

\vec{E} : a "force field" made by charge
a map of the force some other charge
would feel $\vec{F}_E = q \vec{E}$

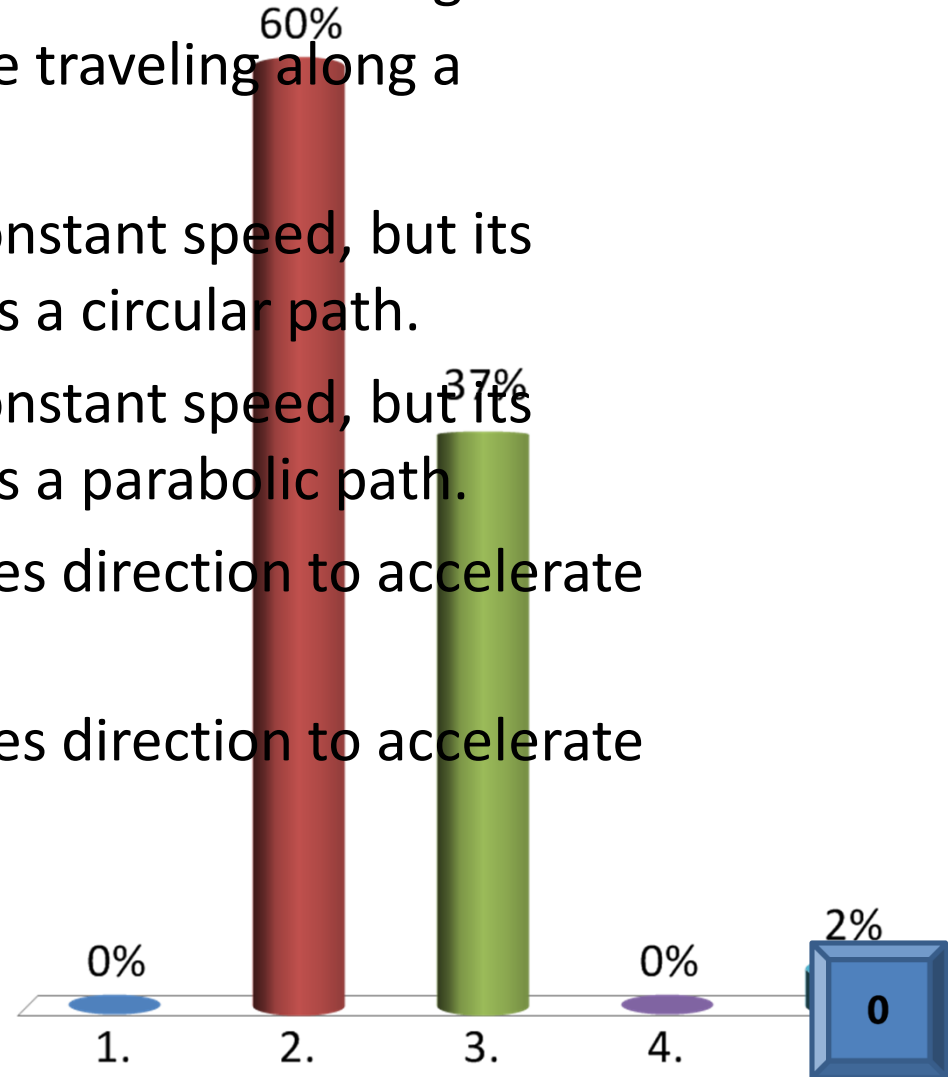
\vec{B} : another force field made by moving charges
(we have never found a stationary "magnetic charge")
charges moving through \vec{B} feel a force

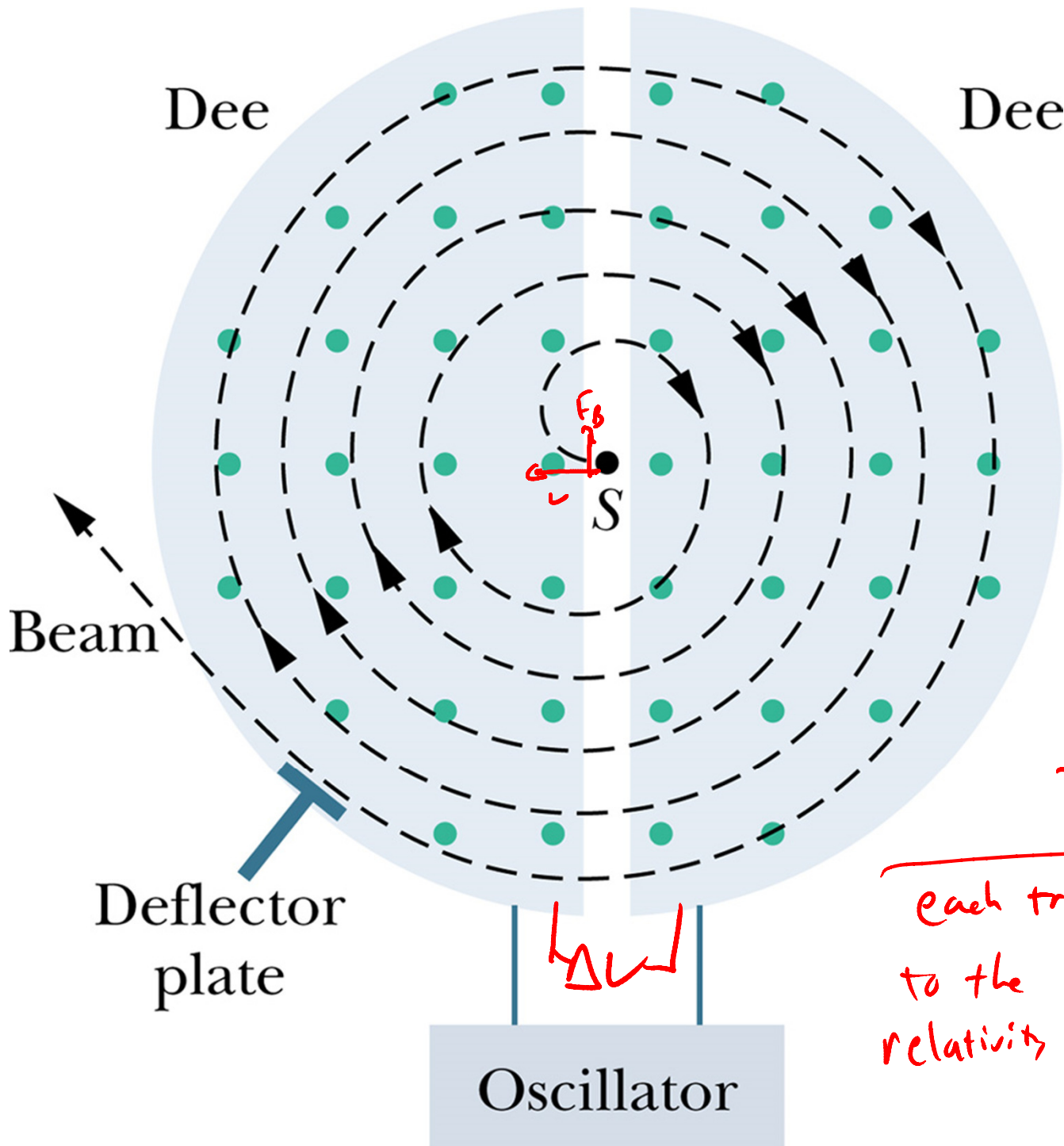
$$\vec{F}_B = q \vec{v} \times \vec{B}$$

$$\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})$$

An alpha particle (an He^{++} ion) is moving east when it enters a magnetic field that is directed north. Which happens to the motion of the alpha particle after entering the field?

1. The particle decelerates while traveling along a straight line until it stops.
- ✓ 2. The particle continues at a constant speed, but its direction changes as it follows a circular path.
3. The particle continues at a constant speed, but its direction changes as it follows a parabolic path.
4. The particle slows and changes direction to accelerate to move due north.
5. The particle slows and changes direction to accelerate to move directly upward.





Cyclotron

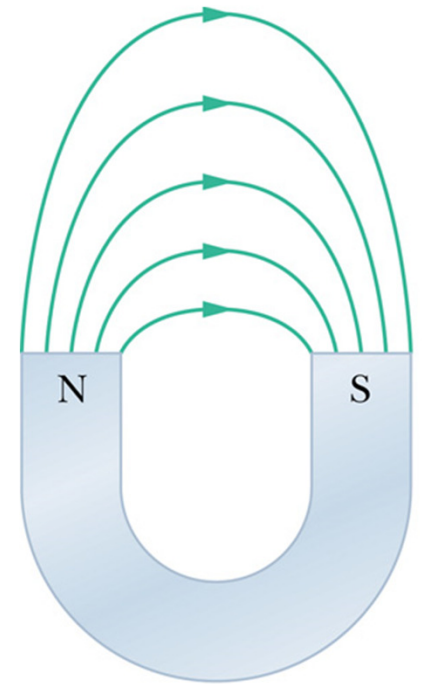
flip ΔV between
sides, boost
again - repeat.

