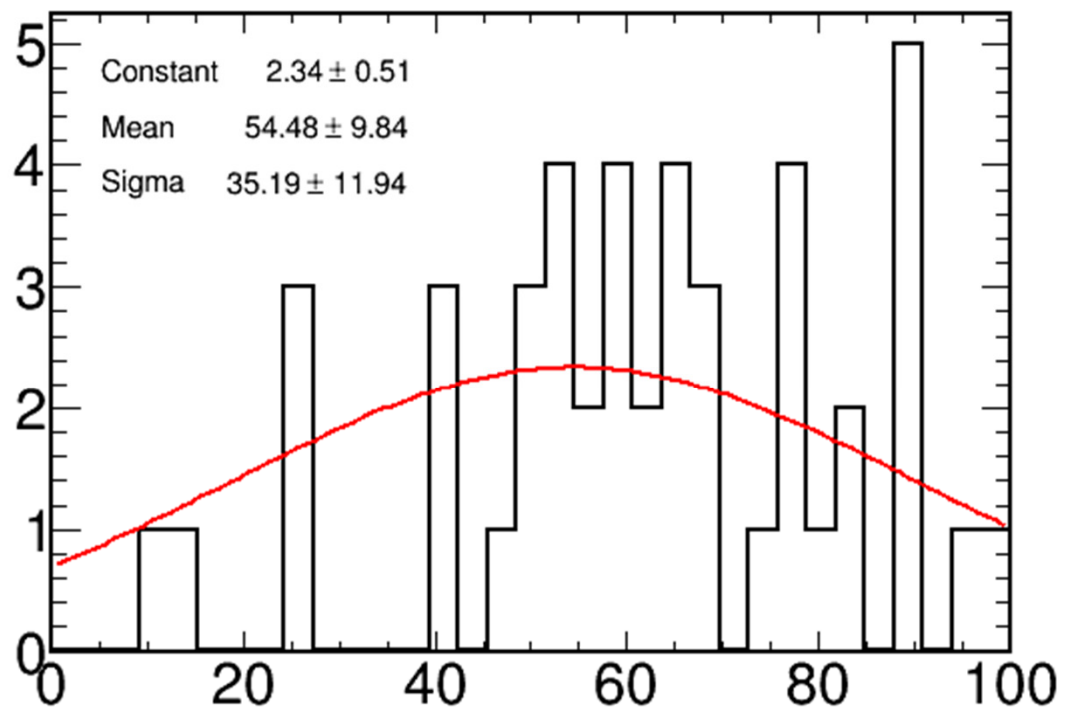
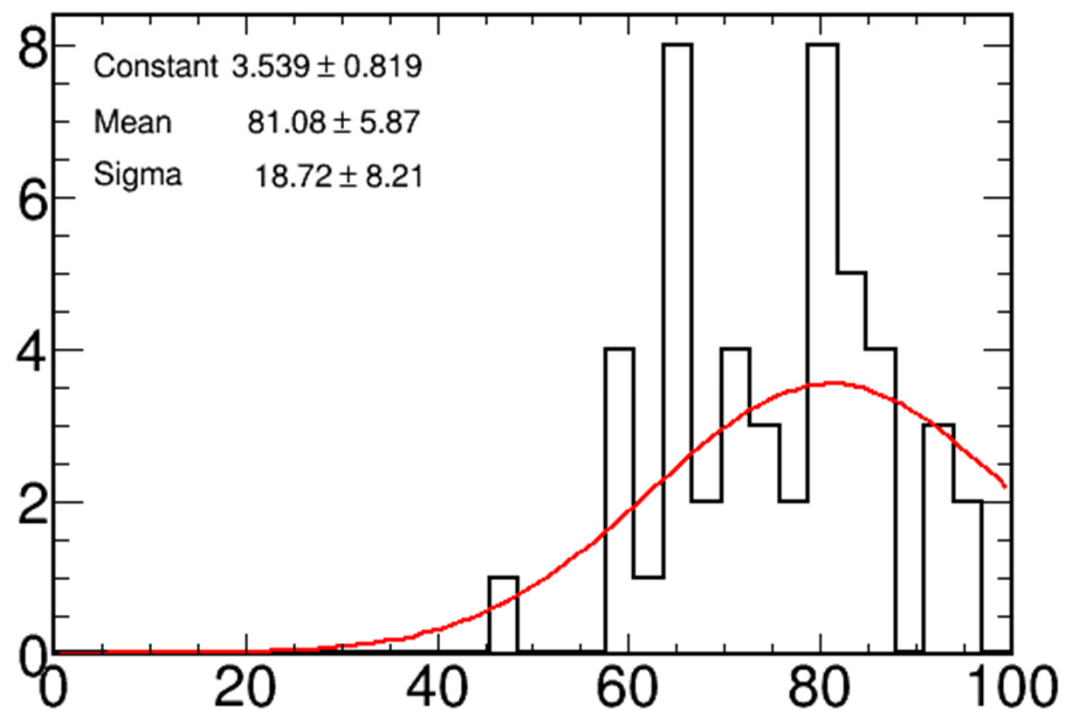


Scores on Test



Total Scores



Energy stored in magnetic field


$$u_B = \frac{U_L}{\text{Volume}} = \frac{\frac{1}{2} L I^2}{A l}$$

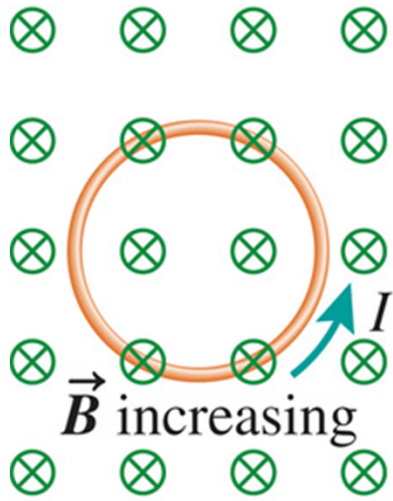
since $\frac{L}{l} = \mu_0 n^2 A$

$$u_B = (\mu_0 n^2 A) \frac{\frac{1}{2} I^2}{A} = \frac{1}{2} \mu_0 n^2 I^2$$

B in solenoid is $\mu_0 n I$

$$u_B = \frac{1}{2} \frac{B^2}{\mu_0}$$

	$U_C = \frac{1}{2} C V^2$
	$u_C = \frac{1}{2} \epsilon_0 E^2$

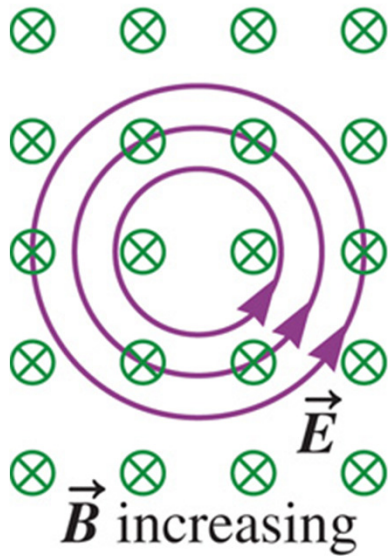


A.

Practical way to look at Faraday's law: Changing magnetic flux induces current in conducting loop, so an emf is induced in conductor.

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

$$\oint \vec{E} \cdot d\vec{e} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$



B.

General way to look at Faraday's law: Changing magnetic flux induces an electric field whether a conductor is present or not.

Maxwell's Equations

$$\oint = \int$$

Below we summarize the four equations on which electromagnetic theory is based. We use here the complete form of Ampere's law as modified by Maxwell:

Gauss' law for \vec{E} :

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Gauss' law for \vec{B} :

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

Faraday's law :

$$\oint \vec{E} \cdot d\vec{S} = -\frac{d\Phi_B}{dt}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

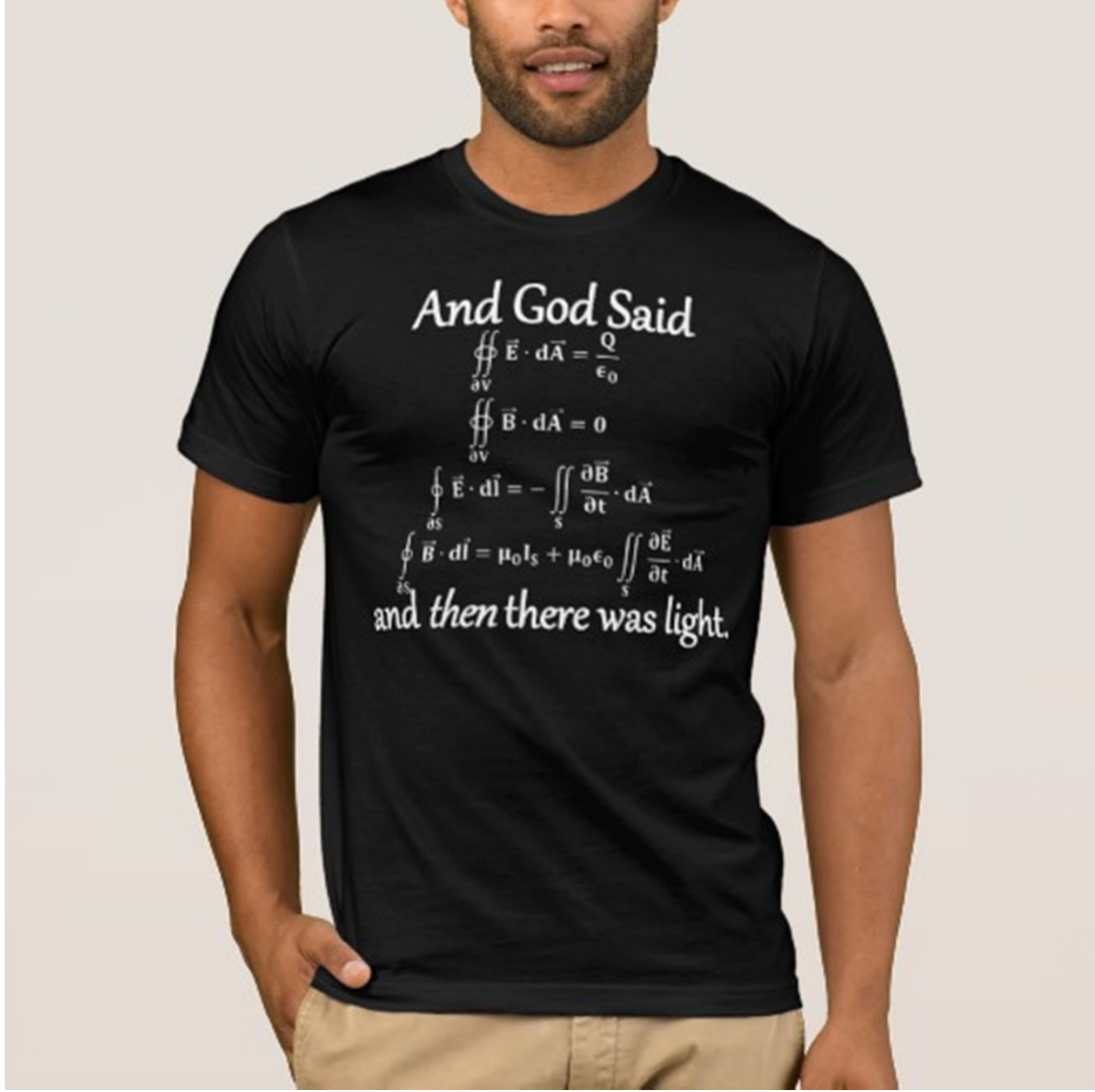
Ampere's law :

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 i_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

These equations describe a group of diverse phenomena and devices based on them such as the magnetic compass, electric motors, electric generators, radio, television, radar, x-rays, and all optical effects.

All these in just four equations!



And God Said

$$\oiint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oiint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{\partial S} \vec{E} \cdot d\vec{l} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

$$\oint_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 I_s + \mu_0 \epsilon_0 \iint_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

and then there was light.

God said,

$$\nabla \cdot \mathbf{D} = \rho$$

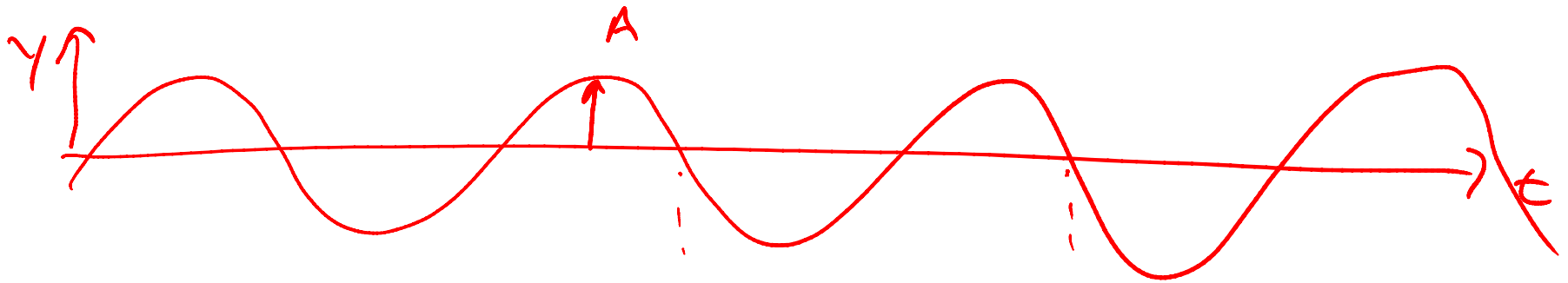
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

And there was light.

