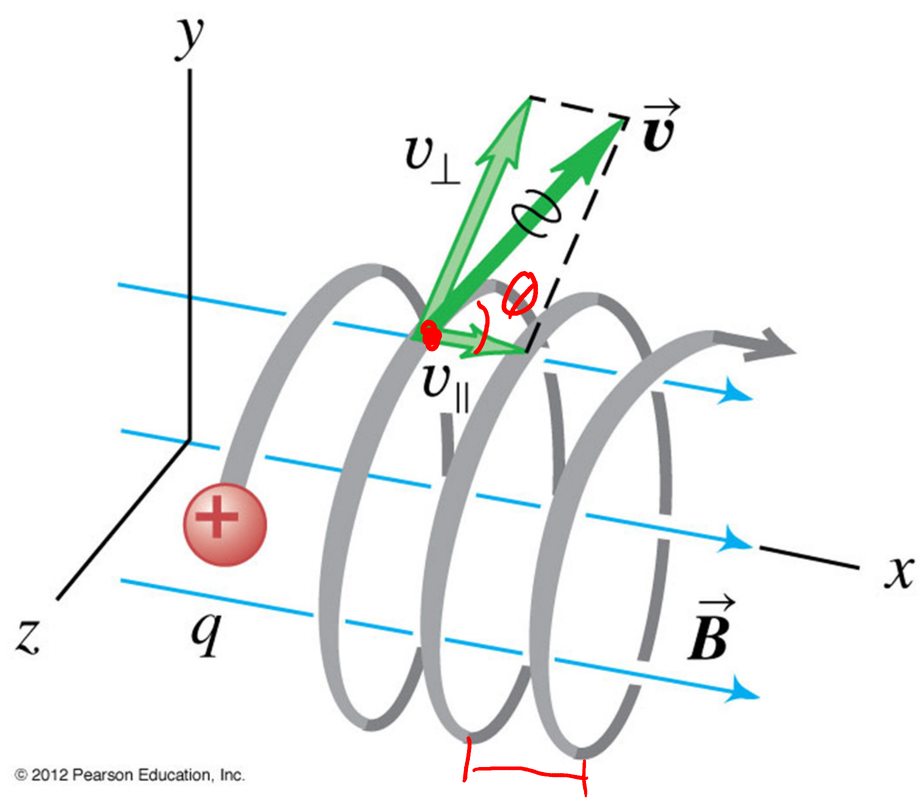
$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$\omega = 2\pi f$$

$$\omega = \frac{qB}{m}$$


What's with the radius of curvature?
(and why is it changing?)
e⁻
e⁺
e⁻

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



"pitch"