T spent in $1/2$ loop

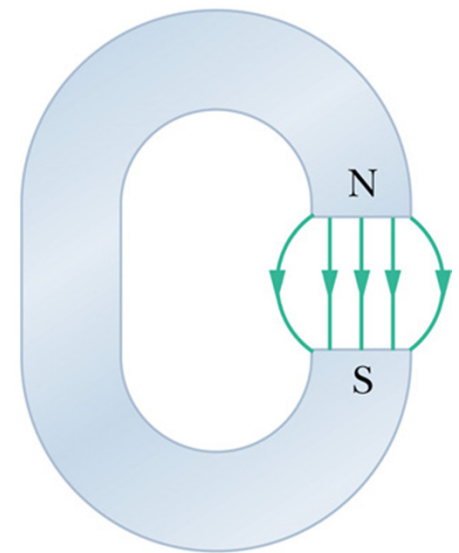
$$\frac{2\pi m}{qB} = T_{\text{full}}$$

$$f = \frac{1}{T} = \text{frequency}$$

each trip you add $U = qV$
to the $\frac{1}{2}mv^2$
relativity caps this out $\sim 50 \text{ MeV}$



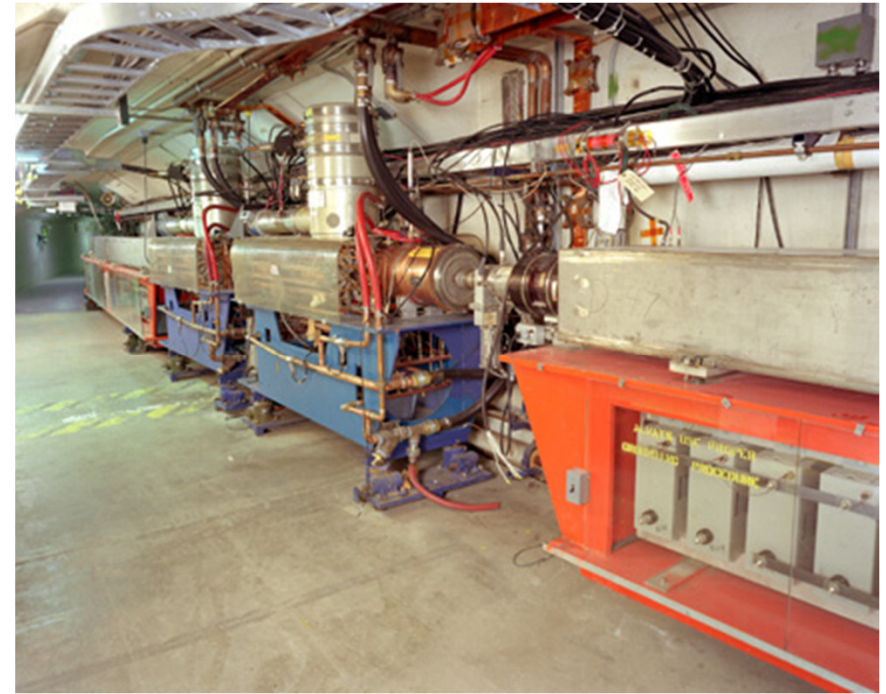
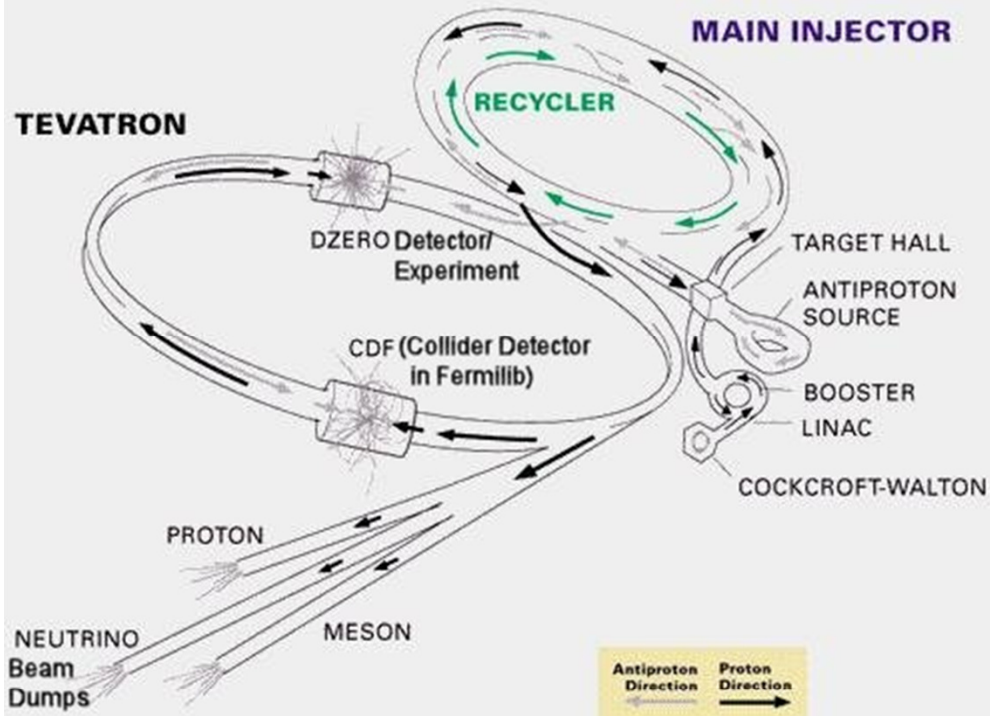
(a)



(b)