Wave review



$$y(t) = A \sin(\omega t - \phi) \quad T \quad \omega = 2\pi f \quad f = \frac{1}{T}$$

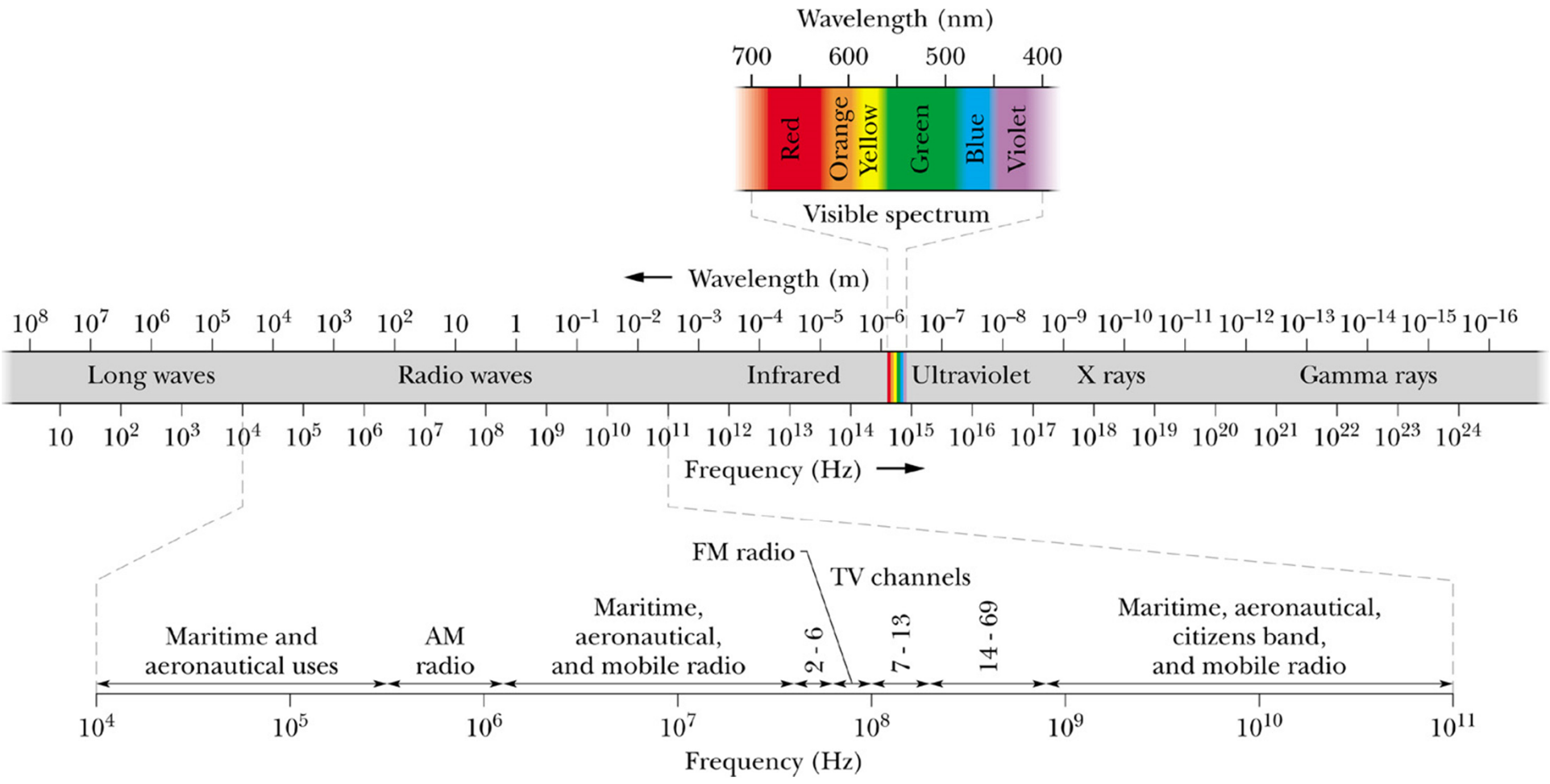
$$y(x, t) = A \sin(kx - \omega t)$$

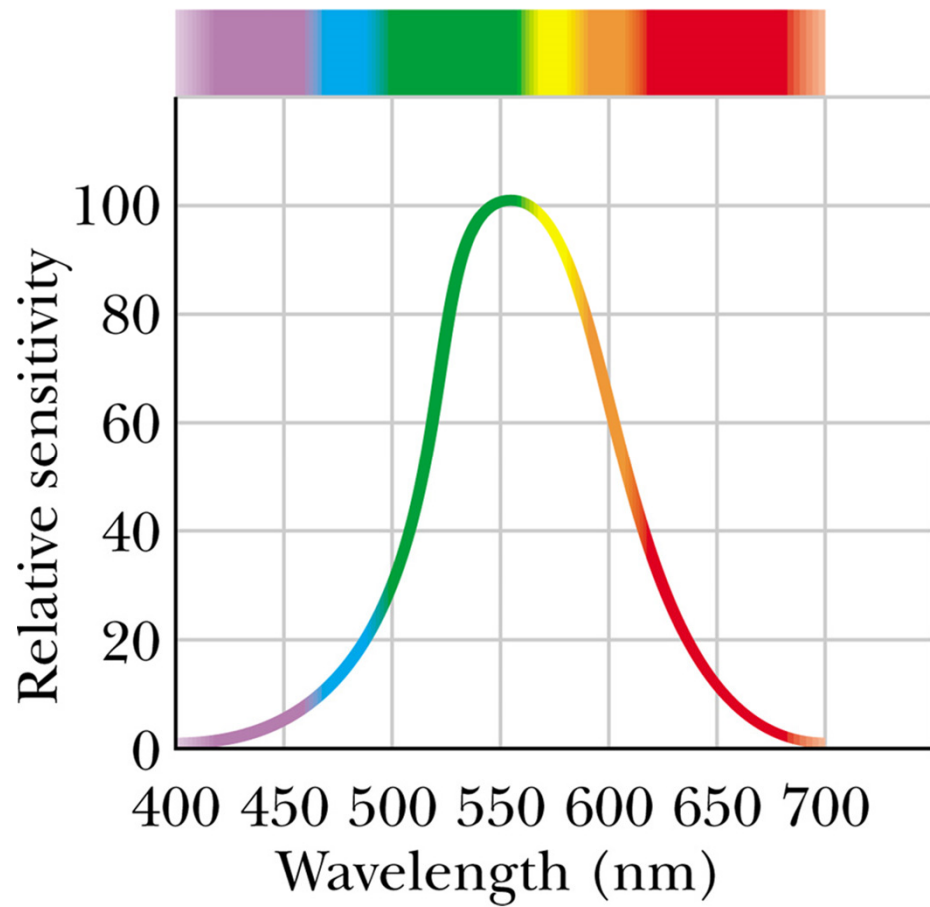
$$\text{speed } c = \frac{\lambda}{T} = \frac{\text{meters/cycle}}{\text{sec/cycle}} = \lambda f = \frac{\omega}{k} \quad \frac{\text{rad/sec}}{\text{rad/m}}$$

$$\text{Energy} \propto A^2$$

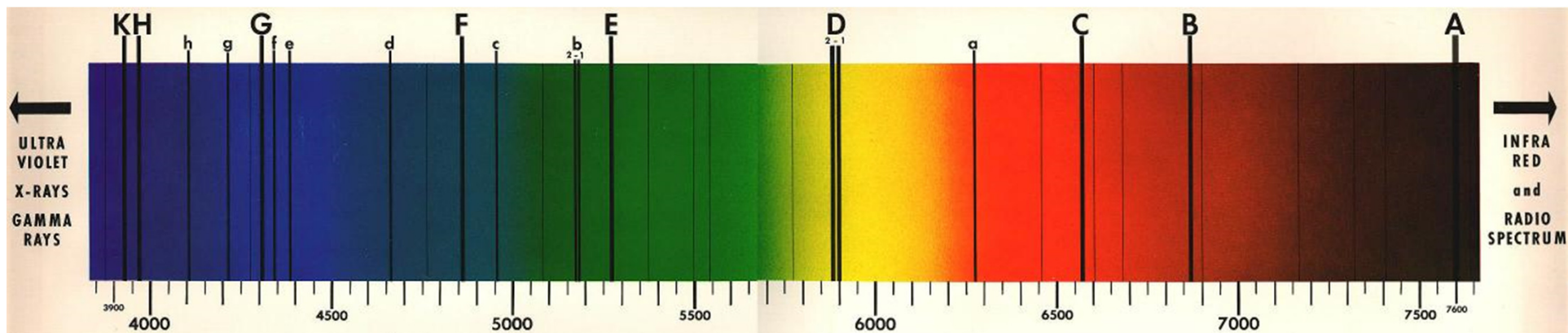
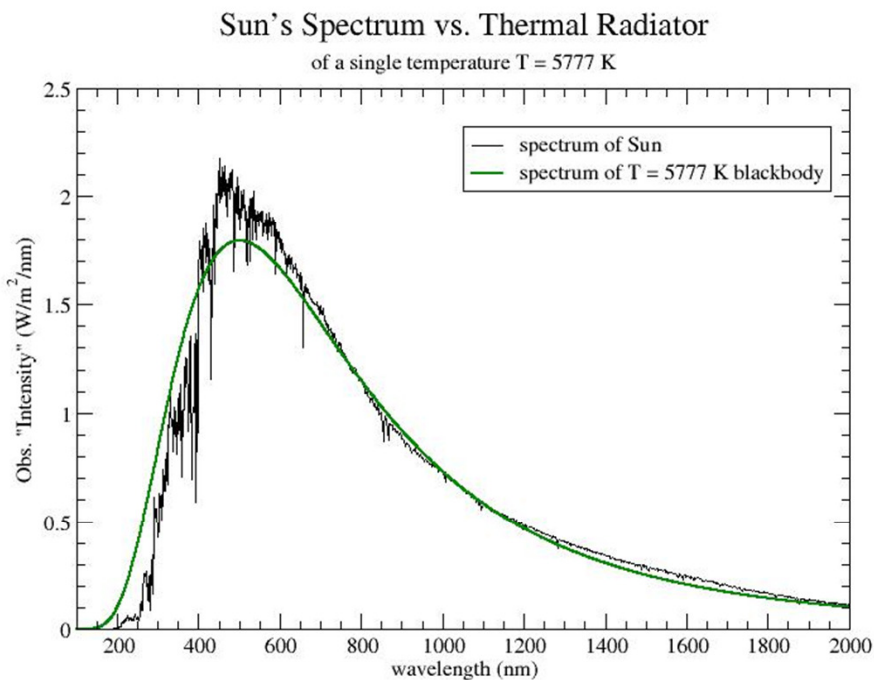
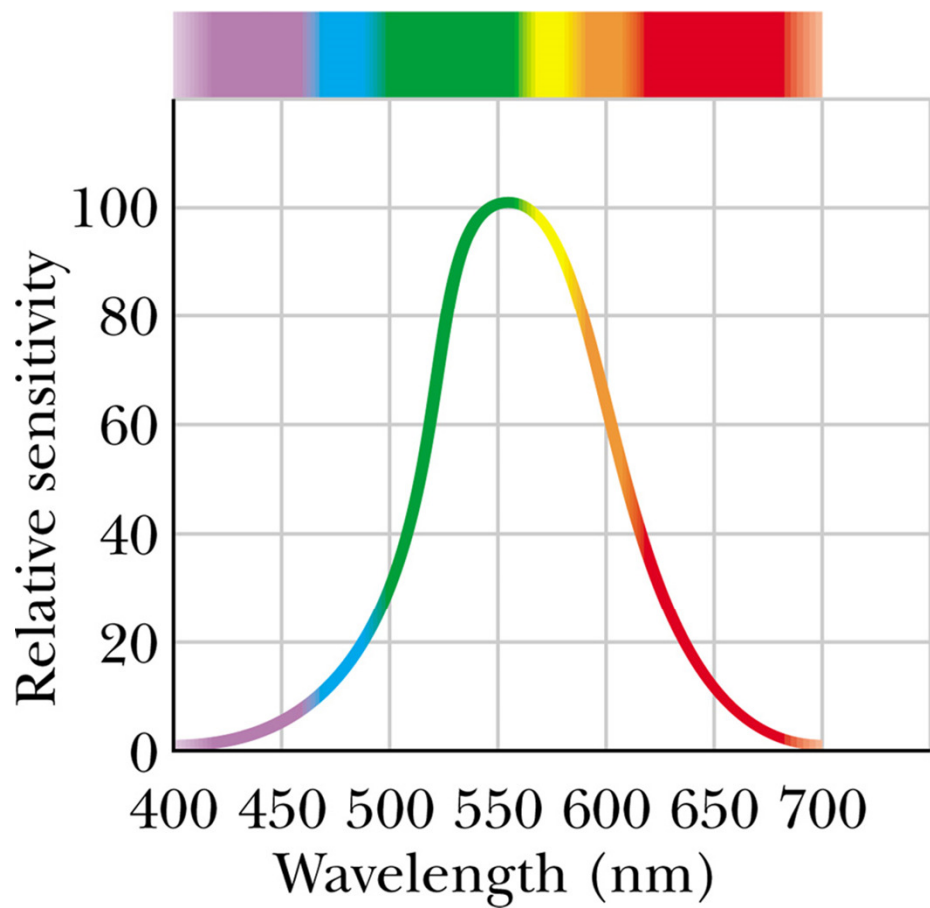
Wave review