Work out components, pitch

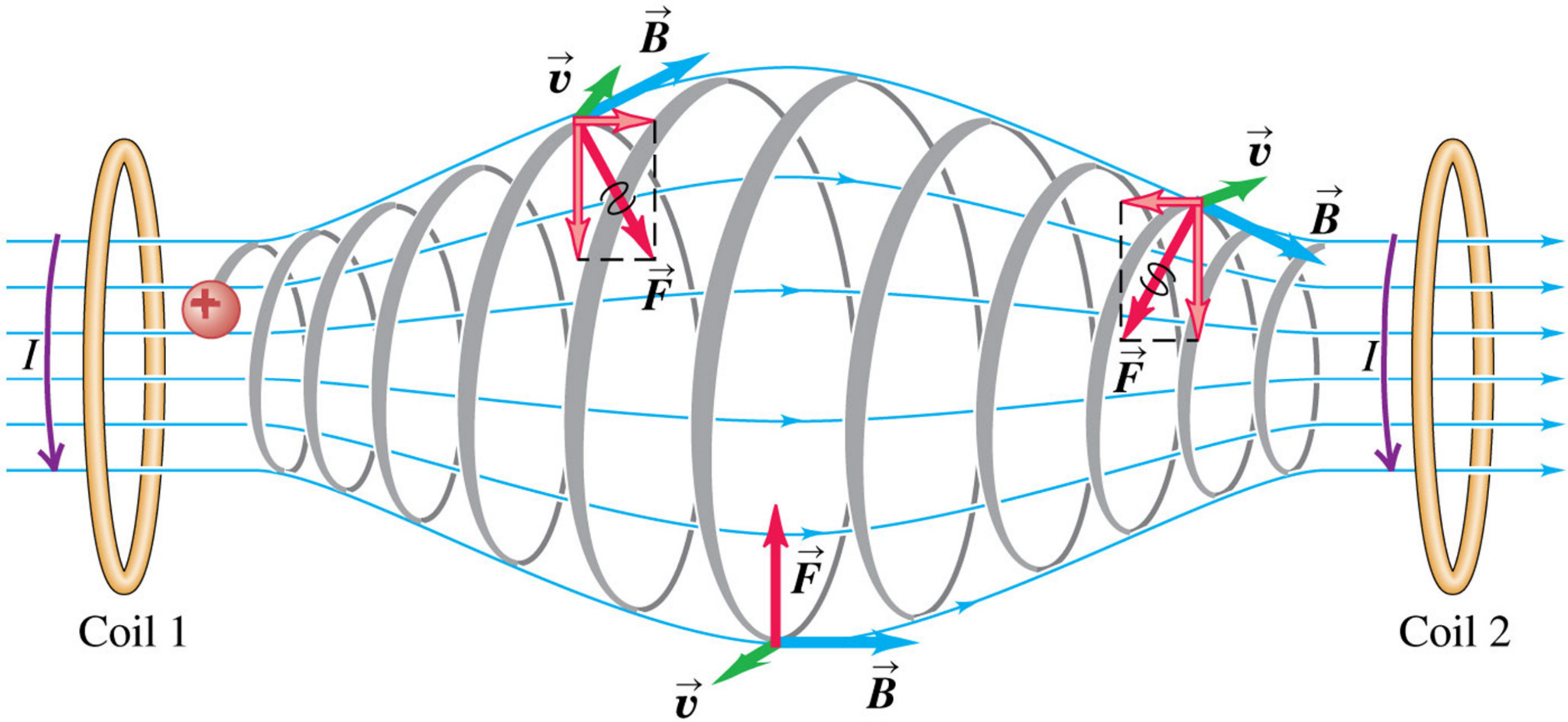
$v_{\perp} = |v| \sin\phi$

$v_{\parallel} = |v| \cos\phi$

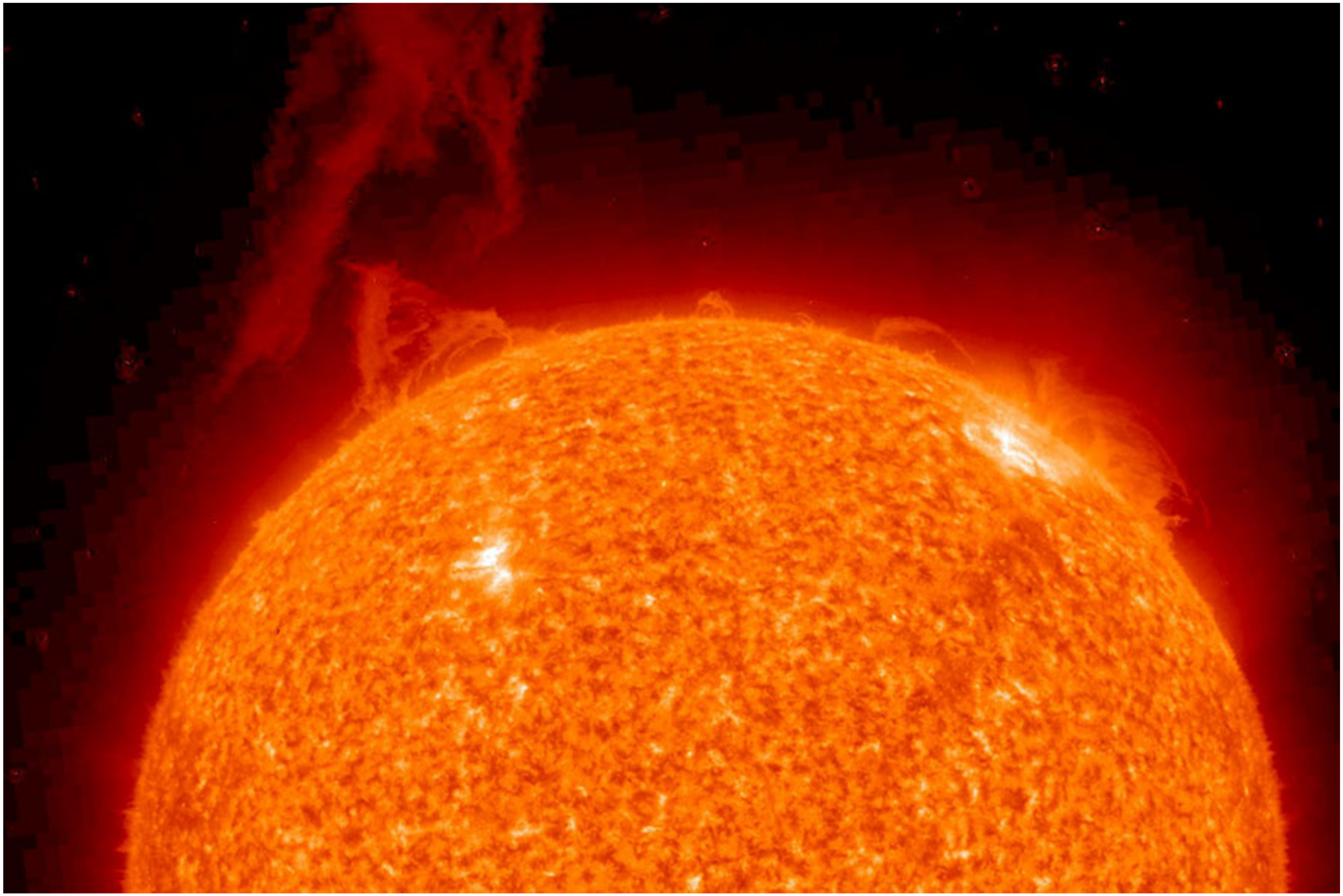