Synchrotron

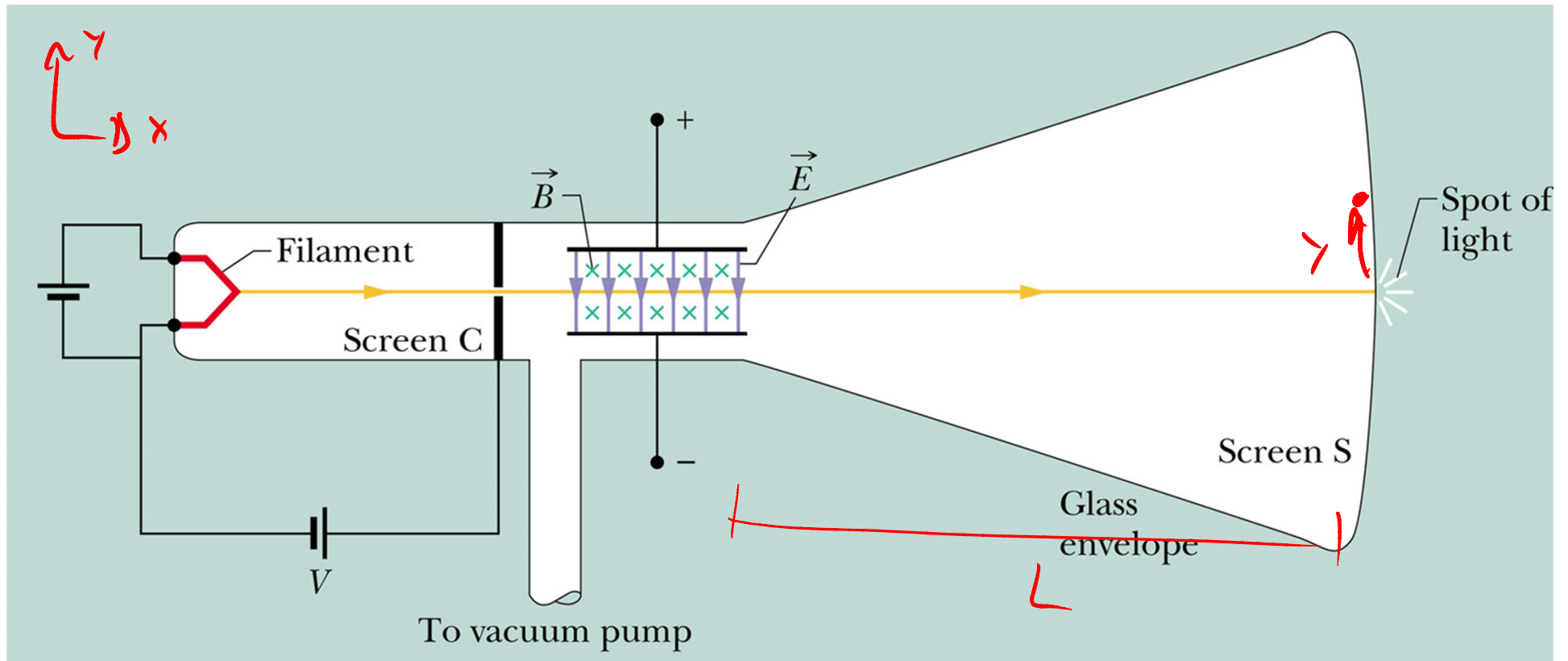
Fermilab's ACCELERATOR CHAIN





Start with E deflecting
a beam of electrons
JJ Thomsen, 1897

$$\vec{F}_E = q\vec{E} = ma$$
$$y = \frac{qEL^2}{2mv^2}$$



$$qE = qvB \sin \theta$$

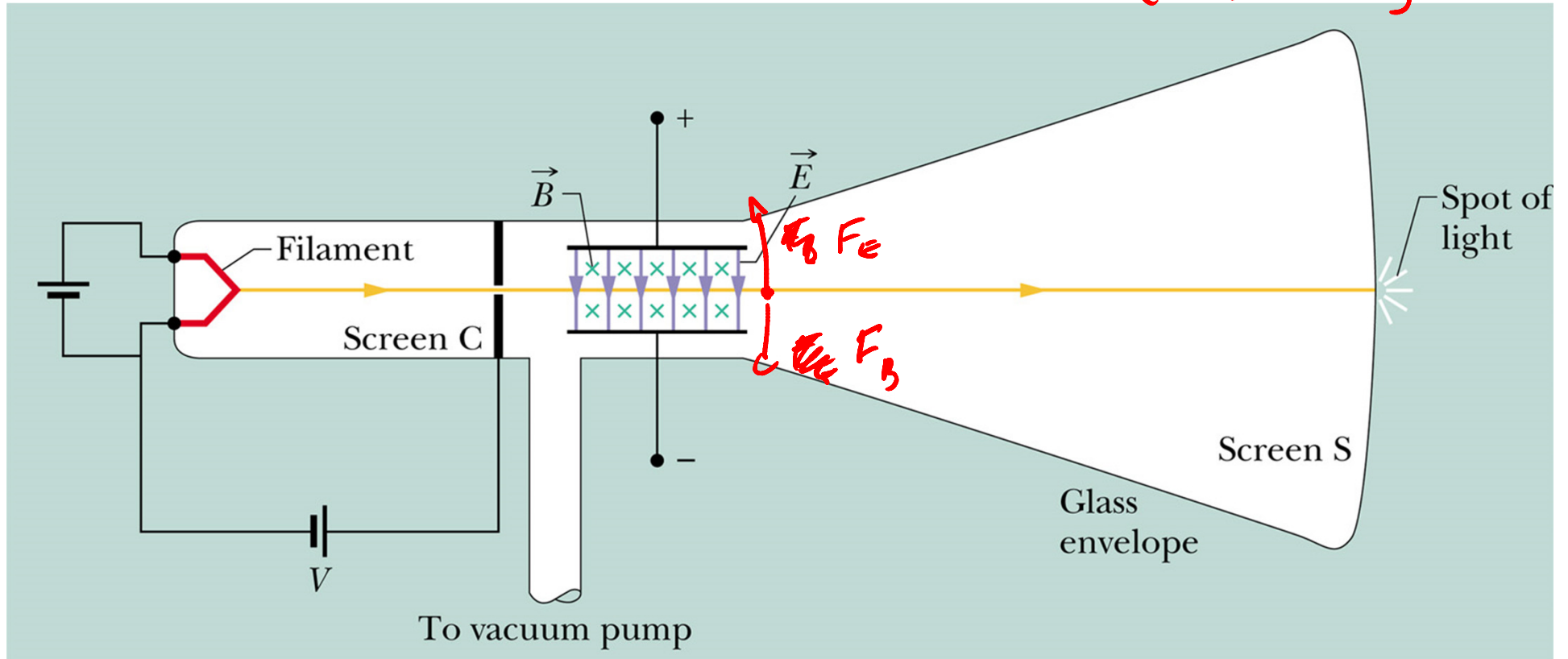
$$v = E/B$$

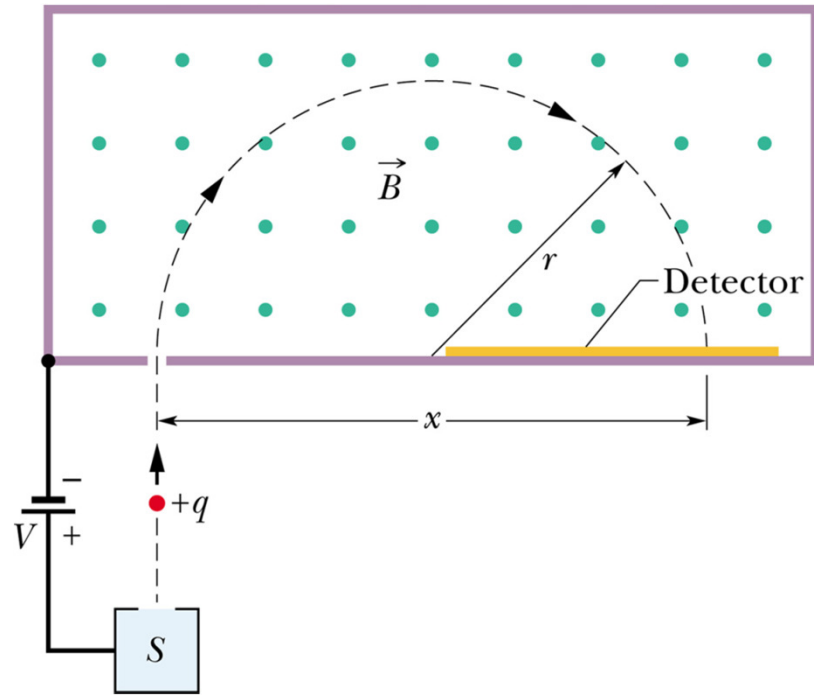
$$U = eV \rightarrow \frac{1}{2}mv^2 \quad v = \sqrt{\frac{2eV}{m}}$$

$$\frac{E}{B} = \sqrt{\frac{2eV}{m}} \quad \text{or} \quad \frac{e}{m} = \frac{E^2}{2VB^2} = 1.75892015 \times 10^{11} \frac{C}{hg}$$

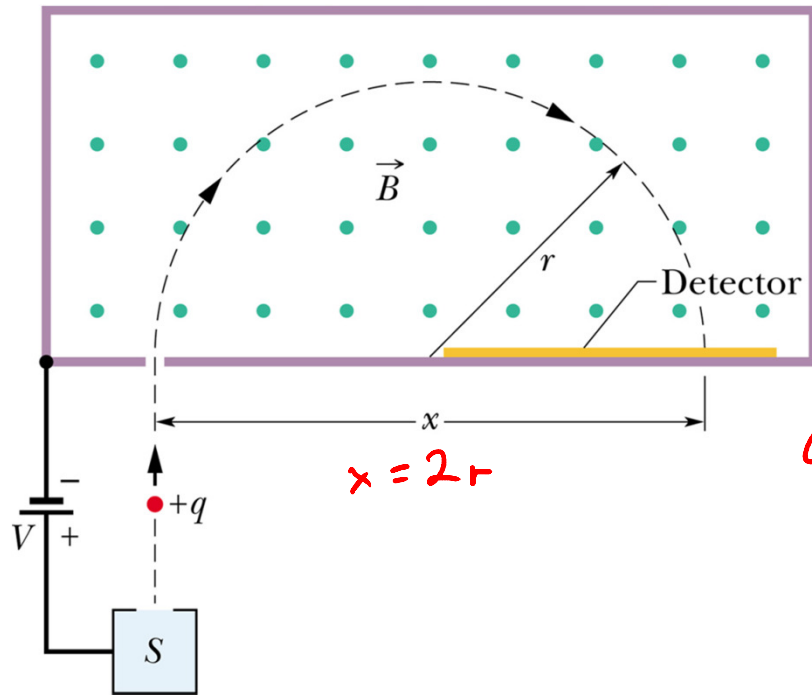
$$m_e = 9.31 \times 10^{-31} \text{ kg}$$

Now add a B





Mass spectrometer



Mass spectrometer

What mass thing hits
at x

What happens when a q cruises
around in a B ? gyroradius $r = \frac{mv}{qB}$
goes in a circle.

What do you know there

$$\text{so } r = \frac{mv}{qB} = \frac{m}{qB} \sqrt{\frac{2qV}{m}} = \frac{1}{B} \sqrt{\frac{2mV}{q}}$$

$$r = 2x, \text{ so } x = \frac{2}{B} \sqrt{\frac{2mV}{q}}$$

$$\text{solve for } m = \frac{B^2 q x^2}{8V}$$

conserve energy

POT \rightarrow kinetic

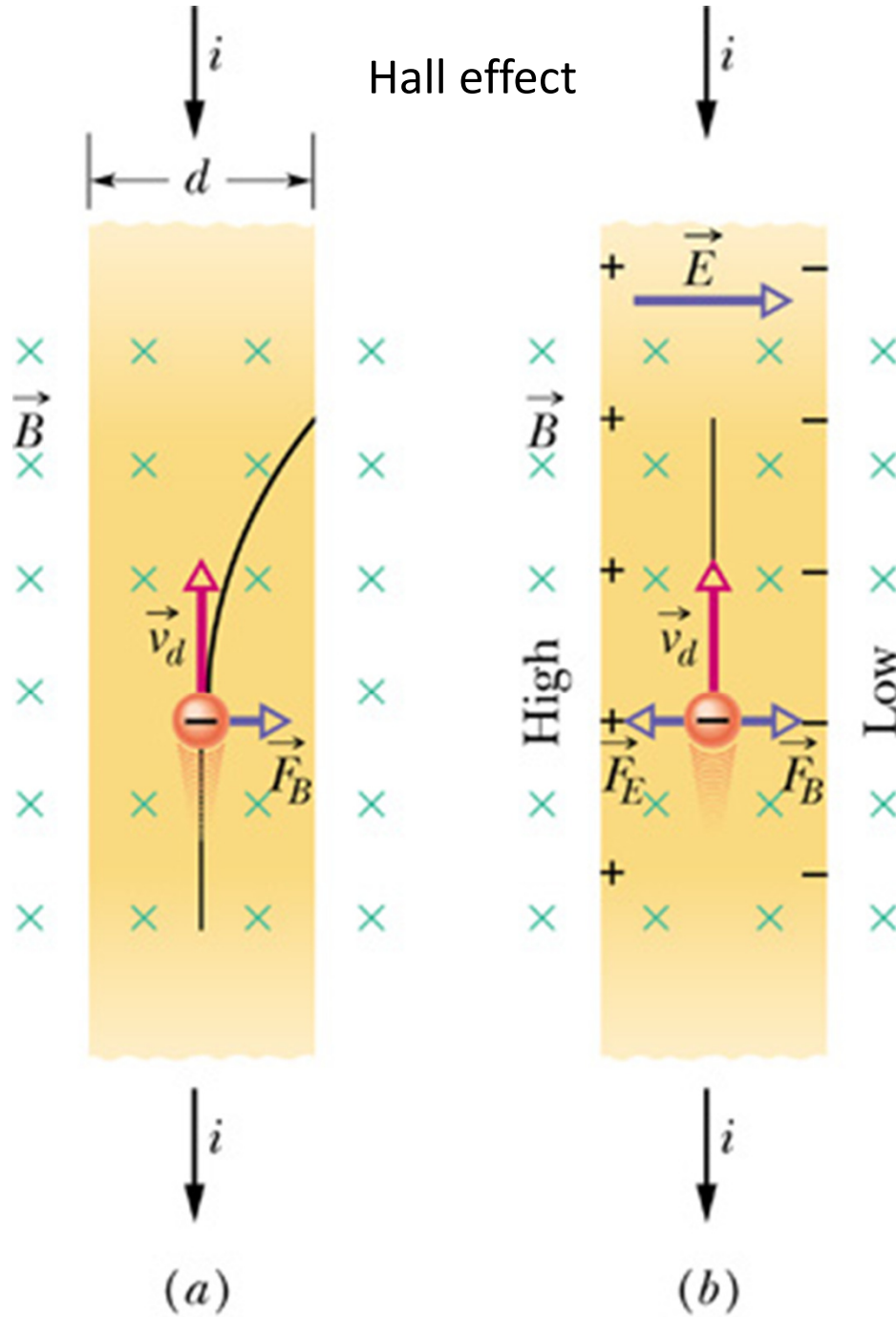
$$U = qV = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$V_{235} = 90,915 \text{ u/s} \quad \left| \quad 1.32 \text{ m} \right.$$