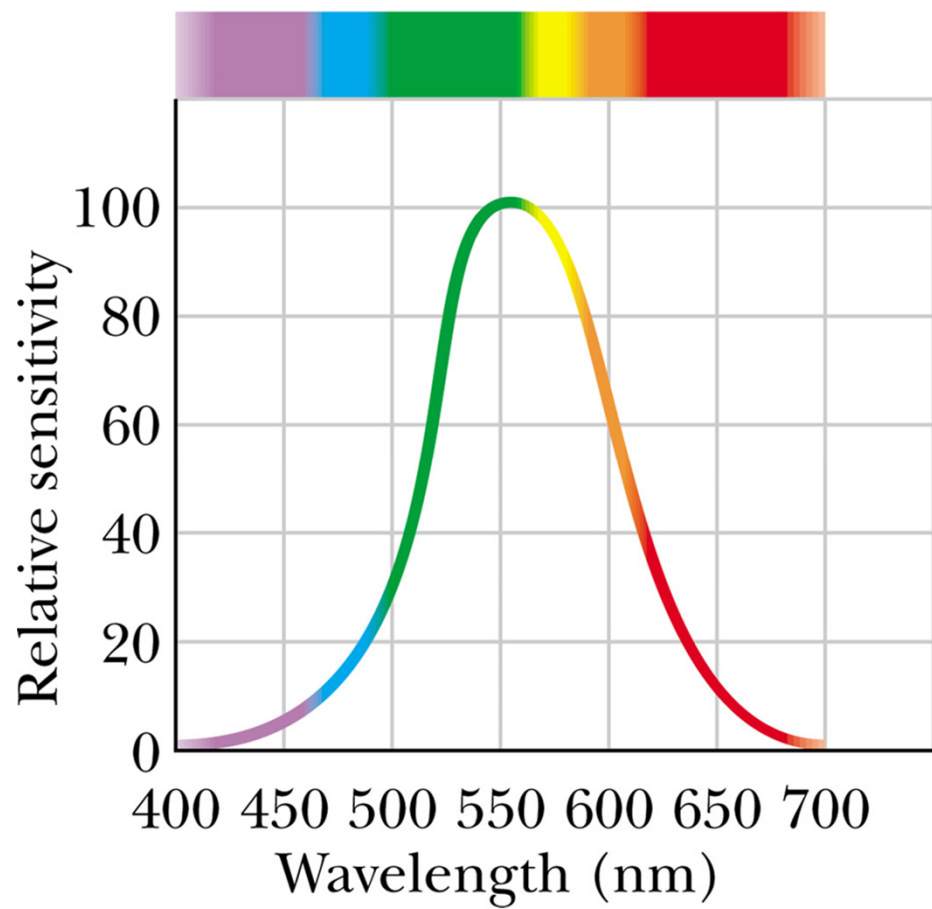
$$c = \lambda f$$

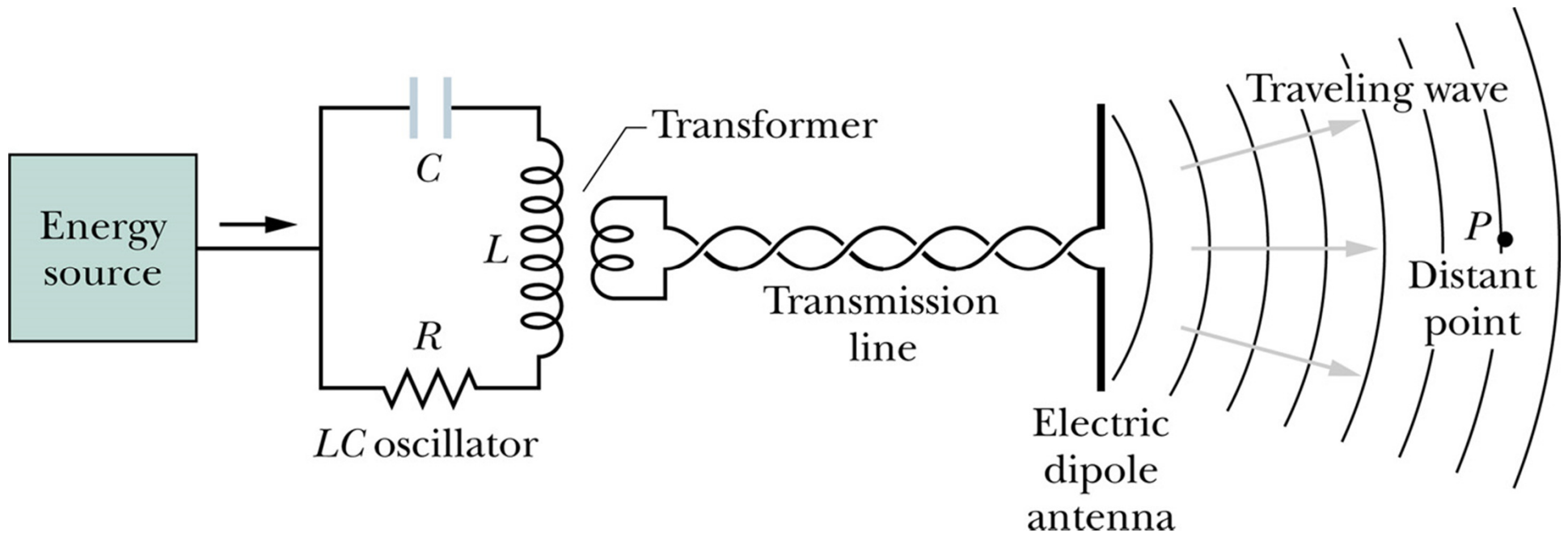




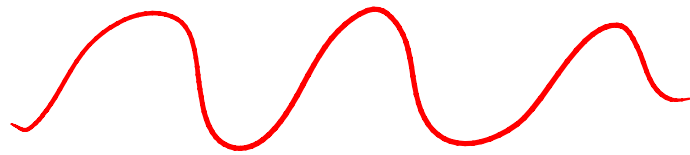
Response of your eye



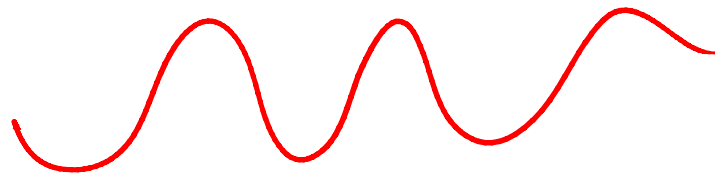


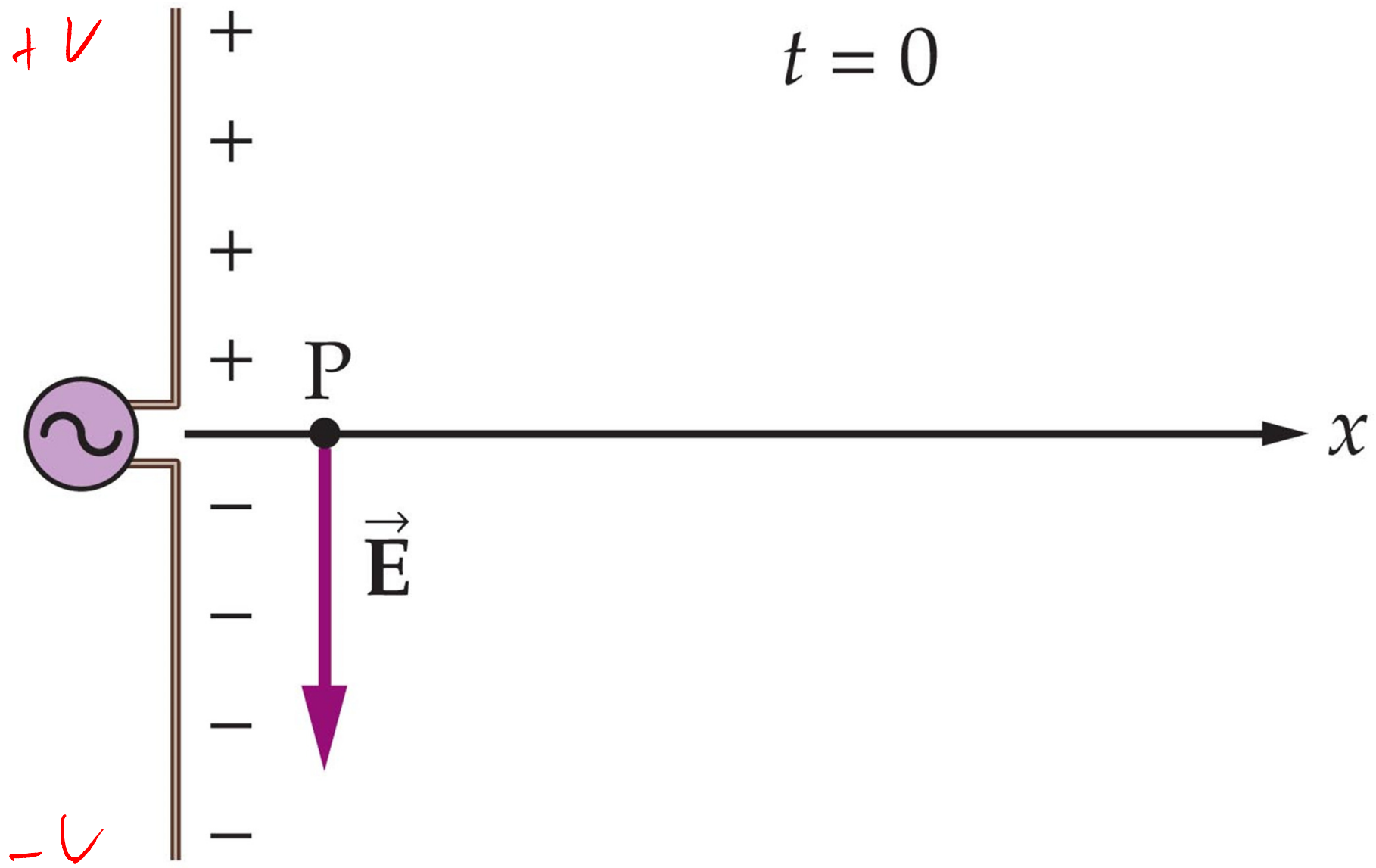


$$V(t)$$

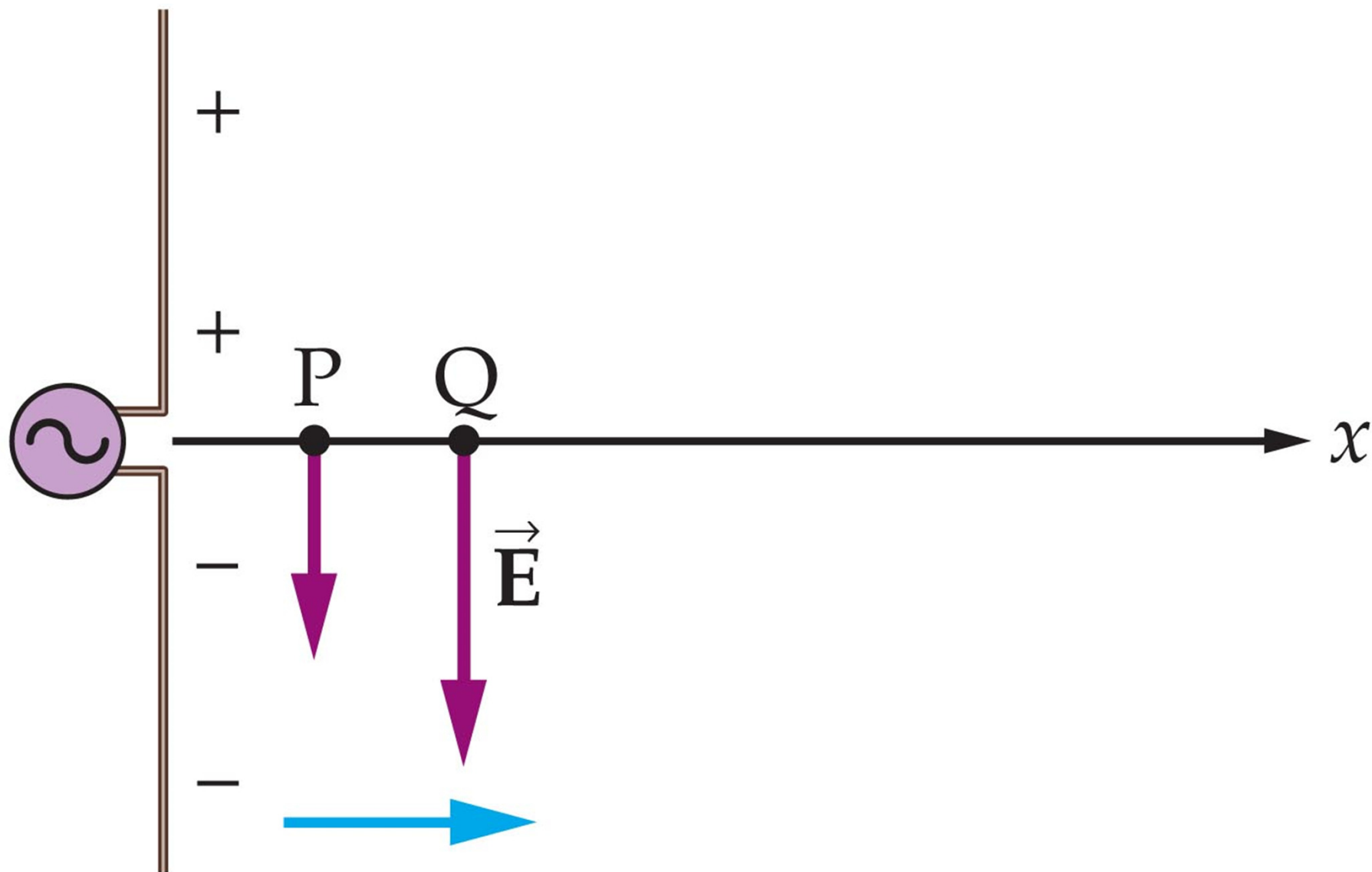


$$I(t)$$



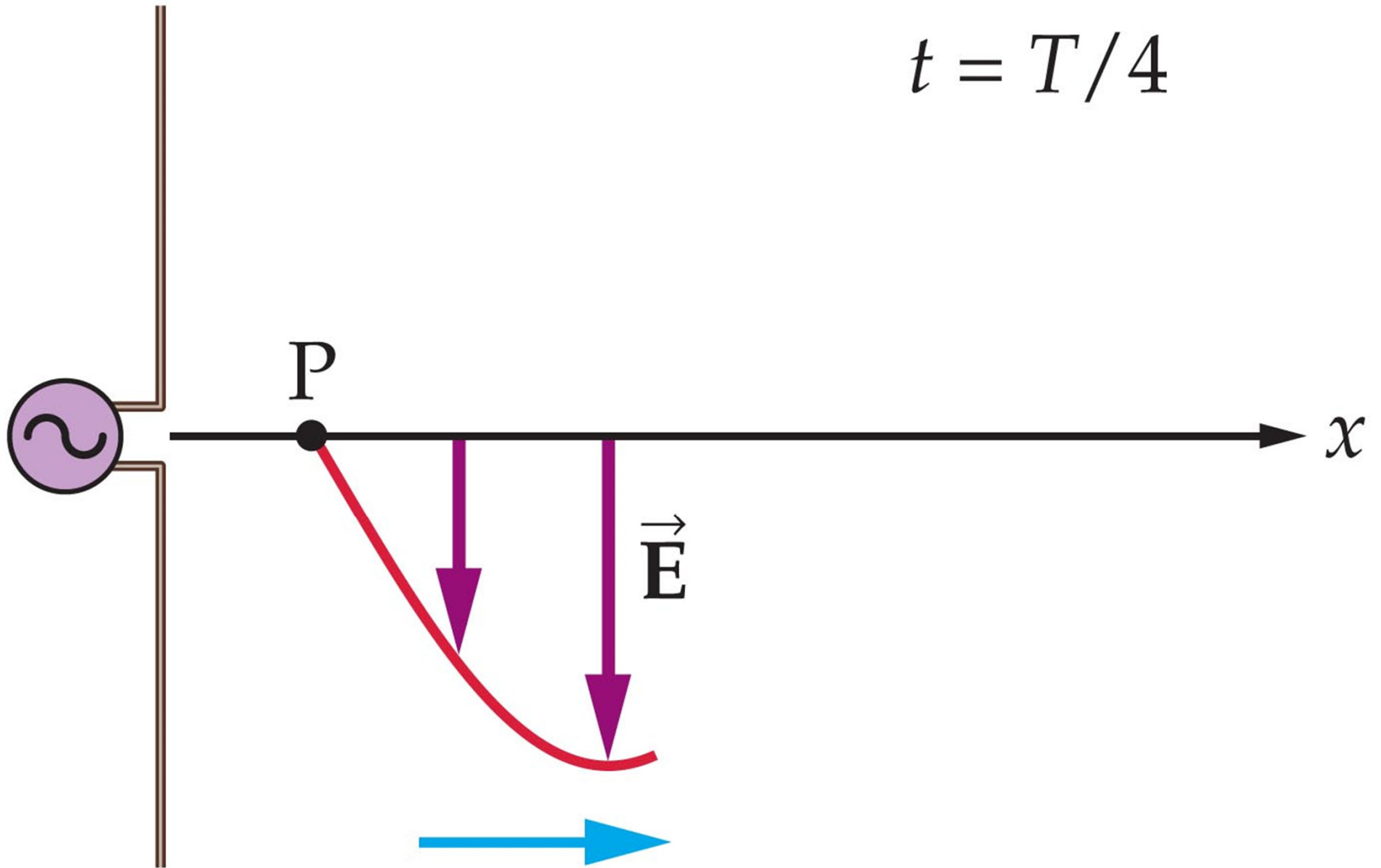


(a)

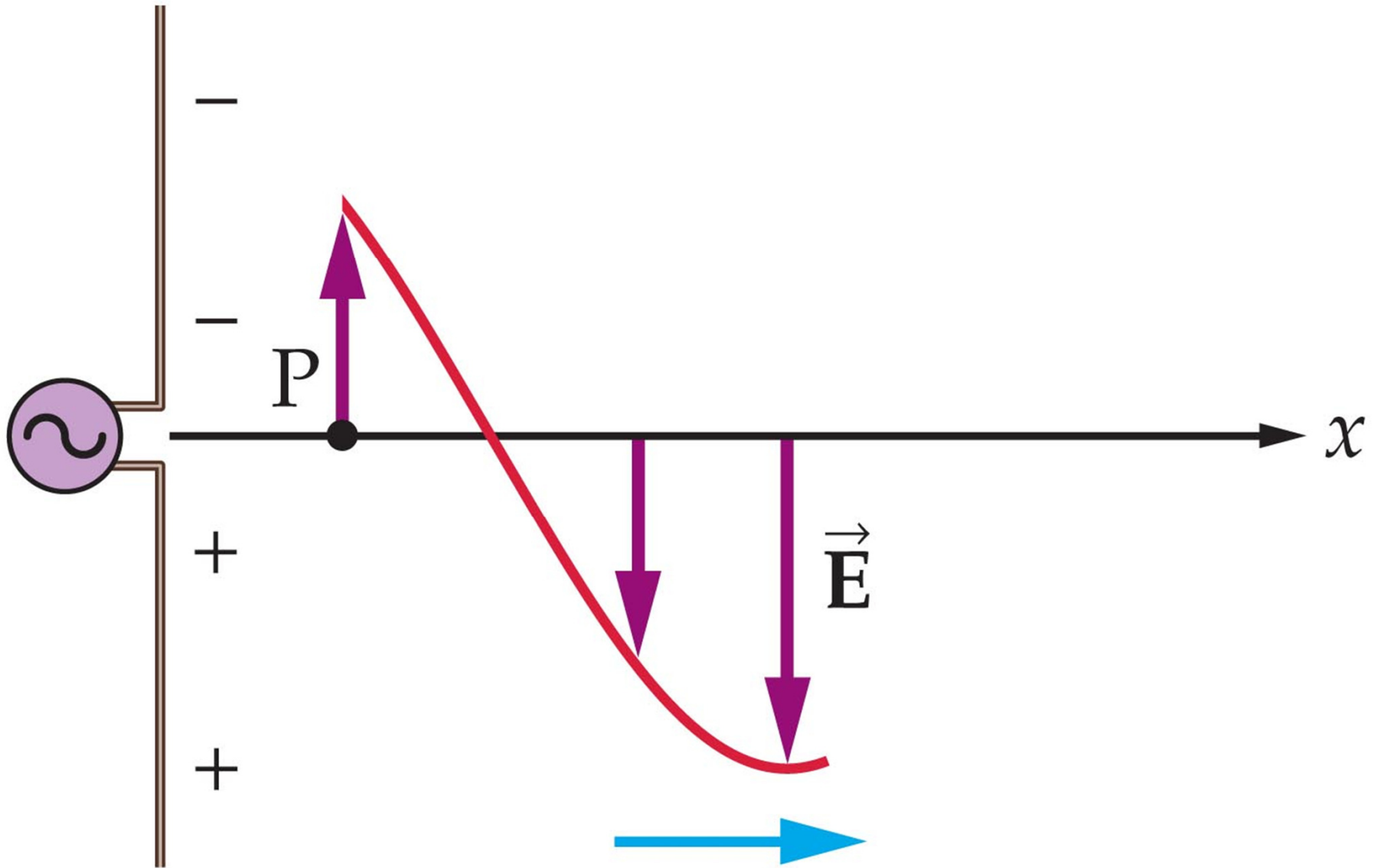


(b)