$r = \frac{m v_{\perp}}{qB}$

$v_{\parallel}$ - no change

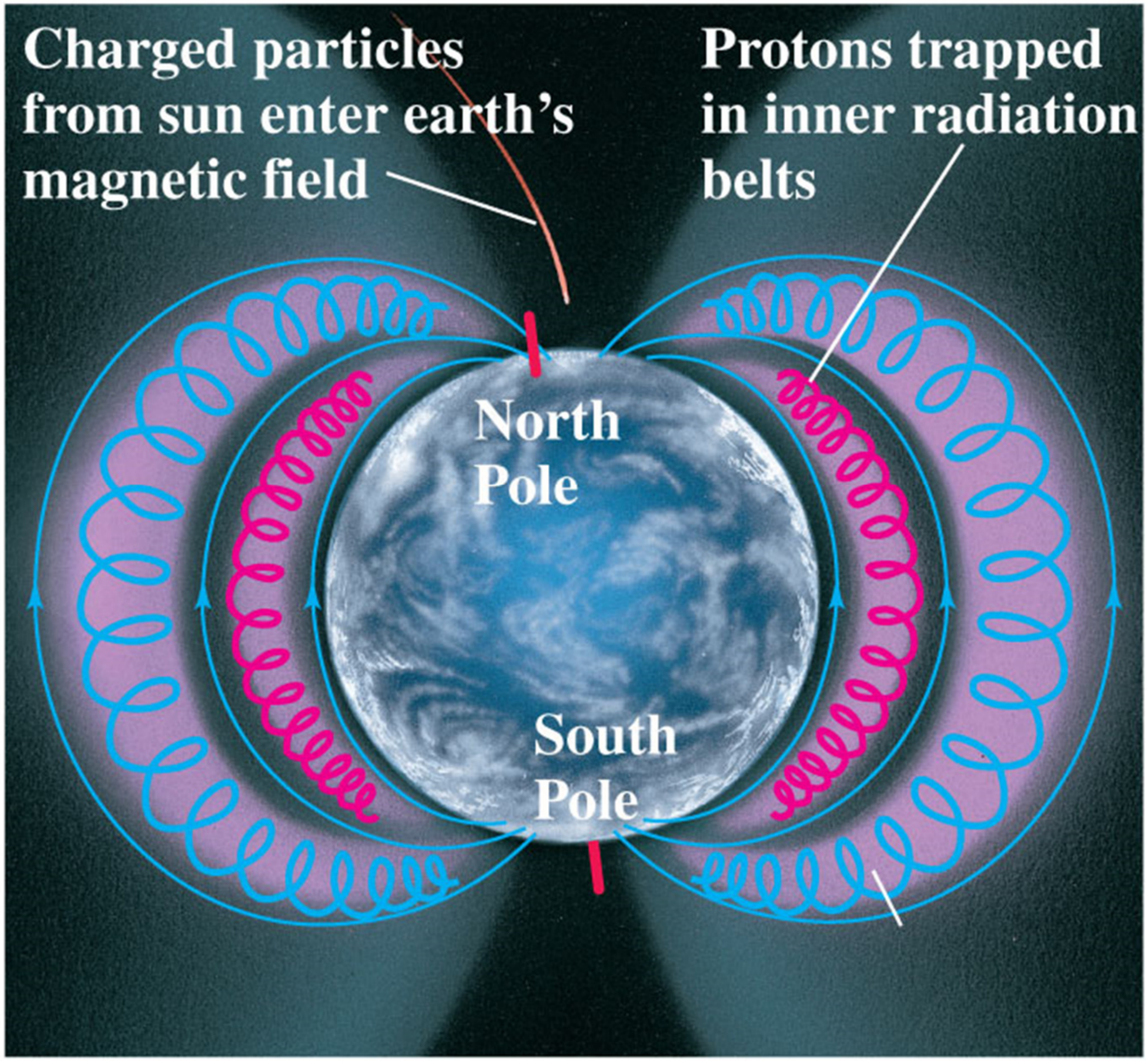
$T v_{\parallel} = \text{pitch}$