$$V_{238} = 90,951 \text{ u/s} \quad \left| \quad 1.33 \text{ m} \right.$$

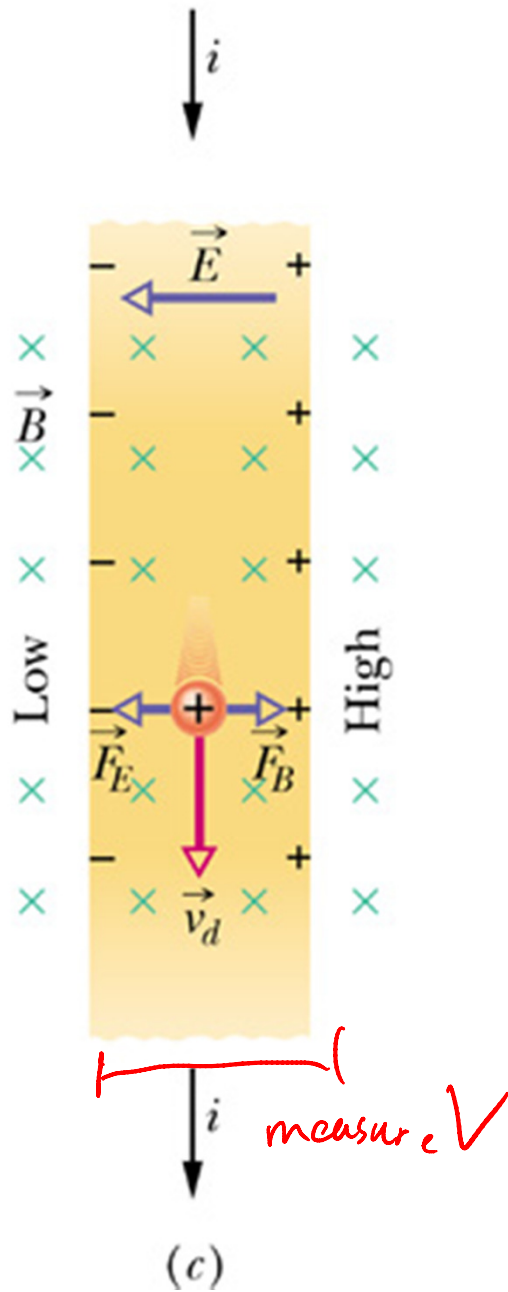
Hall effect



$$F_B = q(\vec{v}_d \times \vec{B})$$

pushes e to right.

$$\vec{F}_E = q\vec{E}$$



Balance when $F_E = F_B$

$$qE = q |v_d| B \sin 90^\circ$$

$$v_d = \frac{J}{ne} = \frac{I}{neA}$$

$$E = \frac{V}{d}$$

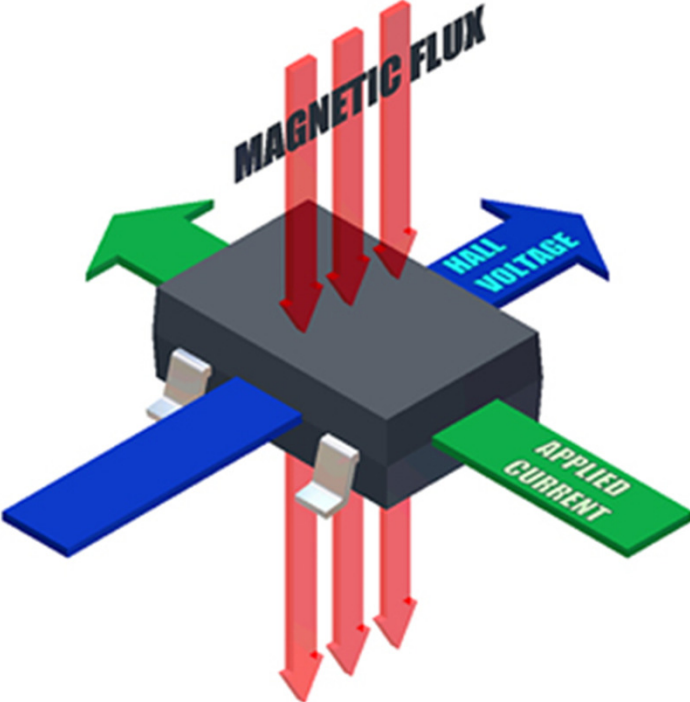
$$\frac{V}{d} = \frac{BI}{neA}$$

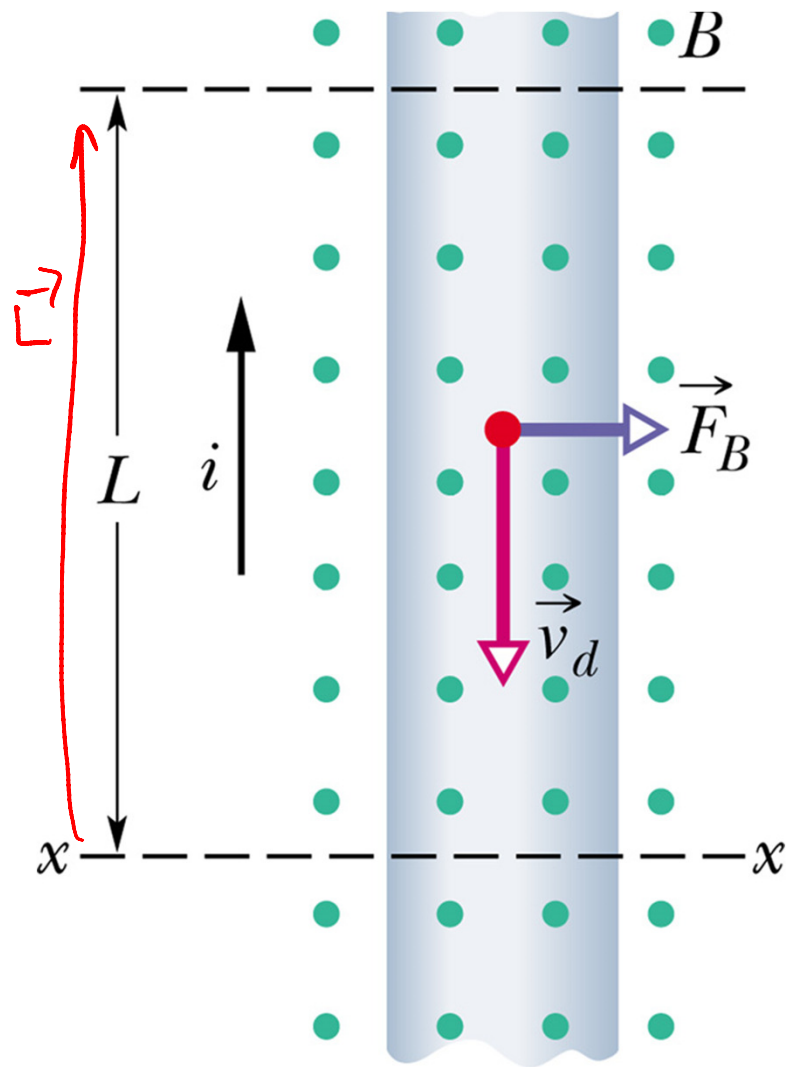
$A/d = Q$
(depth)