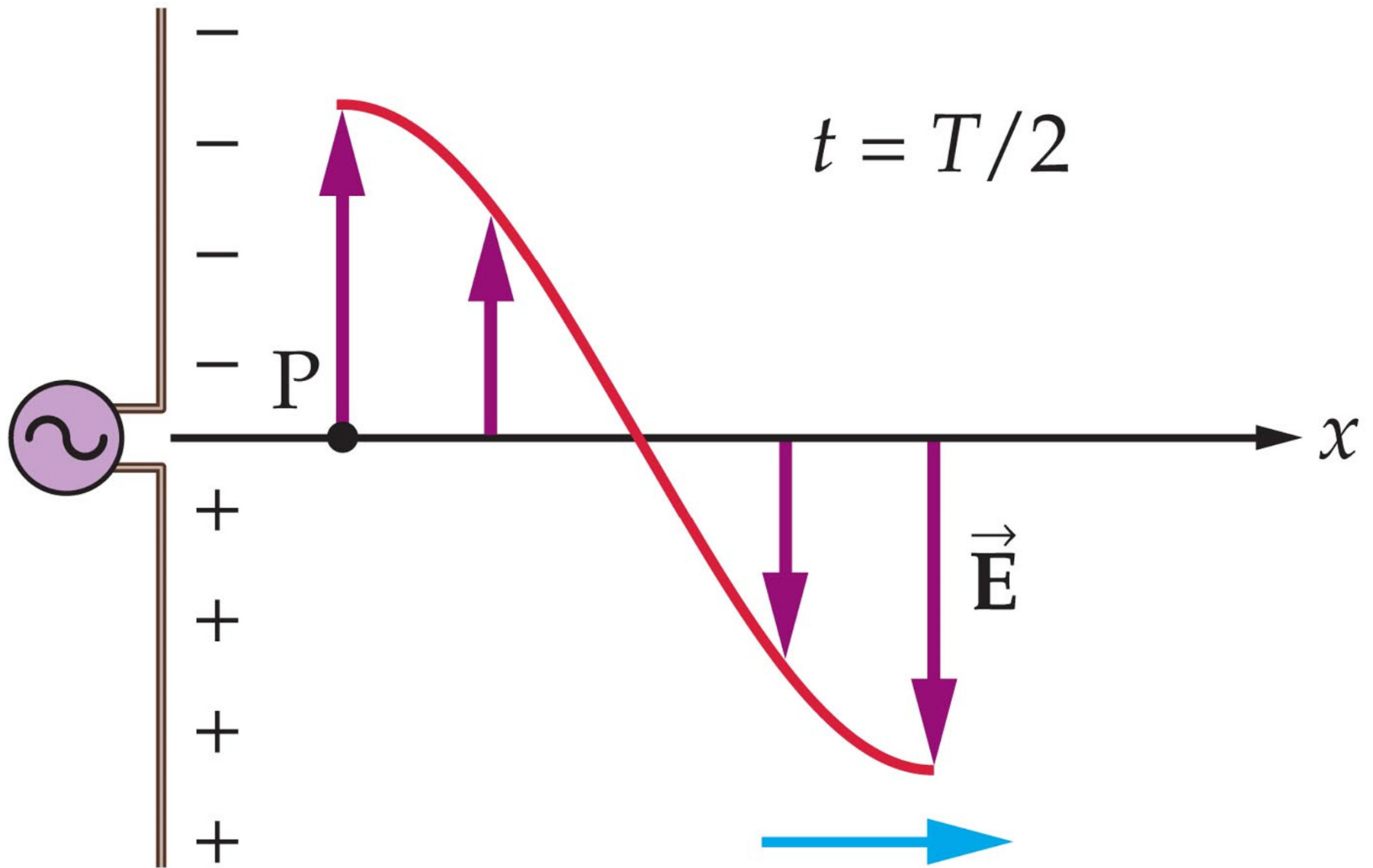
$$t = T/4$$



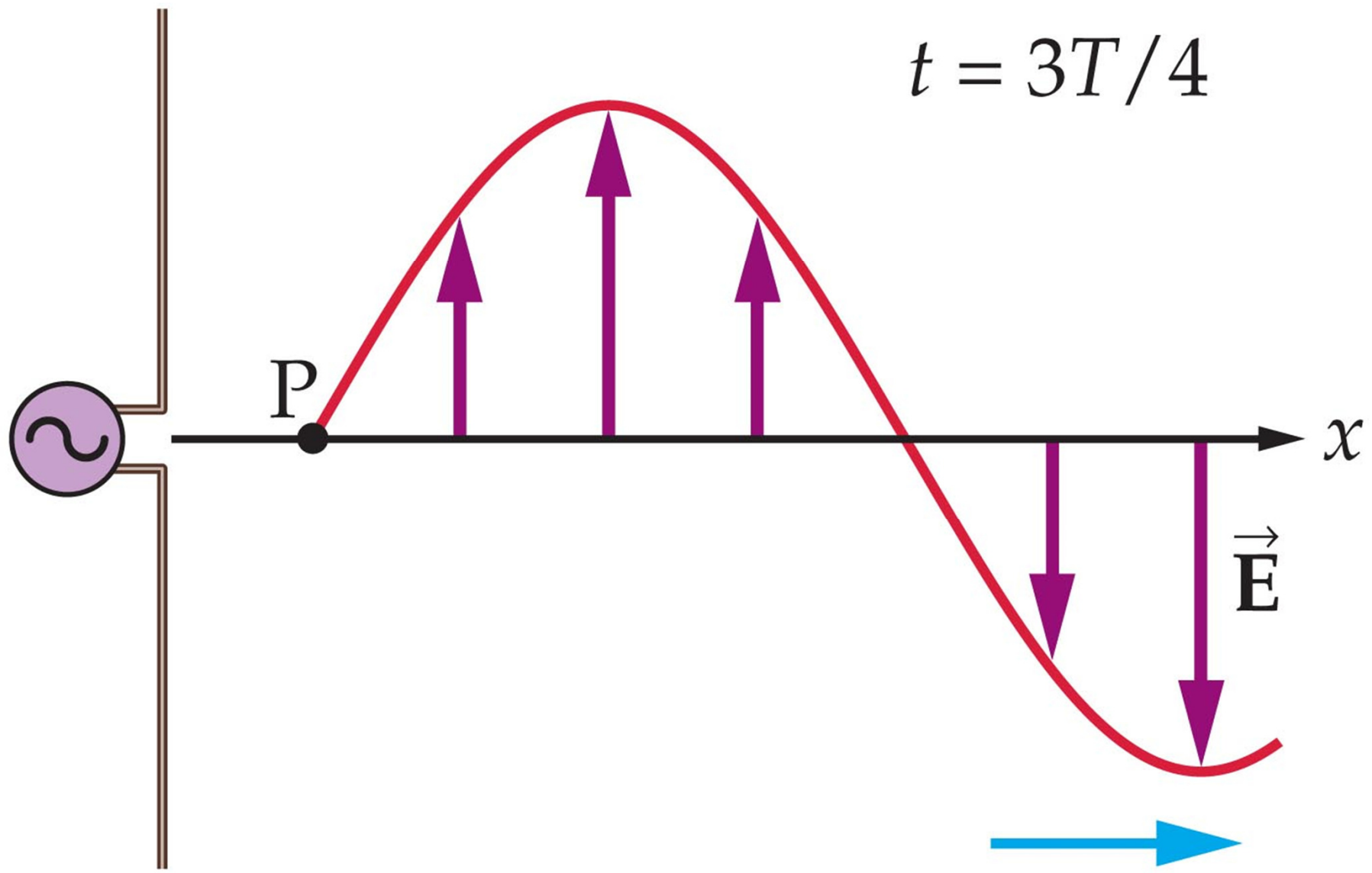
(c)



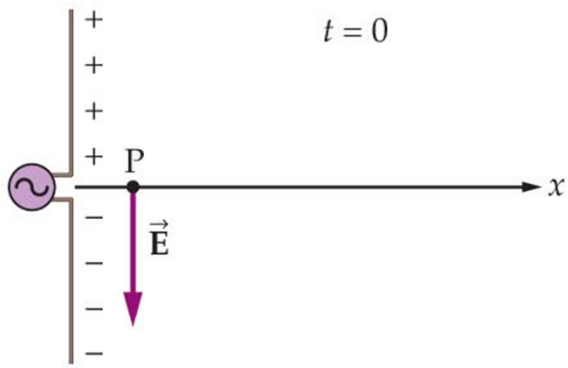
(d)



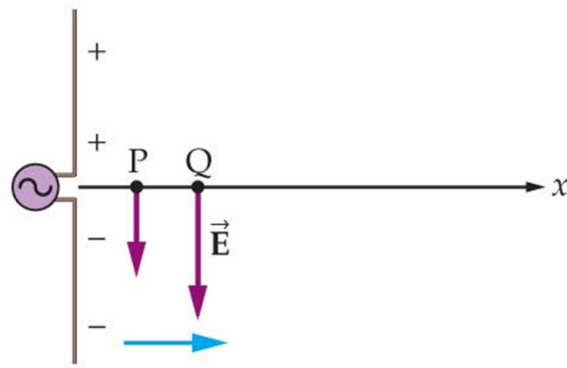
(e)



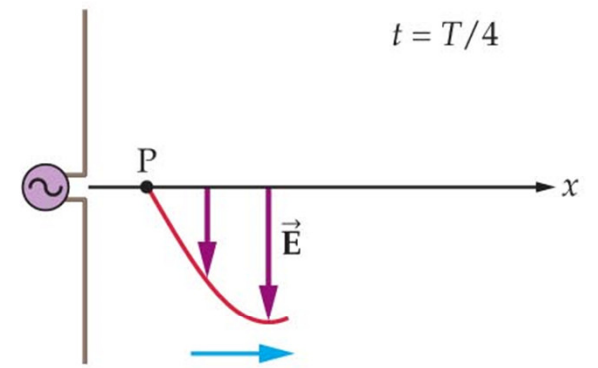
(f)



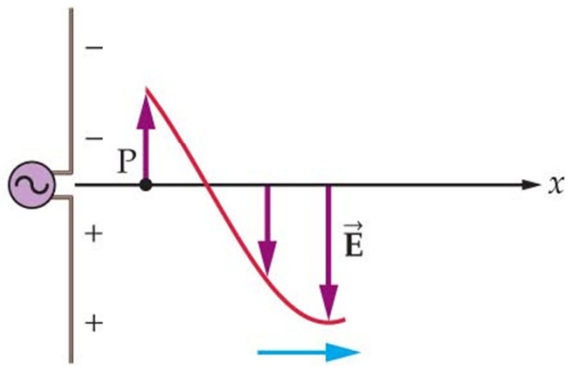
(a)



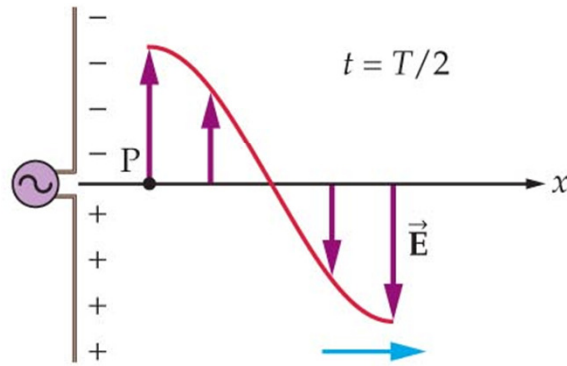
(b)



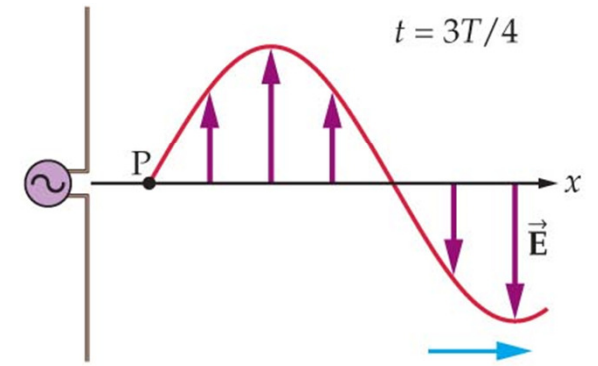
(c)



(d)

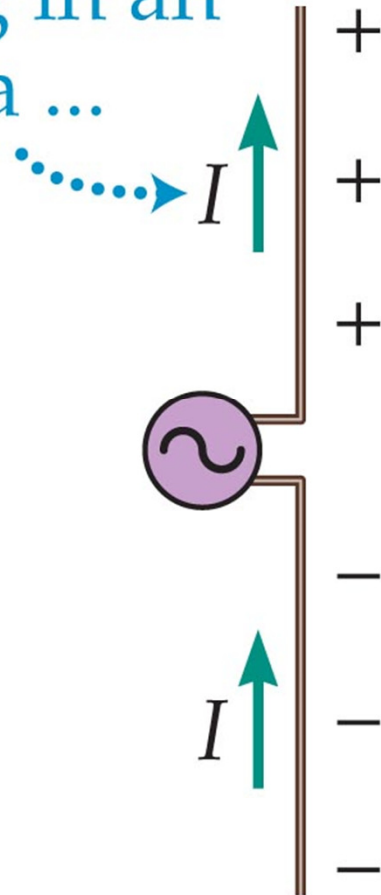


(e)



(f)

A current
flowing in an
antenna ...