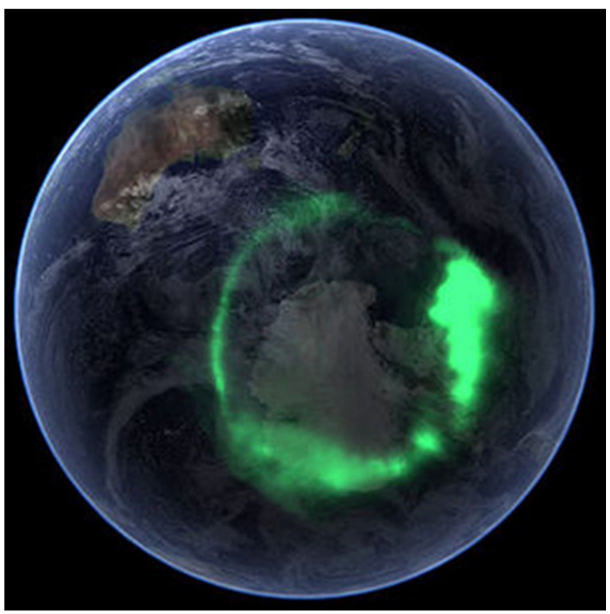
$$r = \frac{mv}{qB}$$

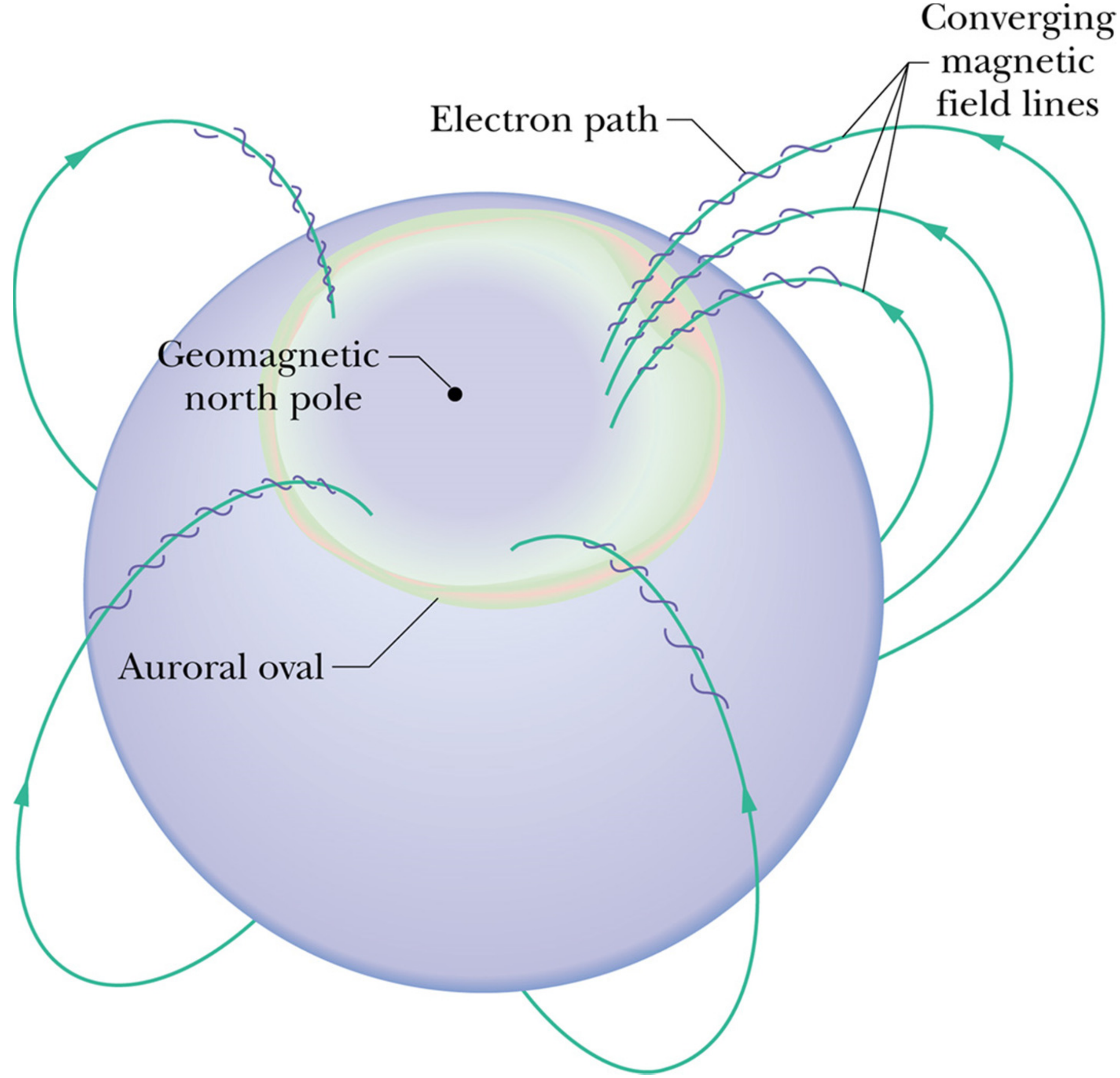


(a)



Charged particles from sun enter earth's magnetic field
Protons trapped in inner radiation belts
North Pole
South Pole





Converging magnetic field lines
Electron path
Geomagnetic north pole
Auroral oval