$$n = \frac{dIB}{VeA}$$

Or, solve for B

Get all 3 components with three chips





$$I = \frac{q}{t} \quad q = I t = I \frac{L}{v_d}$$

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

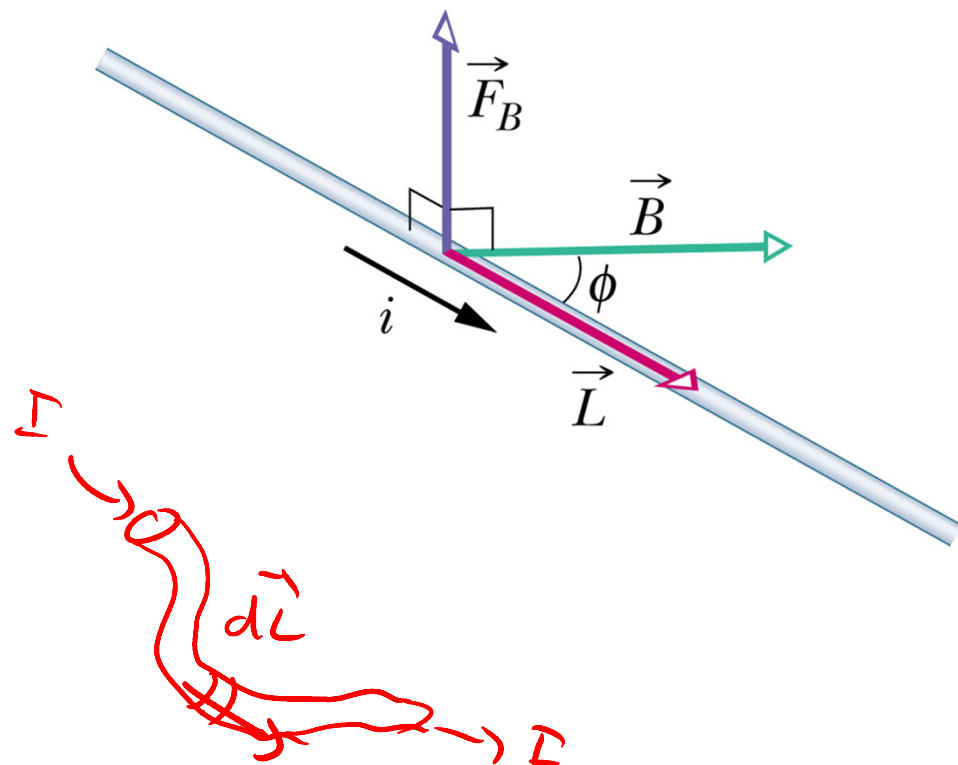
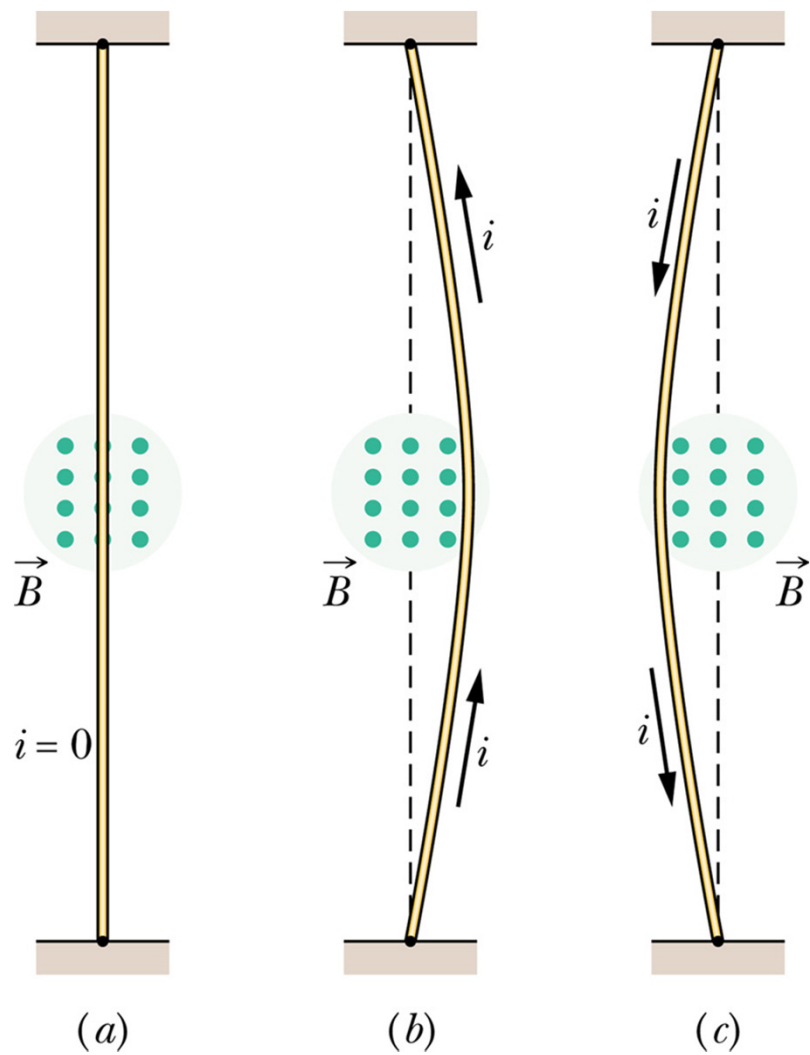
$$= \left(\frac{I L}{v_d} \right) \vec{v} \times \vec{B}$$

if \perp , $\frac{v}{v_d} = 1$

$$\vec{F}_B = I L \vec{B}$$

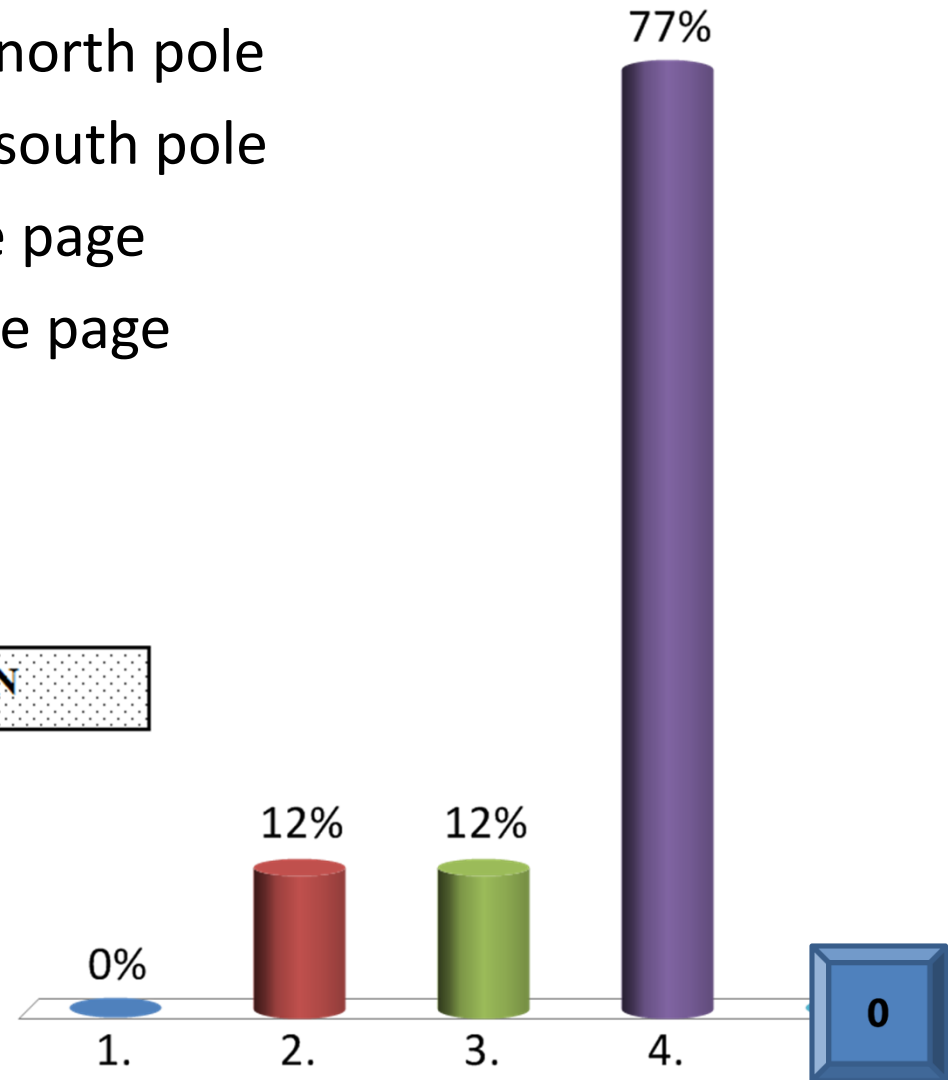
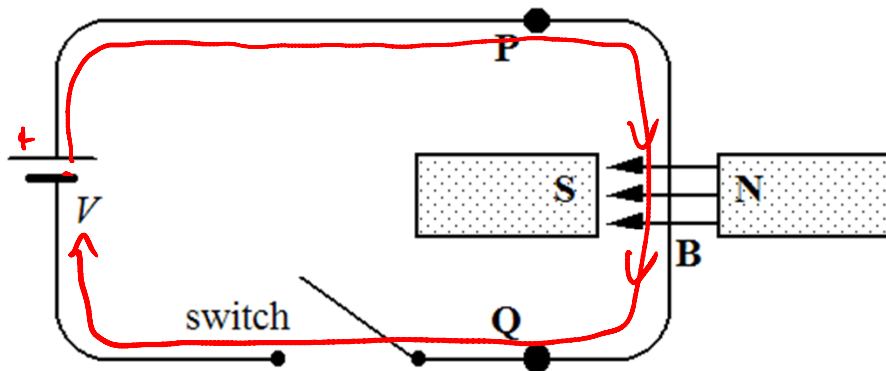
$$\vec{F}_B = I \vec{L} \times \vec{B}$$