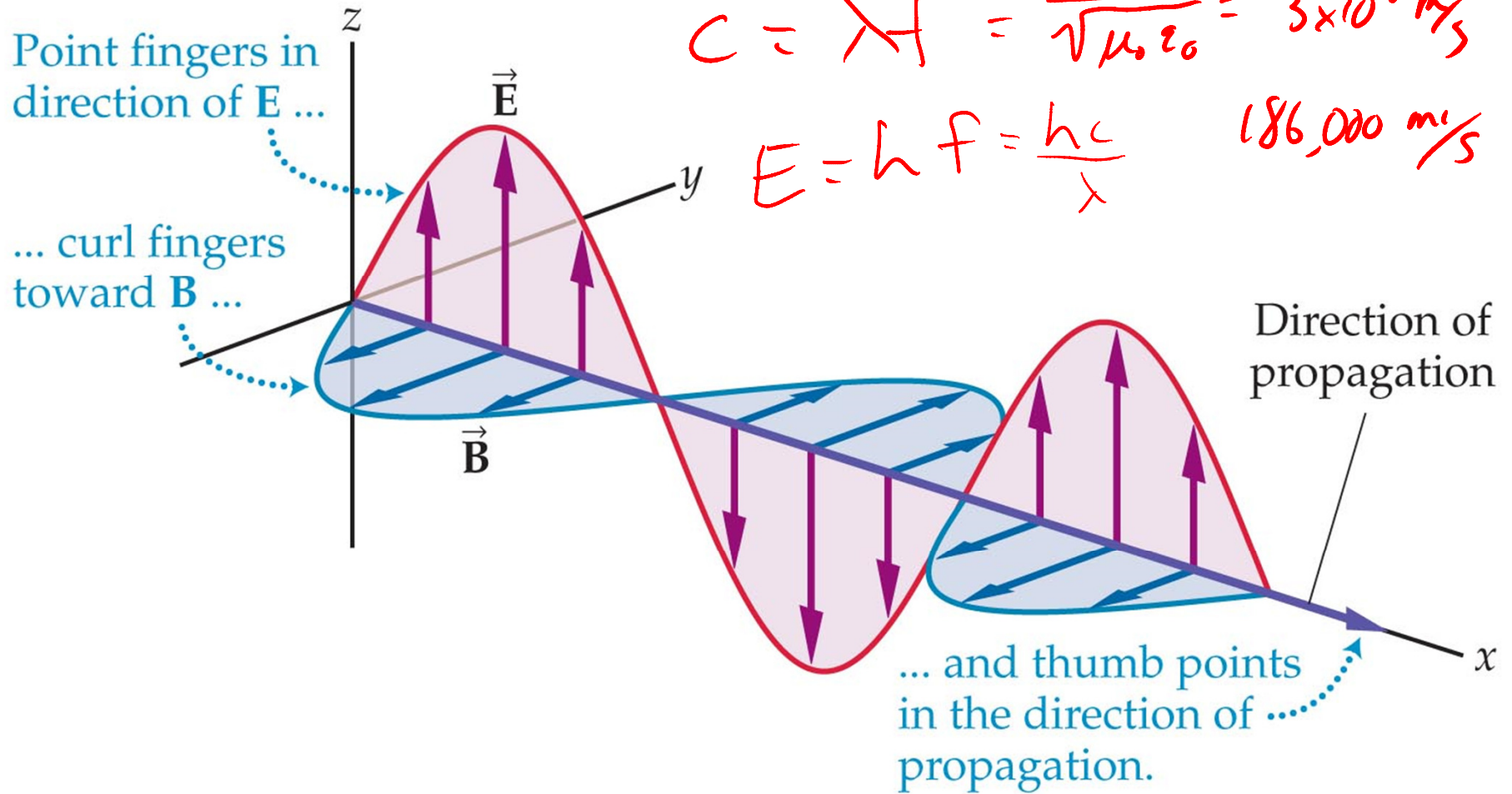


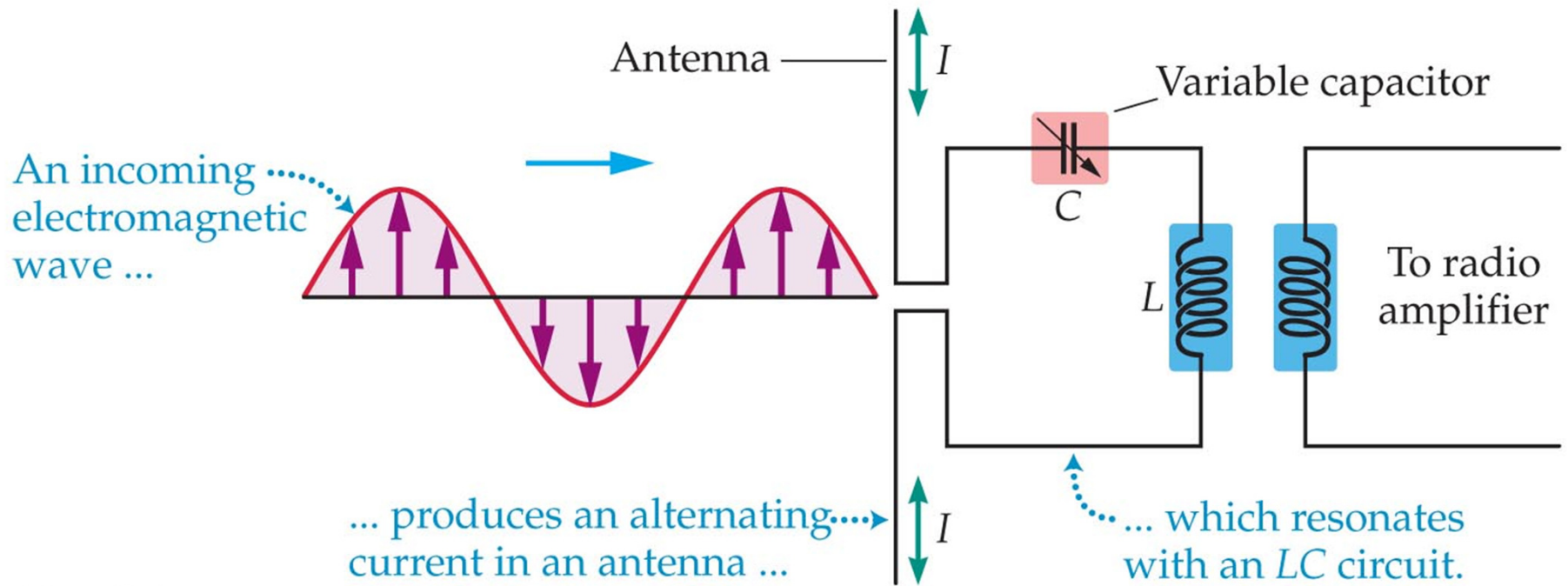
... produces
both electric
and magnetic
fields.

$\vec{E} \times \vec{B}$ gives direction of motion

$$c = \lambda f = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

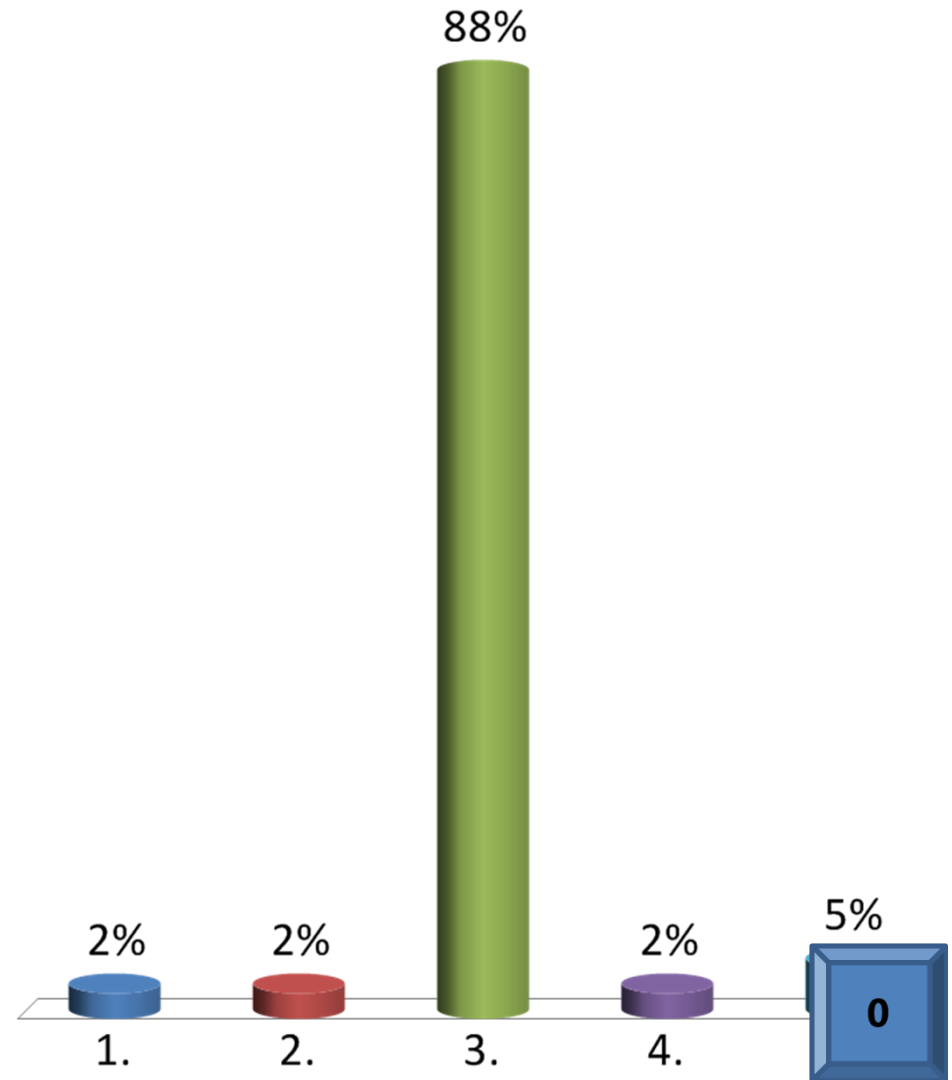
$$E = hf = \frac{hc}{\lambda} \quad 186,000 \text{ m/s}$$



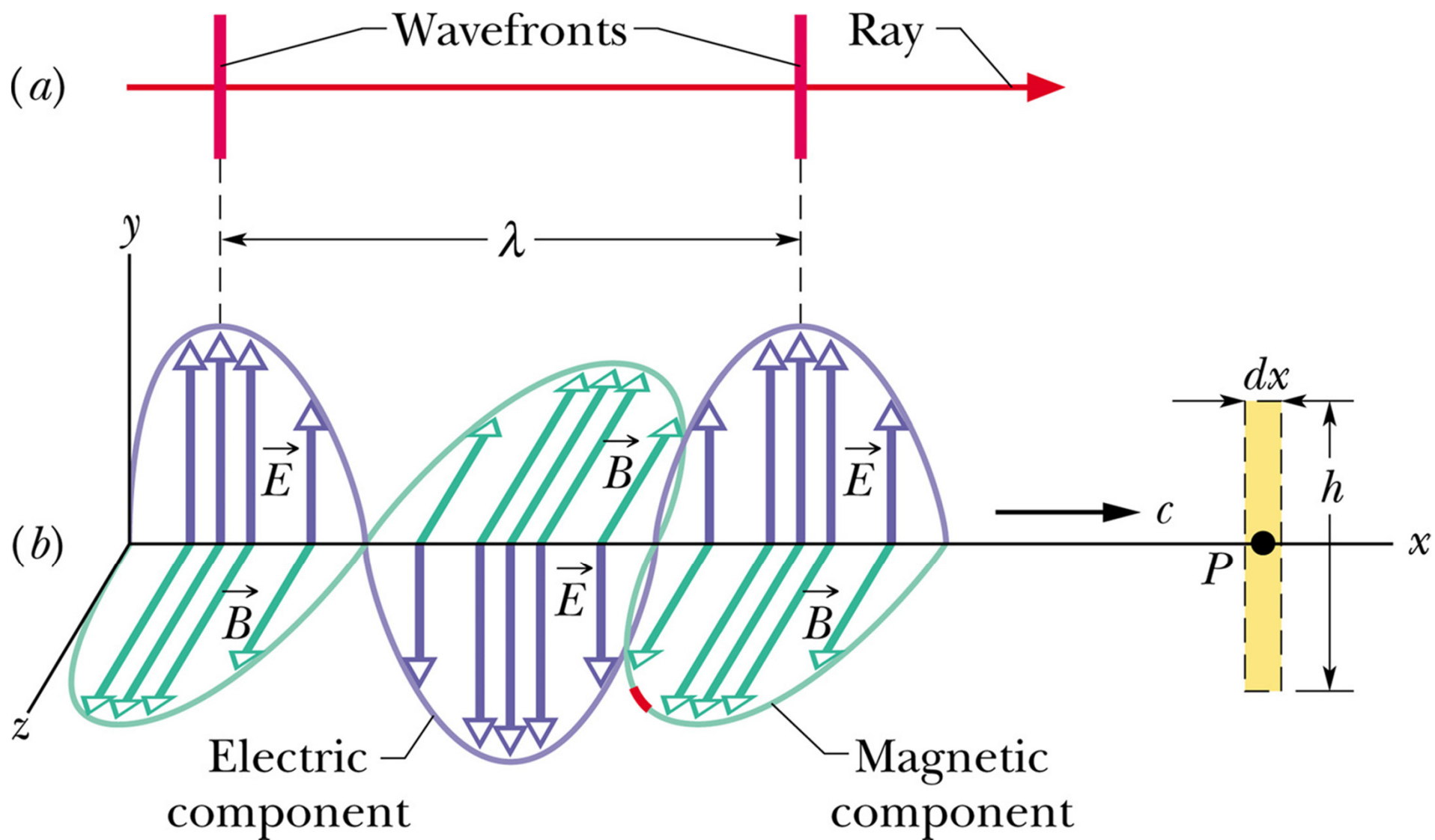


For which one of the following properties do visible light and ultraviolet waves have the same value?

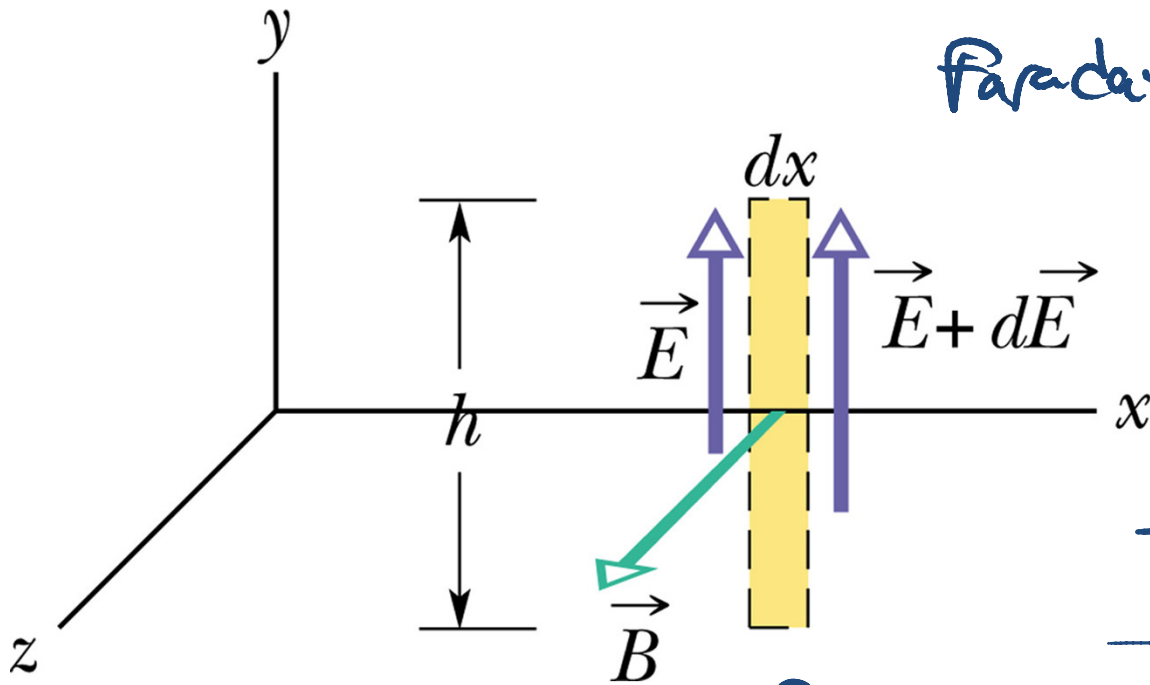
1. Wavelength
2. Frequency
- ✓ 3. Speed
4. Energy
5. Period