$$\vec{F}_B = I \vec{L} \times \vec{B}$$

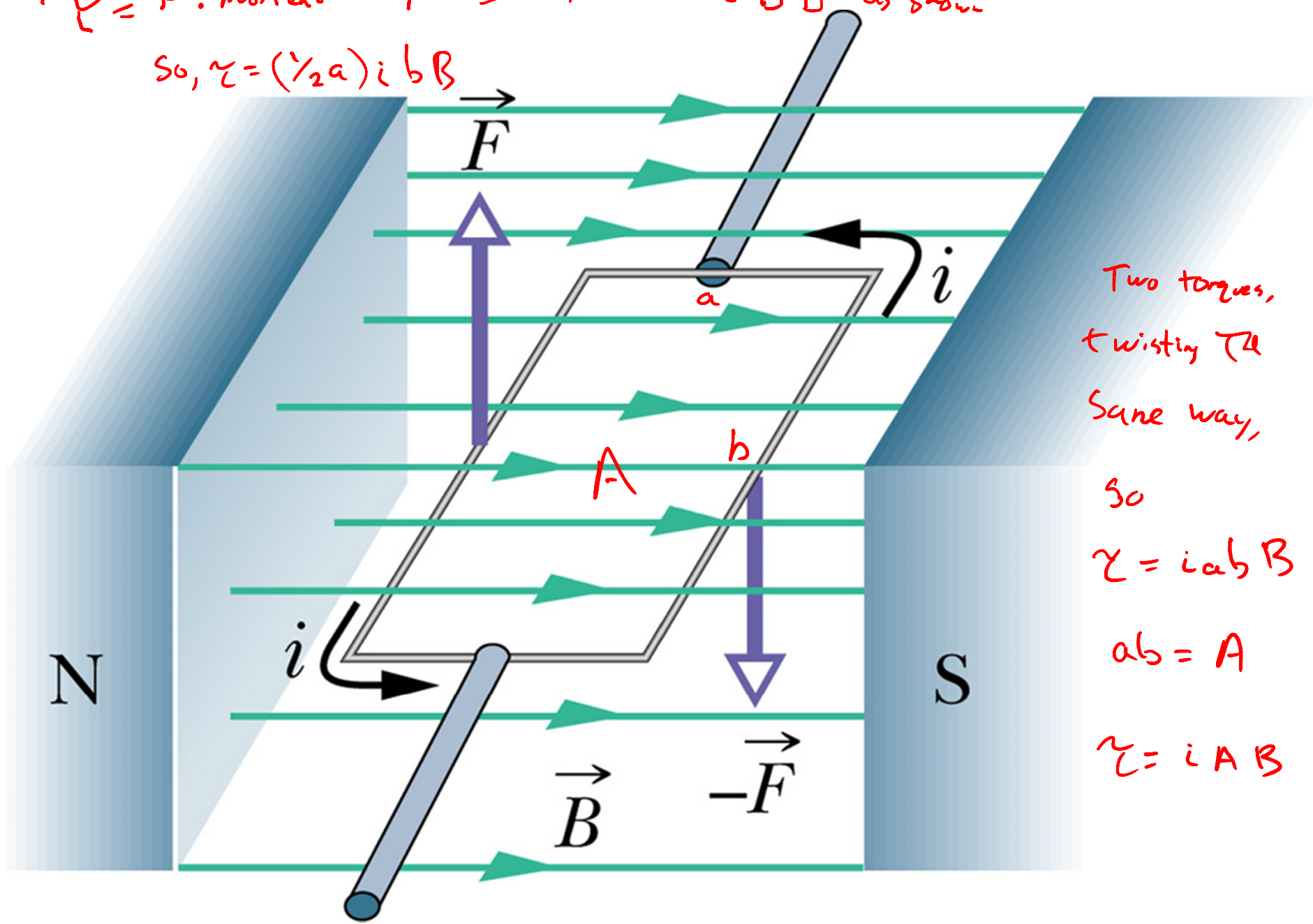


Part of a loop of wire passes between the poles of a magnet. When the switch is closed and a current flows, what is the movement of the wire between the poles of the magnet?

1. The wire moves towards the north pole
2. The wire moves towards the south pole
3. The wire moves up out of the page
- ✓ 4. The wire moves down into the page
5. No movement happens



$\tau = F \cdot \text{moment}$ $F = I \vec{L} \times \vec{B} \approx i b B$ as shown
 So, $\tau = (\frac{1}{2} a) i b B$



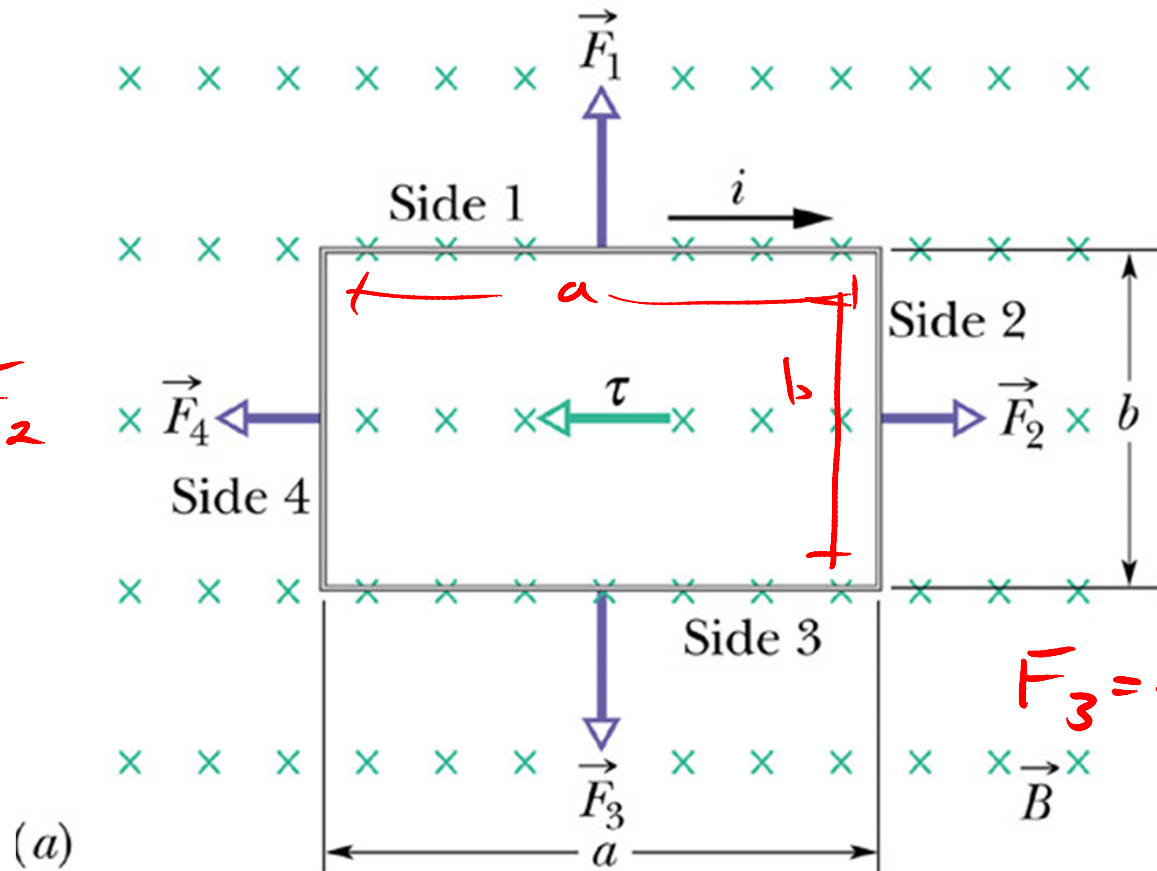
Two torques,
 twisting τ
 Same way,
 So
 $\tau = i a b B$
 $a b = A$
 $\tau = i A B$

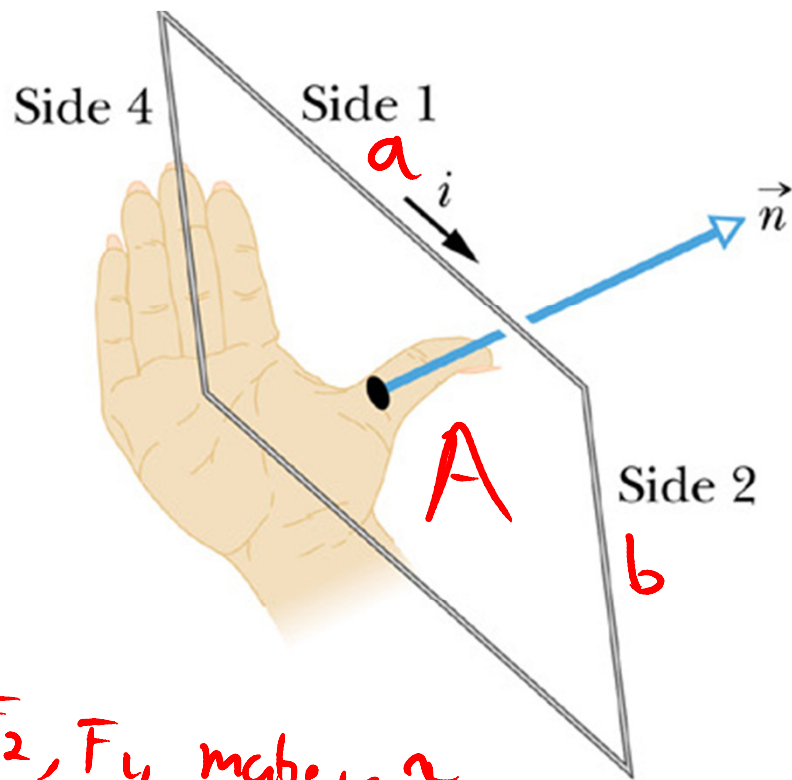
$$F_1 = I \vec{L} \times \vec{B} = I a \sin 90^\circ$$