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Faraday's Law $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$



$$\Phi_B = \text{B Area} = B h dx$$

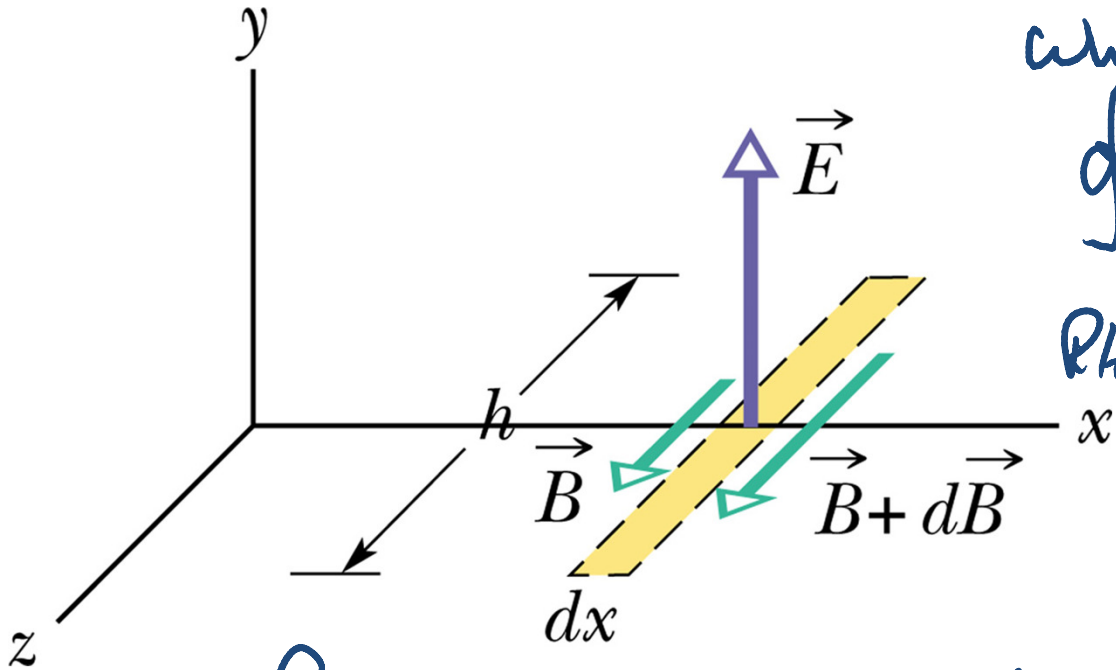
$$\frac{d\Phi_B}{dt} = \frac{dB}{dt} h dx$$

$$\begin{aligned} \oint \vec{E} \cdot d\vec{l} &= (E dx)h + 0 - Eh + 0 \\ &= (dE)h \end{aligned}$$

Combine in Faraday's Law $(dE)h = -h dx \frac{dB}{dt}$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

See page 1096 in your book for the complete workup



why not Ampere's Law?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 \vec{I}_{enclosed}$$

RHS

$$\frac{d\Phi_E}{dt} = \frac{d}{dt} (E h dx)$$

$$= h dx \frac{dE}{dt}$$

LHS

$$\int \vec{B} \cdot d\vec{l} = Bh + 0 - (B + dB)h + 0 = -h dB$$

$$-h dB = \mu_0 \epsilon_0 h dx \frac{dE}{dt}$$

$$- \frac{dB}{dx} = \mu_0 \epsilon_0 \frac{dE}{dt}$$

$$\frac{1}{\epsilon_0 \mu_0} = \left(\frac{\omega}{k} \right)^2 = c^2$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

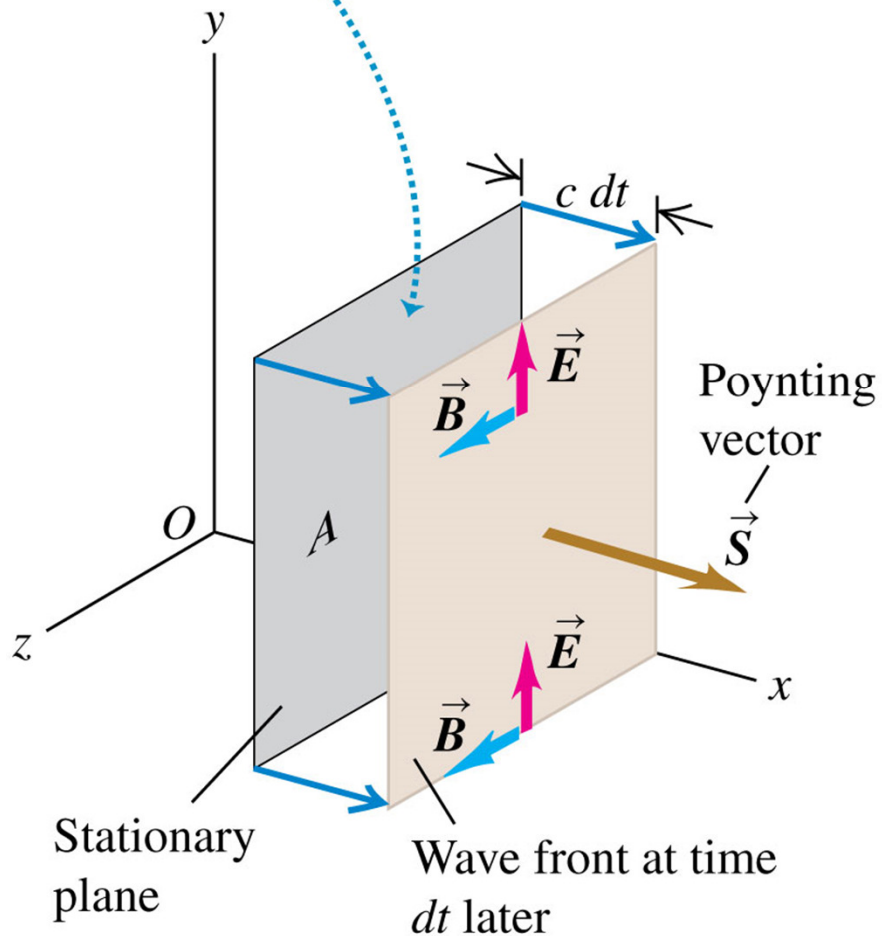
Energy?

$$U_{\text{TOT}} = \underbrace{\frac{1}{2} \epsilon_0 E^2}_{u_E} + \underbrace{\frac{1}{2\mu_0} B^2}_{u_B}$$

$$B = E/c = \sqrt{\epsilon_0 \mu_0} E$$

$$u_{\text{TOT}} = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} (\sqrt{\epsilon_0 \mu_0} E)^2 = \epsilon_0 E^2$$

At time dt , the volume between the stationary plane and the wave front contains an amount of electromagnetic energy $dU = uAc dt$.



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$$dU = u_{\text{TOT}} dV$$

$$= (\epsilon_0 E^2) (A c \cdot dt)$$

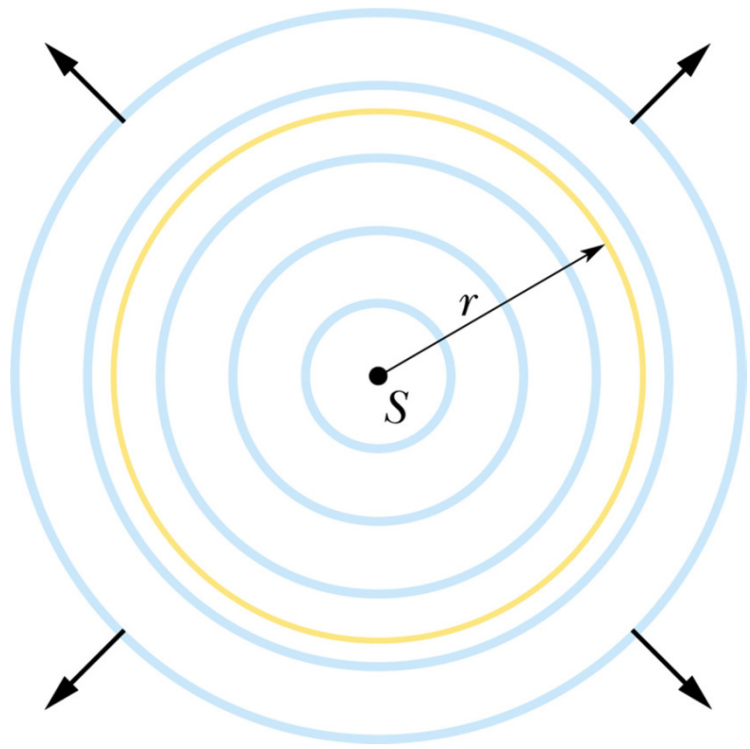
$$S = \frac{dU/dt}{A} = \epsilon_0 c E^2$$

$$S = \frac{\epsilon_0 E^2}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2$$

Or, use $B = E/c$

$$S = \frac{EB}{\mu_0} \quad \left[\frac{J}{s \cdot m^2} = \frac{W}{m^2} \right]$$

$$S = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$



S was $\frac{\text{Power}}{\text{Area}}$

$$\frac{P_{\text{source}}}{4\pi r^2} = \frac{\text{Power}}{\text{Area}} = I$$

Intensity

Poynting Vector $E \times B = \frac{(E_{\text{avg}})^2}{c\mu_0} = \frac{P_{\text{source}}}{4\pi r^2}$

Polaris

431 l.y. away

$L = 2200 L_{\odot}$



Students: work out what is the average electric field produced by light from Polaris in your eye?

Polaris puts out $2200 L_{\odot}$ | $L_{\odot} = 3.90 \times 10^{26} \text{ W}$

it is 431 l.y. away. | $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$

$$I = \frac{E_{\text{avg}}^2}{c \mu_0} \quad I = \frac{\text{Power}}{\text{Area}} = \frac{2200 L_{\odot}}{4\pi (431 \text{ ly})^2}$$

$$\text{so } E_{\text{avg}} = \sqrt{\frac{\text{Power } c \mu_0}{4\pi r^2}} \\ = 1.24 \text{ mV/m}$$

$$\frac{E}{B} = c \Rightarrow B = \frac{E}{c} = 4.1 \text{ pT}$$

Momentum

Shine light for Δt transfers $\Delta p = \frac{\Delta U}{c}$

$$F = \frac{\Delta p}{\Delta t} \quad F = \frac{d(mu)}{dt} = m \frac{du}{dt} = ma$$

$$I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Energy/Time}}{\text{Area}}$$

$$\Delta U = IA \Delta t$$

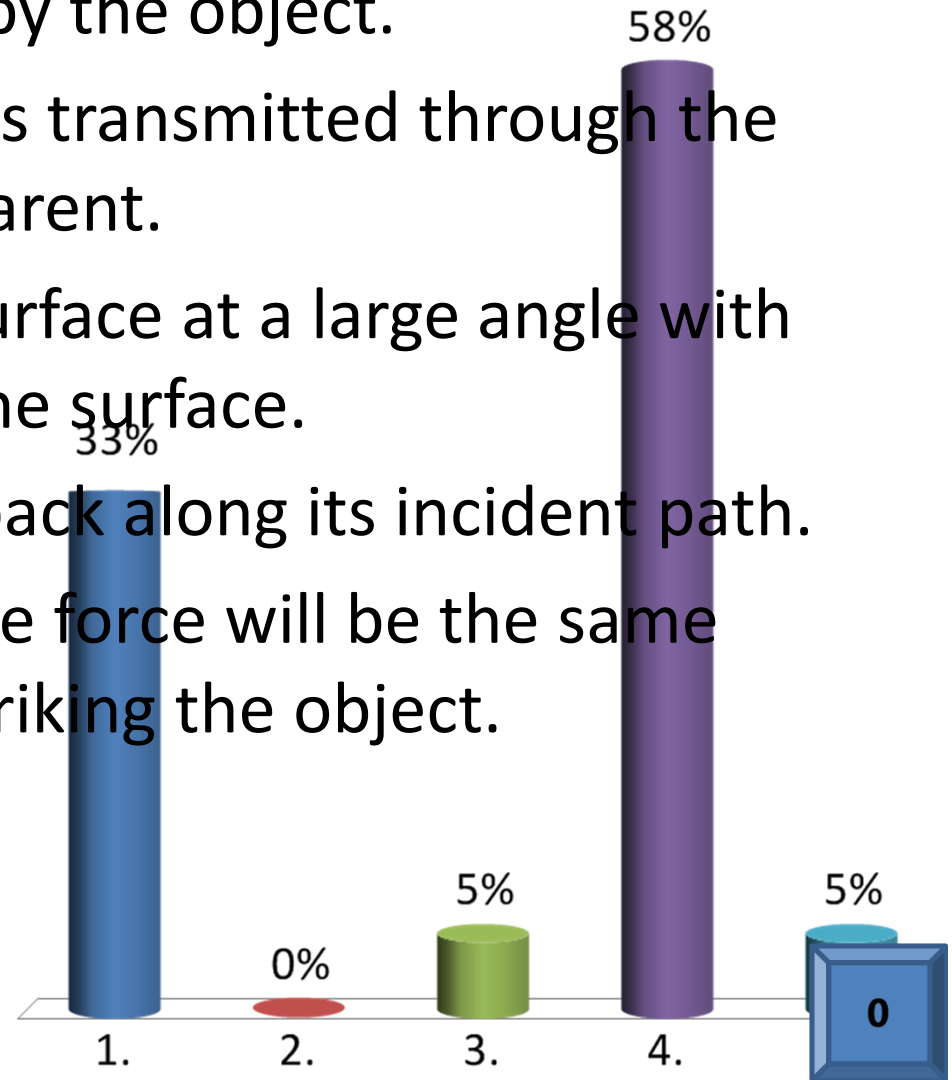
$$\Delta p = \frac{\Delta U}{c} = \frac{IA \Delta t}{c}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{IA}{c}$$

$$F/A = I/c = \text{Pressure}$$

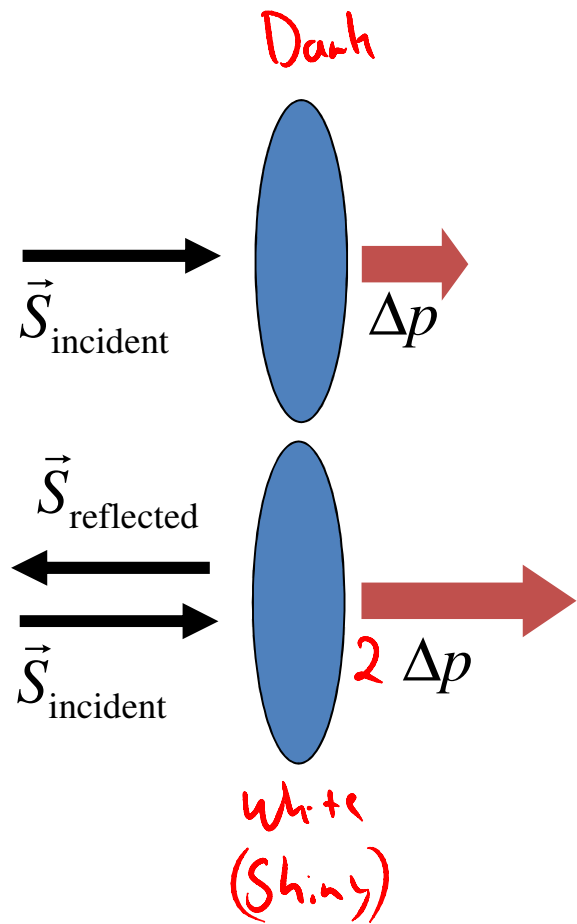
In which of the following cases maximizes the force exerted on an object by electromagnetic radiation?

1. The radiation is absorbed by the object.
2. Nearly all of the radiation is transmitted through the object because it is transparent.
3. The radiation strikes the surface at a large angle with respect to the normal to the surface.
- ✓ 4. The radiation is reflected back along its incident path.
5. In all of the above cases the force will be the same since it is the same light striking the object.



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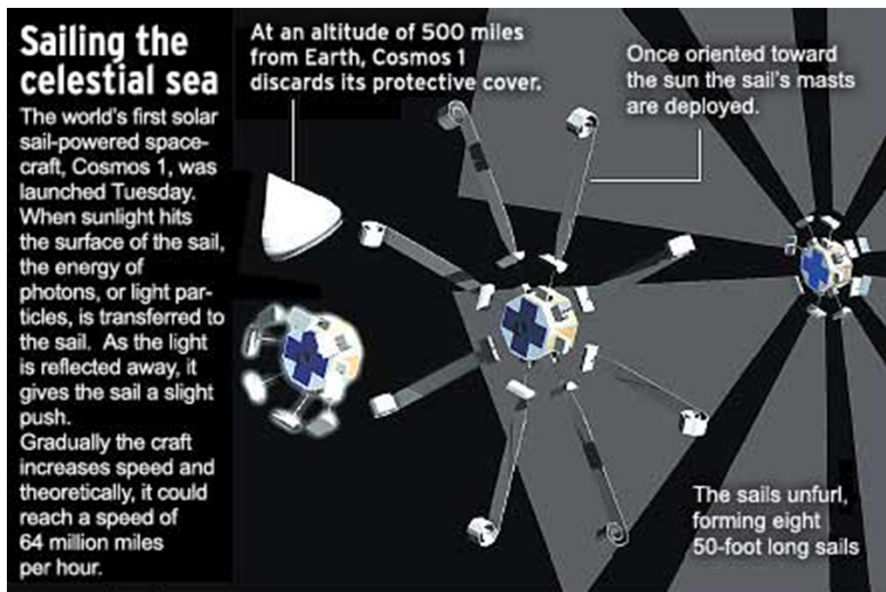
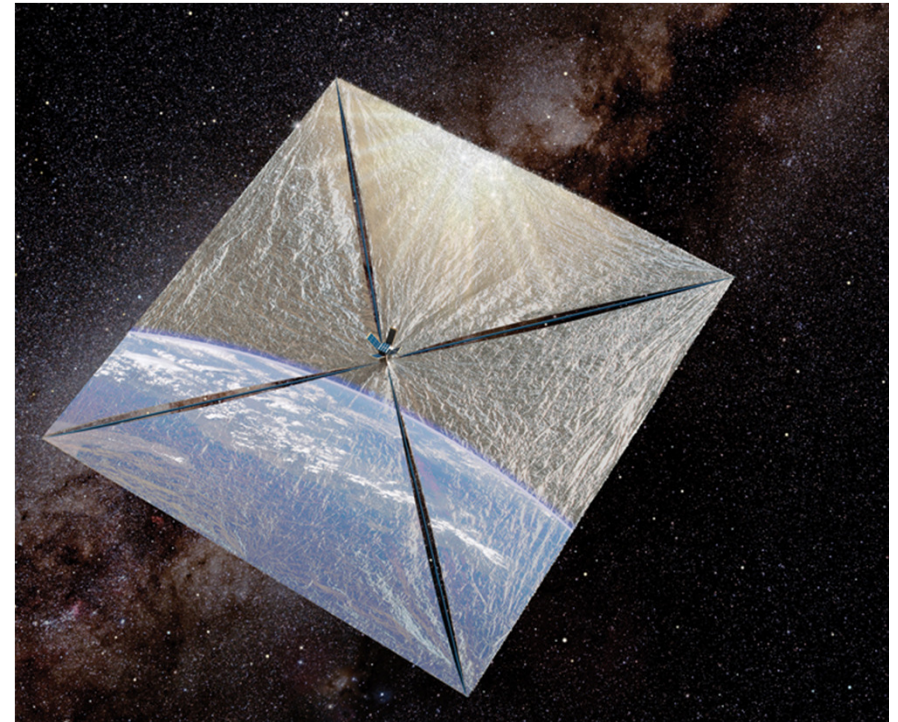
$$\frac{F}{A} = \frac{I}{c}$$



.

Solar Sailing (Planetary Society efforts)

Lightsail-1
(on short list for an
upcoming nanosat launch)

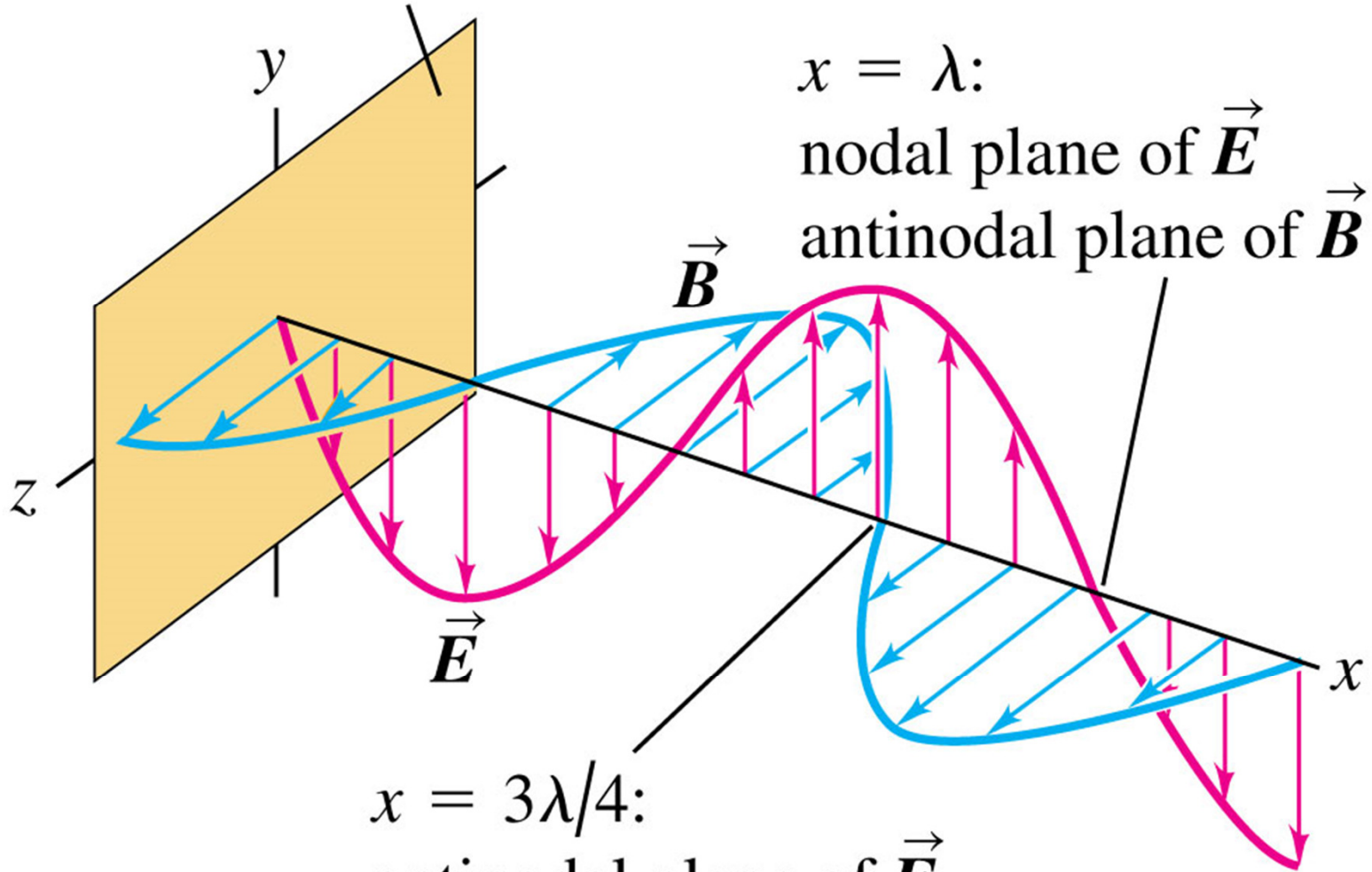


Source: The Planetary Society

AP

COSMOS-I
(lost in launch failure, 2005)

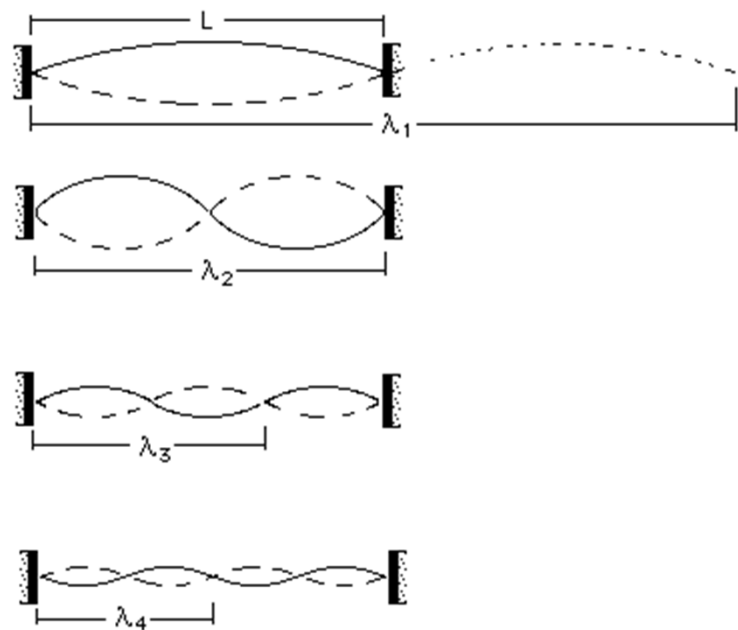
Perfect conductor



$x = \lambda$:
nodal plane of \vec{E}
antinodal plane of \vec{B}

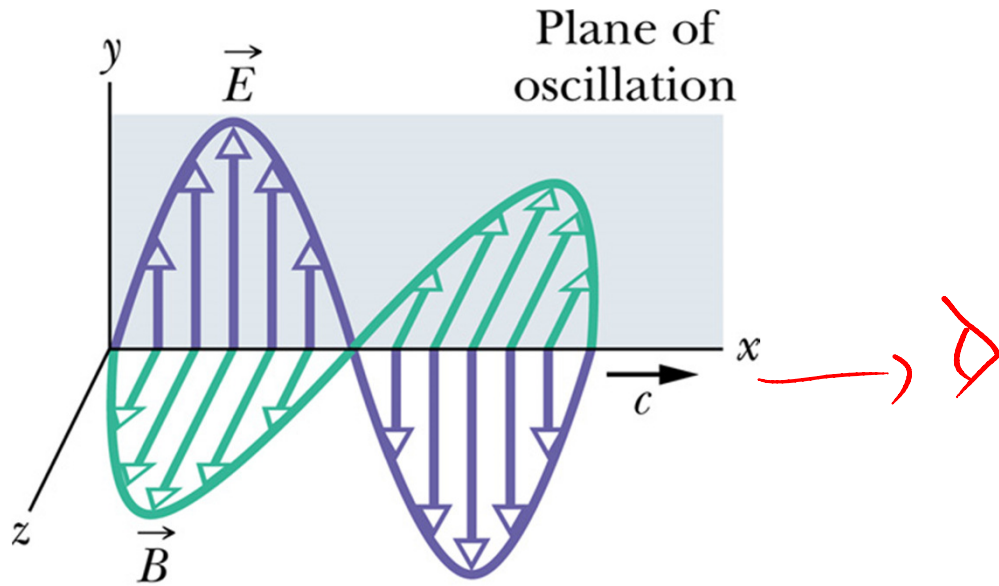
$x = 3\lambda/4$:
antinodal plane of \vec{E}
nodal plane of \vec{B}

$$E(x,t) = \sin(kx - \omega t)$$



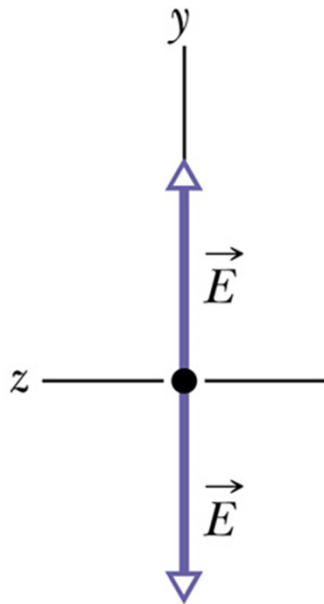
Cartoon swiped from U. of New South Wales' physics webpage





(a)

"Polarization Axis"



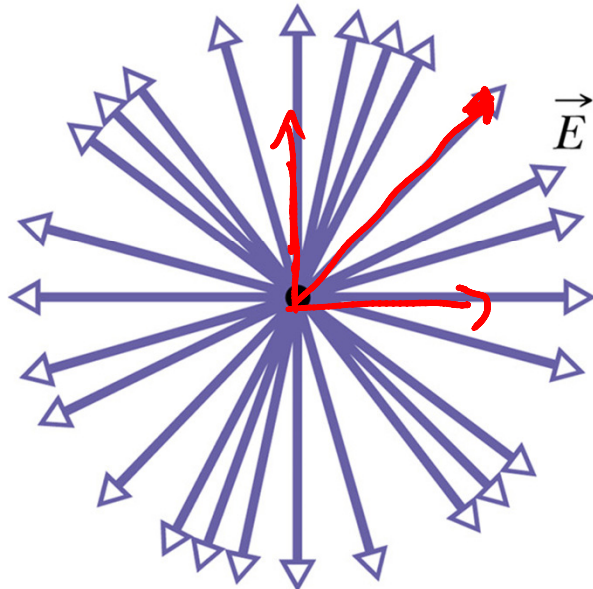
(b)

Linear

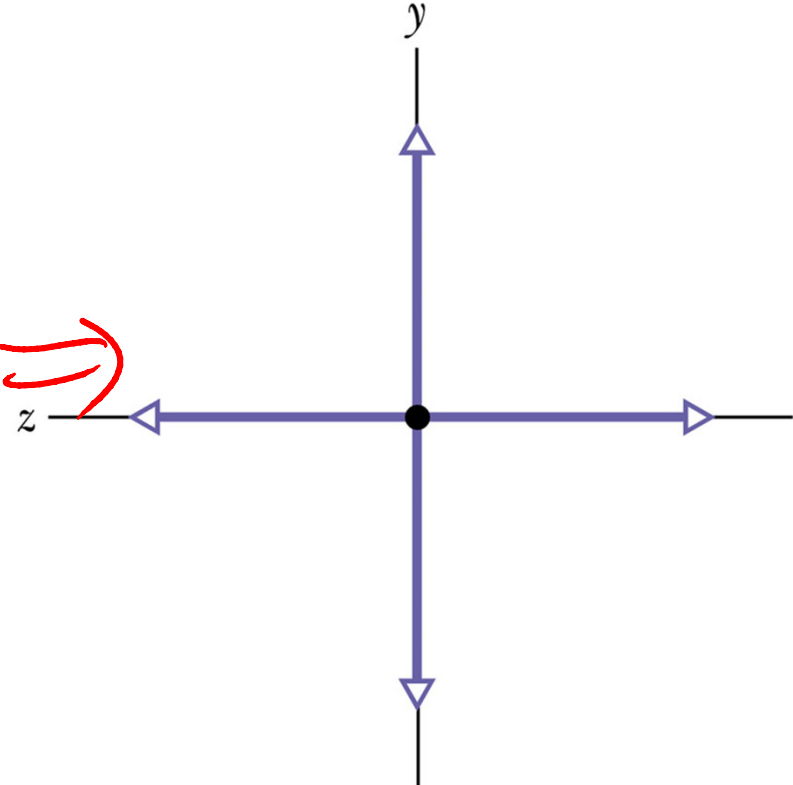
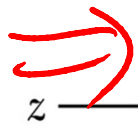
Circular Polarization