$$F_4 = -F_2$$

$$F_2 = I b \sin 90^\circ$$

$$F_3 = -F_1$$

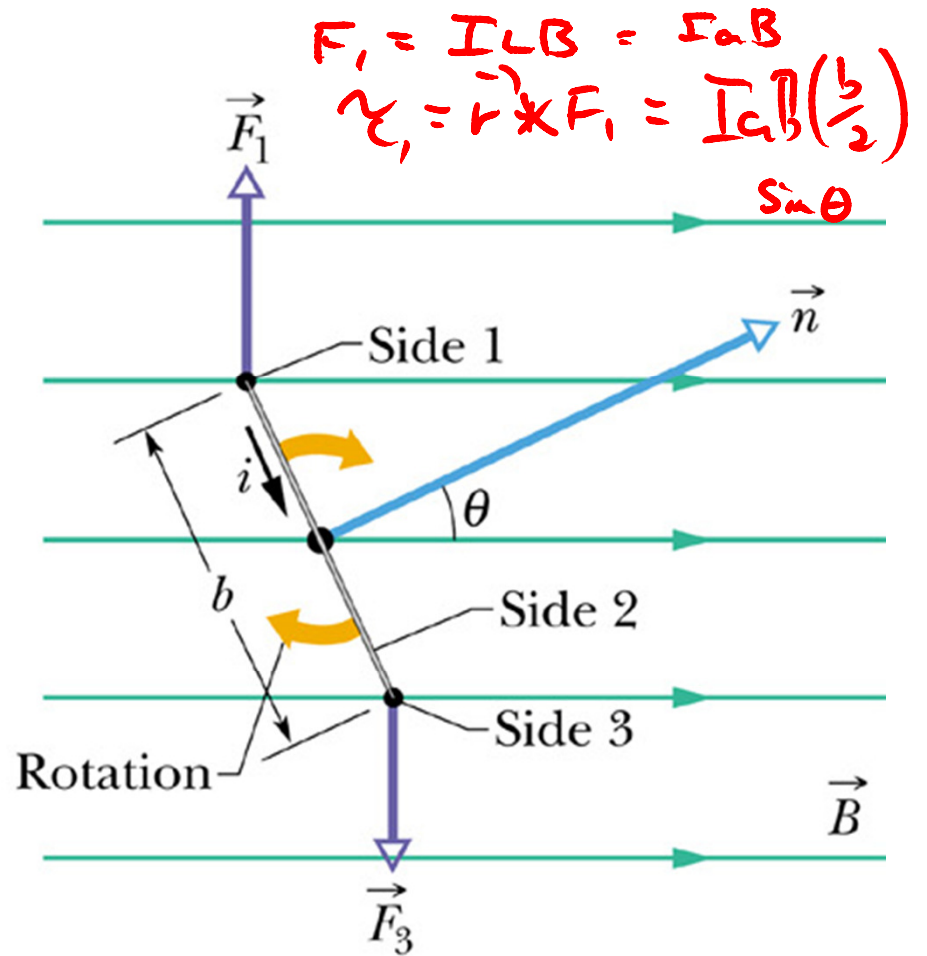




F_2, F_4 make no τ

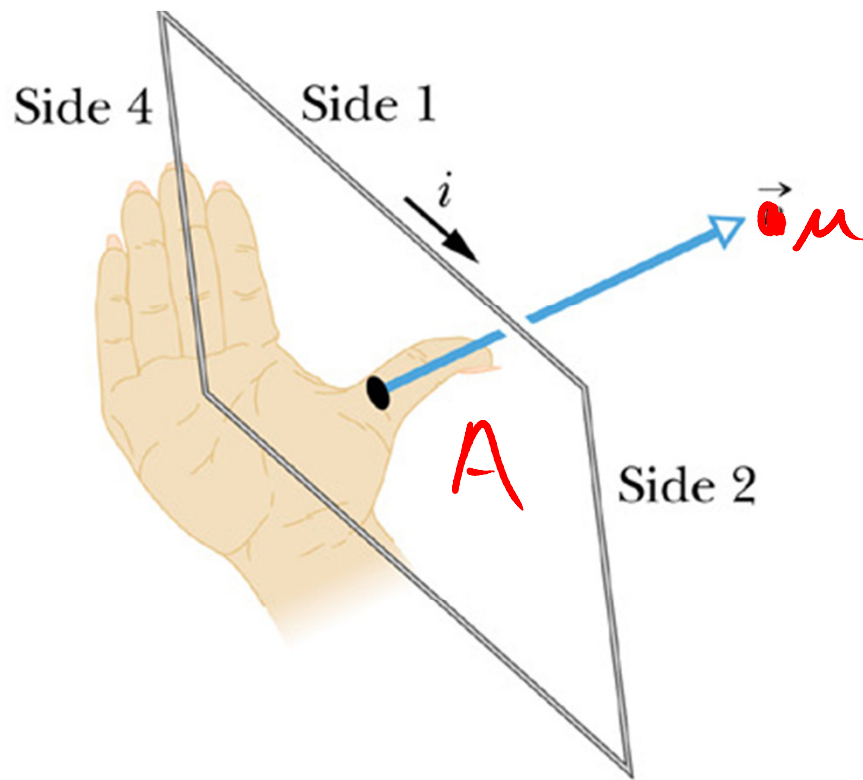
(b)

$$\begin{aligned}\tau_{\text{tot}} &= \tau_1 + \tau_2 \\ &= I a B b \sin\theta \\ \tau &= I A B \sin\theta\end{aligned}$$



(c)

$$\begin{aligned}F_3 &= I L B = I a B \\ \tau_3 &= F_3 \left(\frac{b}{2}\right) \sin\theta \\ &= I a B \left(\frac{b}{2}\right) \sin\theta\end{aligned}$$



(b)

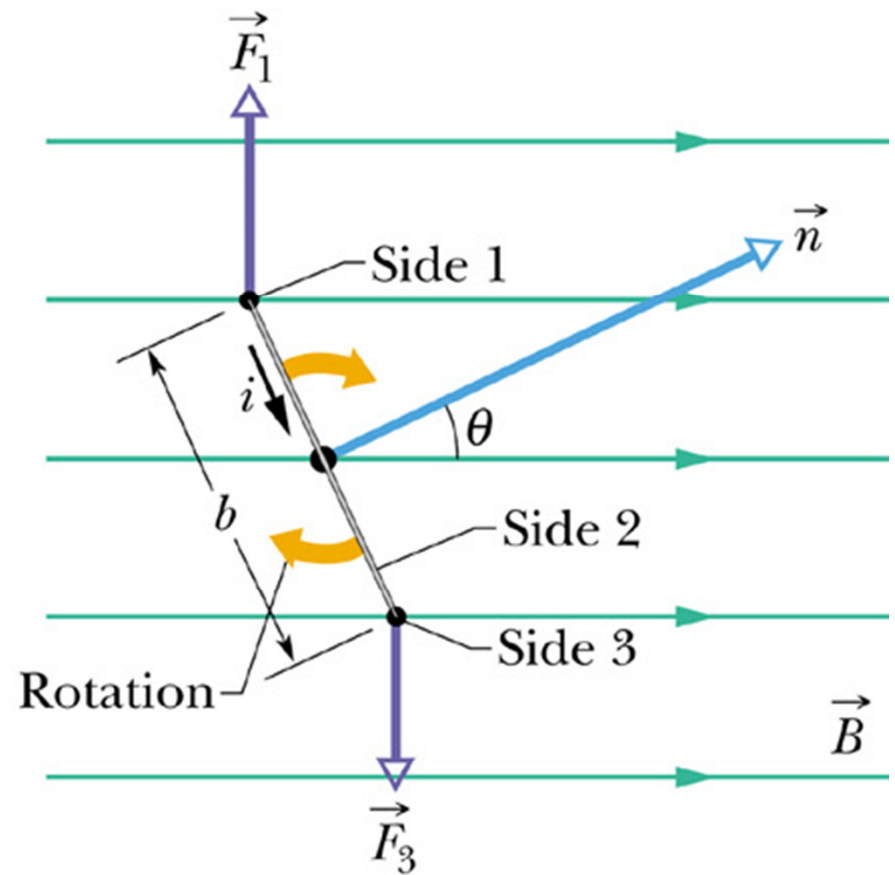
$$\text{let } \mu = NIA$$

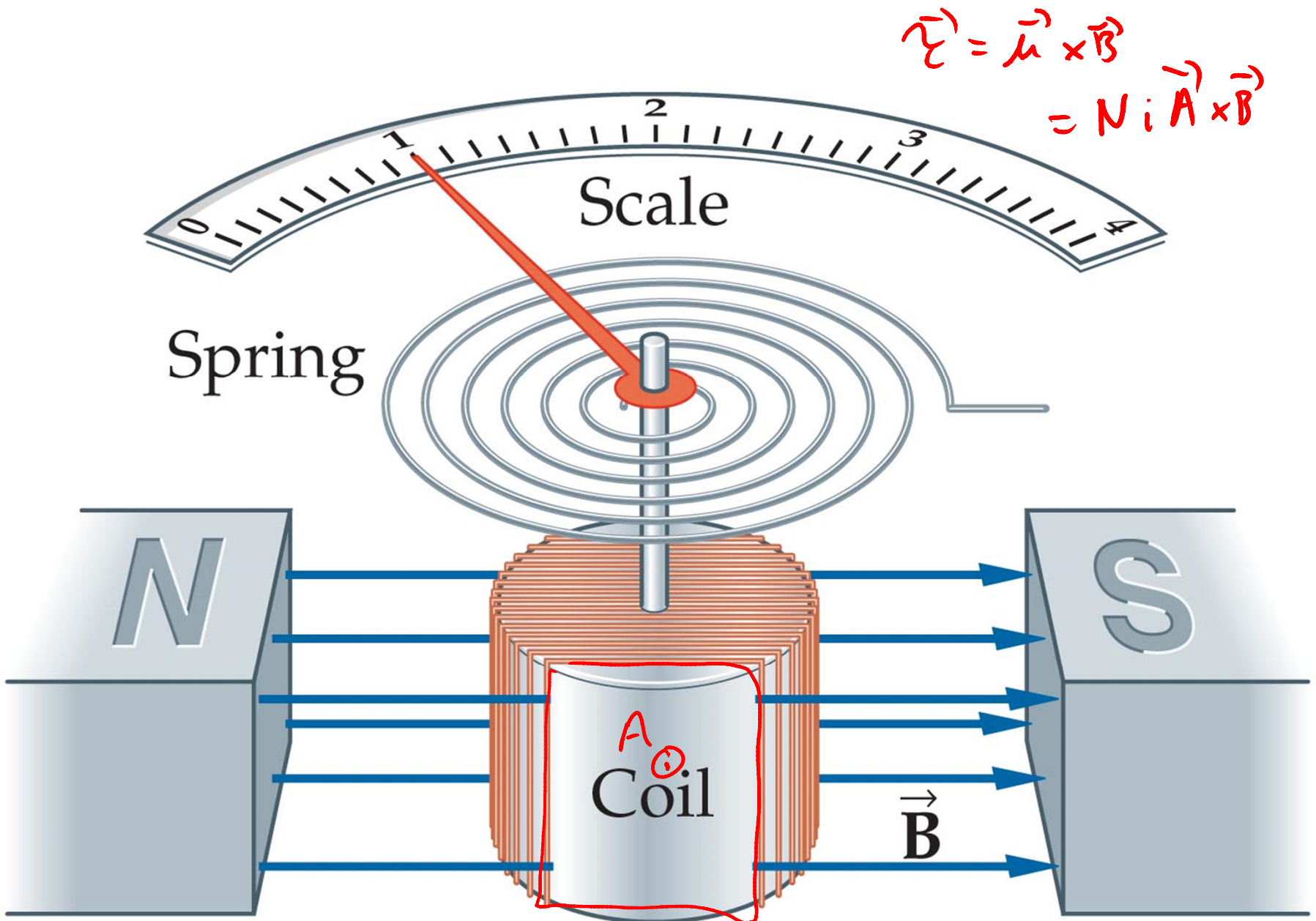
(c)

$N = \# \text{ loops}$

$$\tau = \mu B \sin \theta$$

$$\tau = \vec{\mu} \times \vec{B}$$



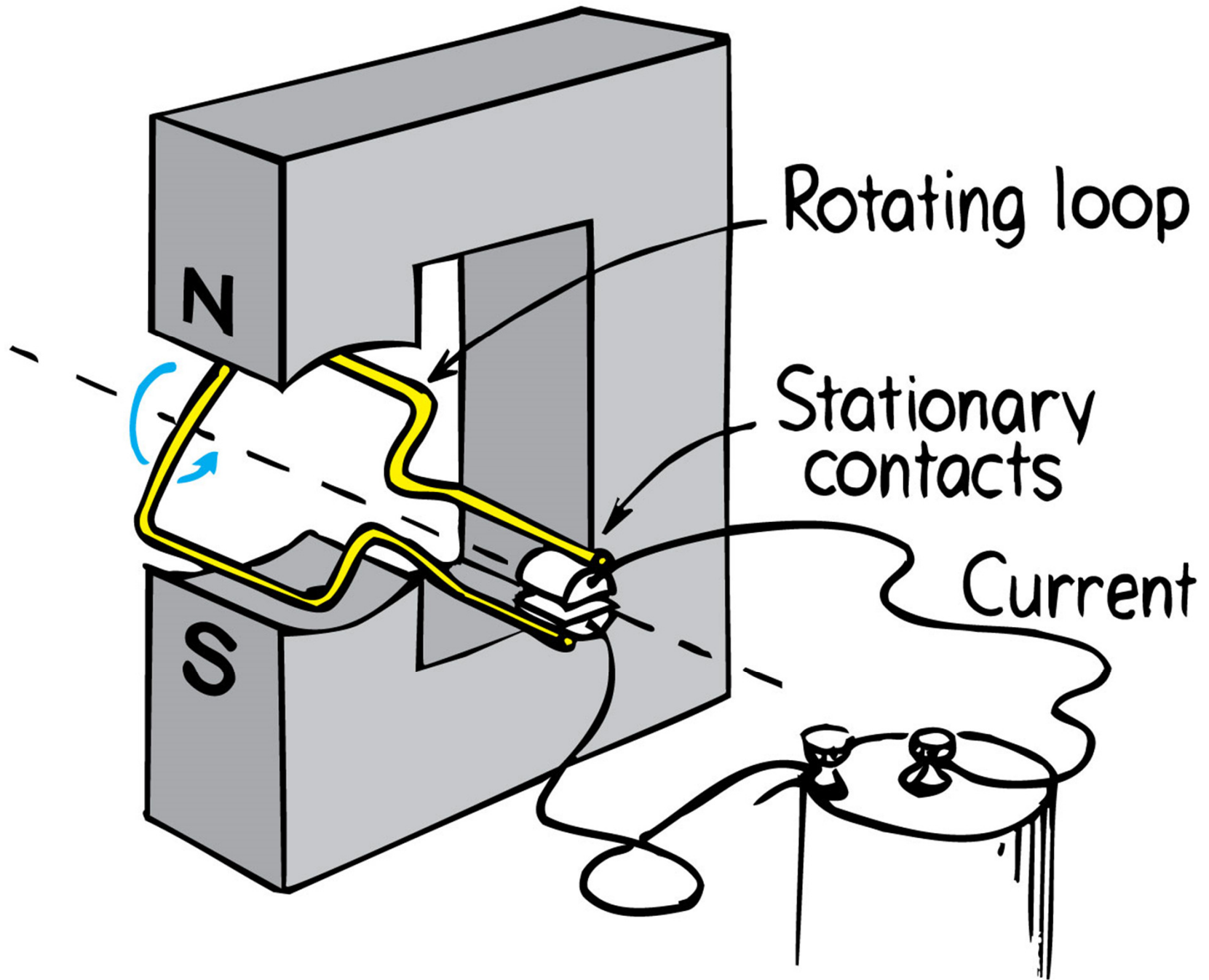


$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

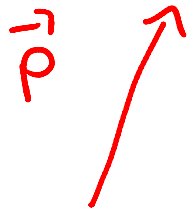
$$= Ni\vec{A} \times \vec{B}$$

$$\vec{\mu} = N \cdot I \cdot \vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



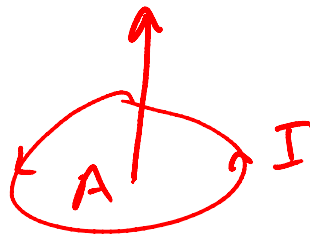
Dipole worksheet
(not to do now, but
keep it as a study aid)



$$\vec{p} = q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

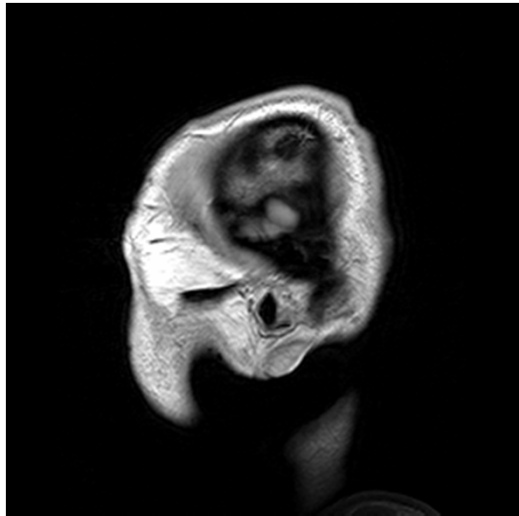
$$U = -\vec{p} \cdot \vec{E}$$



$$\vec{\mu} = NIA\vec{A}$$

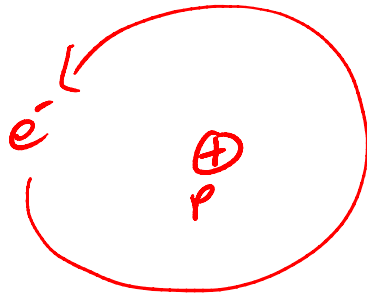
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

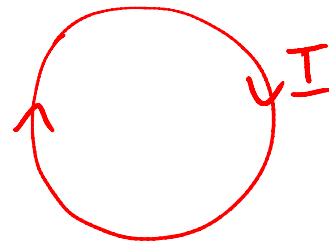


Images swiped from Wikipedia's MRI article

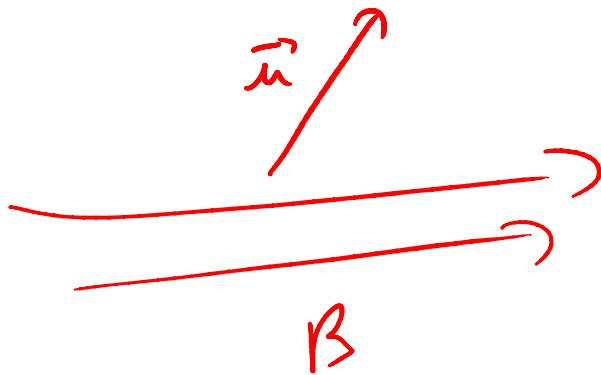
How that MRI makes images by getting H atoms to flip dipoles and change U



H



$\vec{\mu}$



Two Long Parallel Wires

and on the other side,

Forces between parallel wires