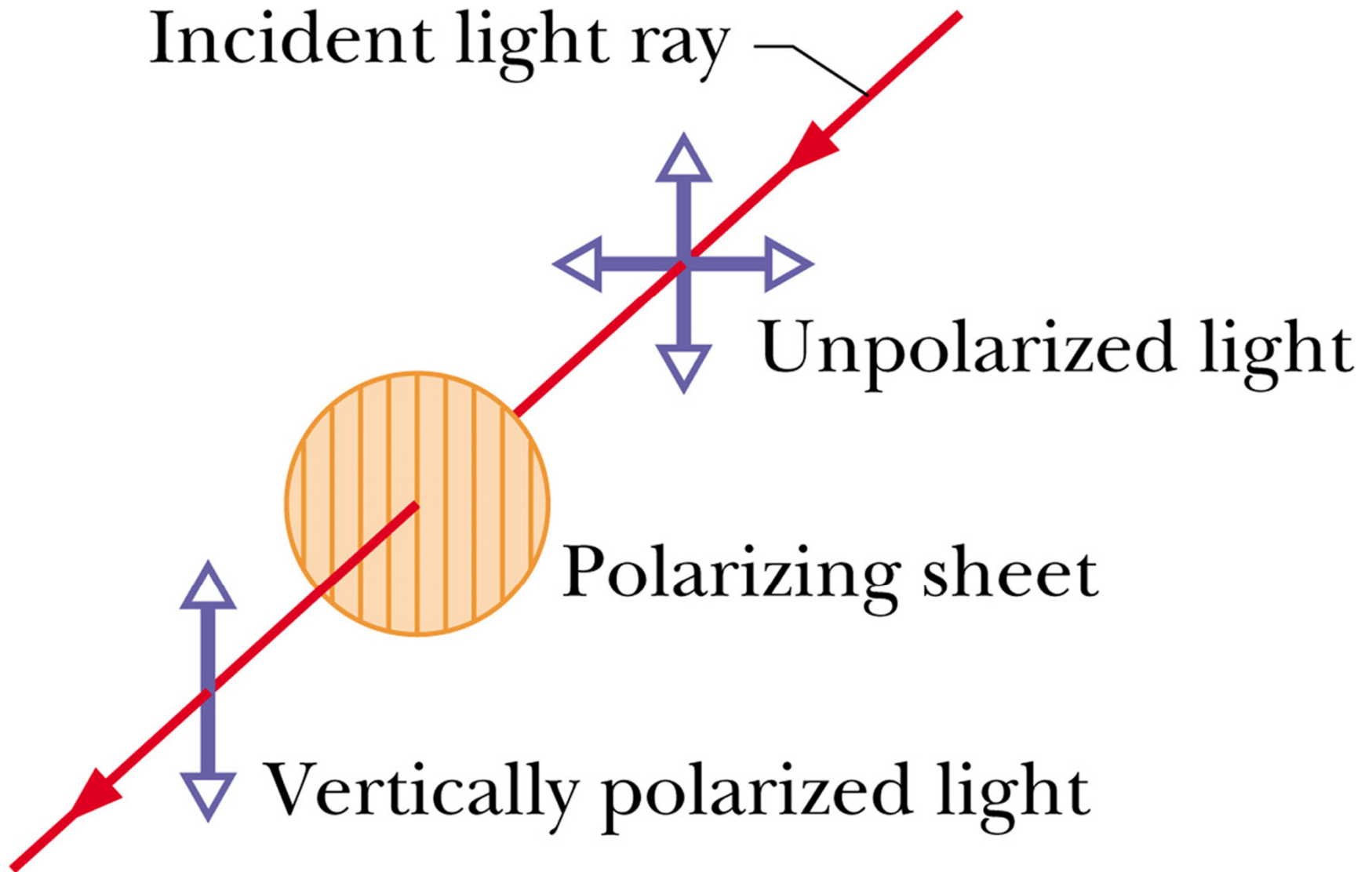
Unpolarized

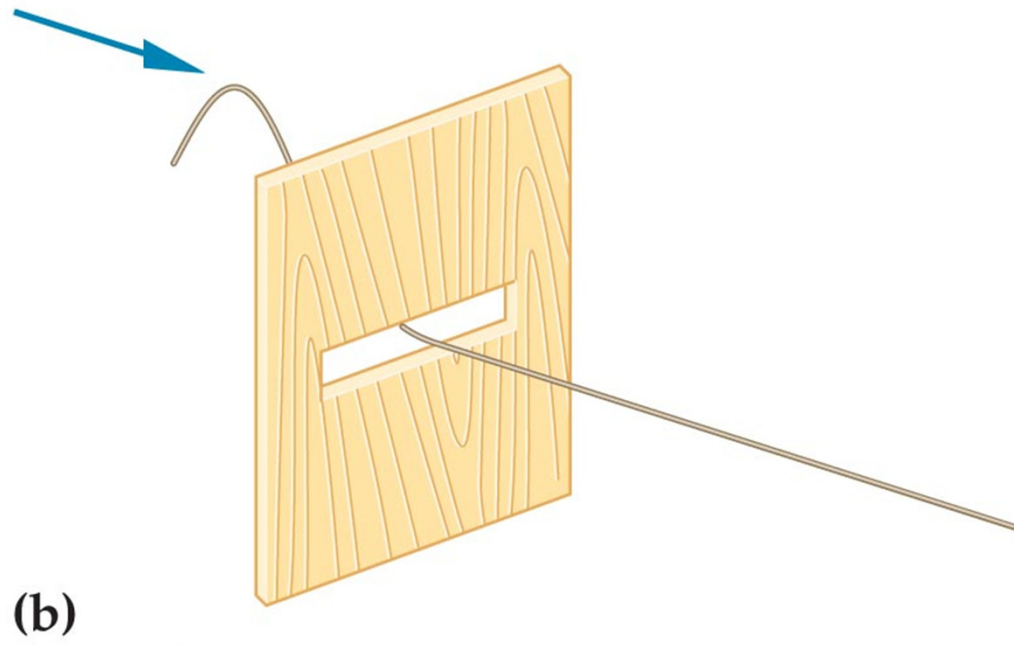
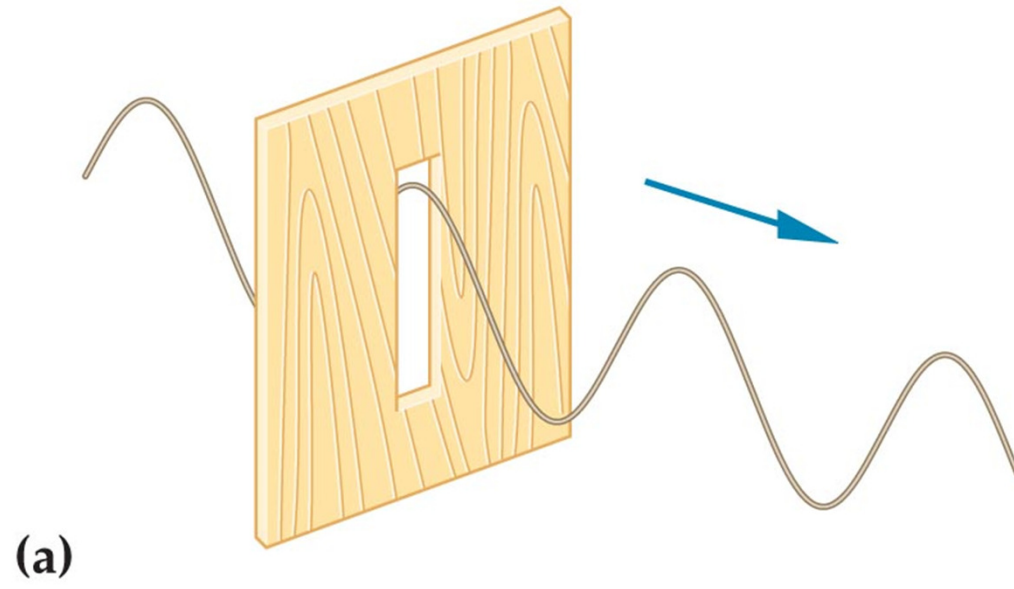


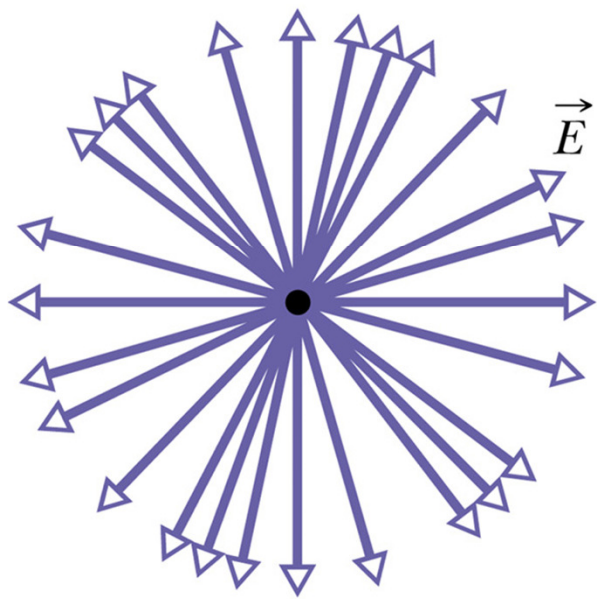
(a)



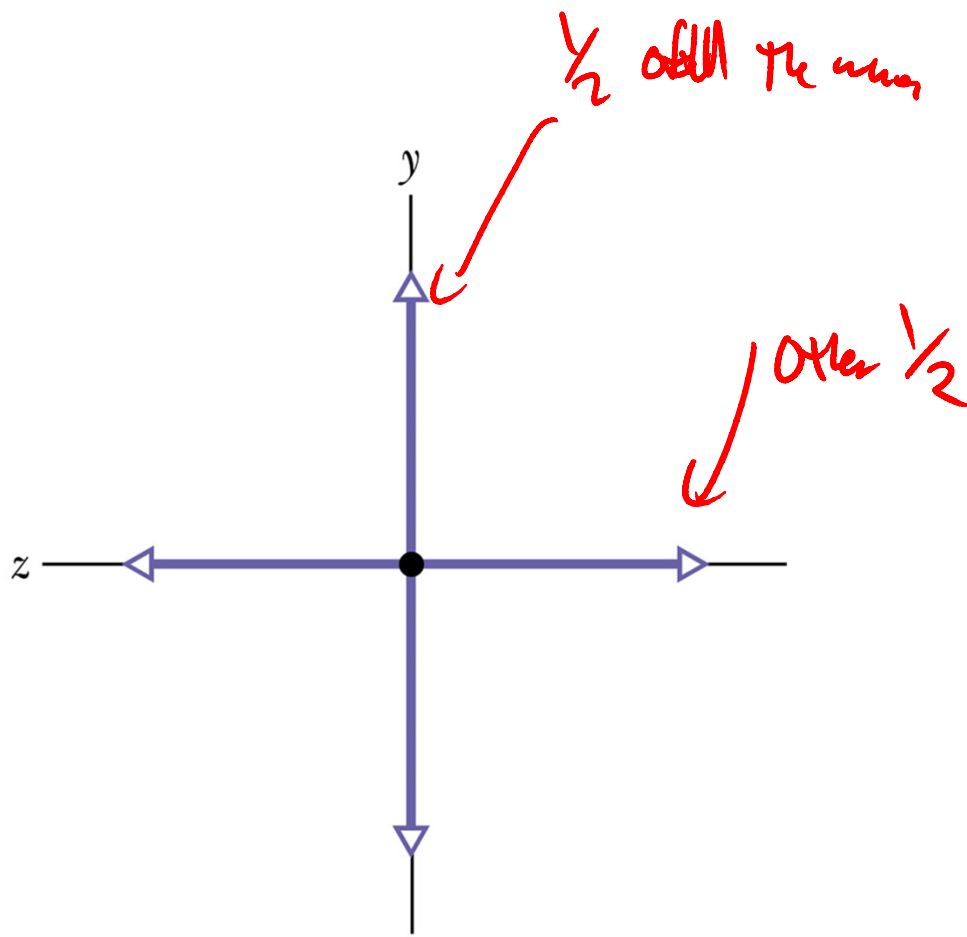
(b)



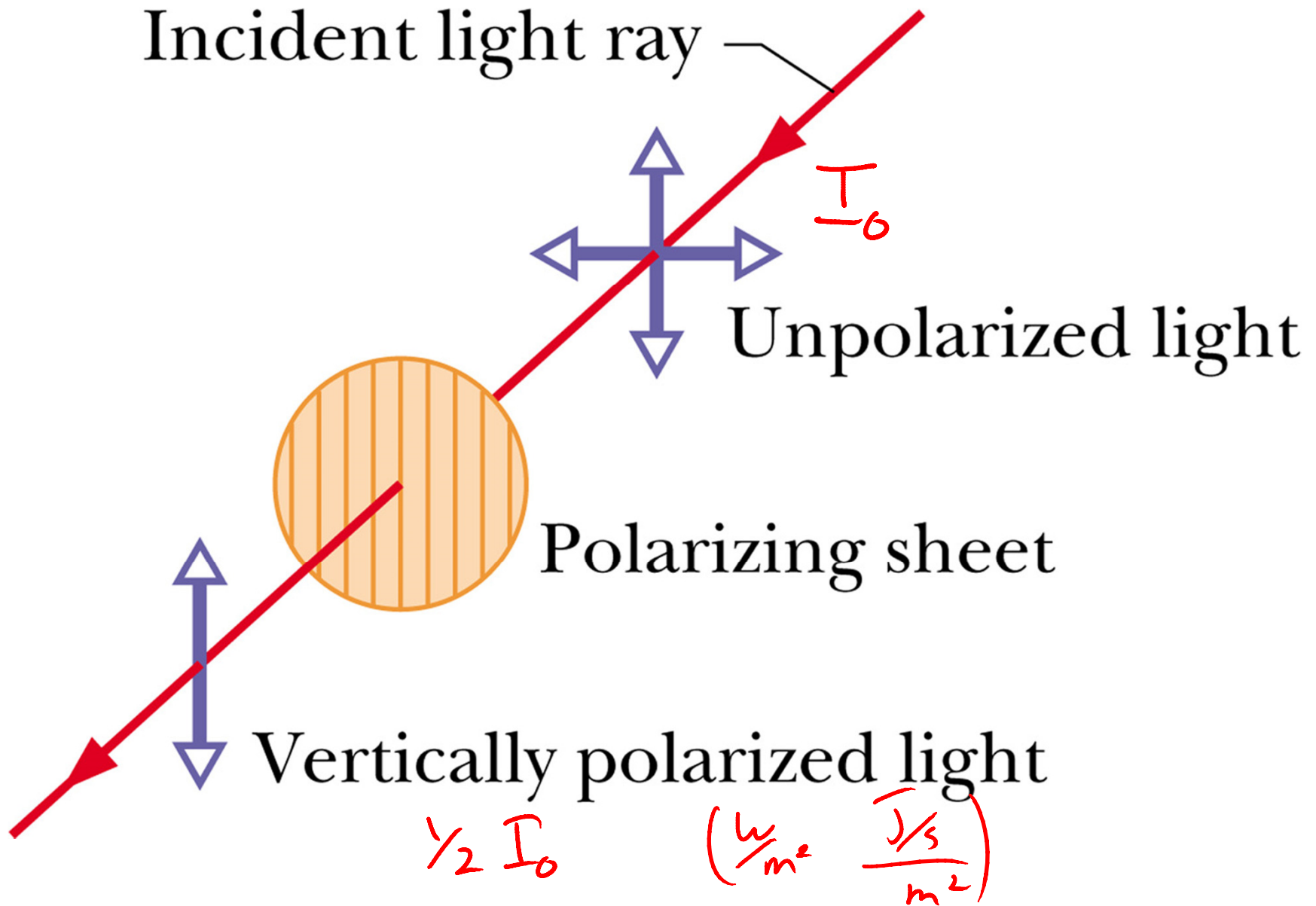




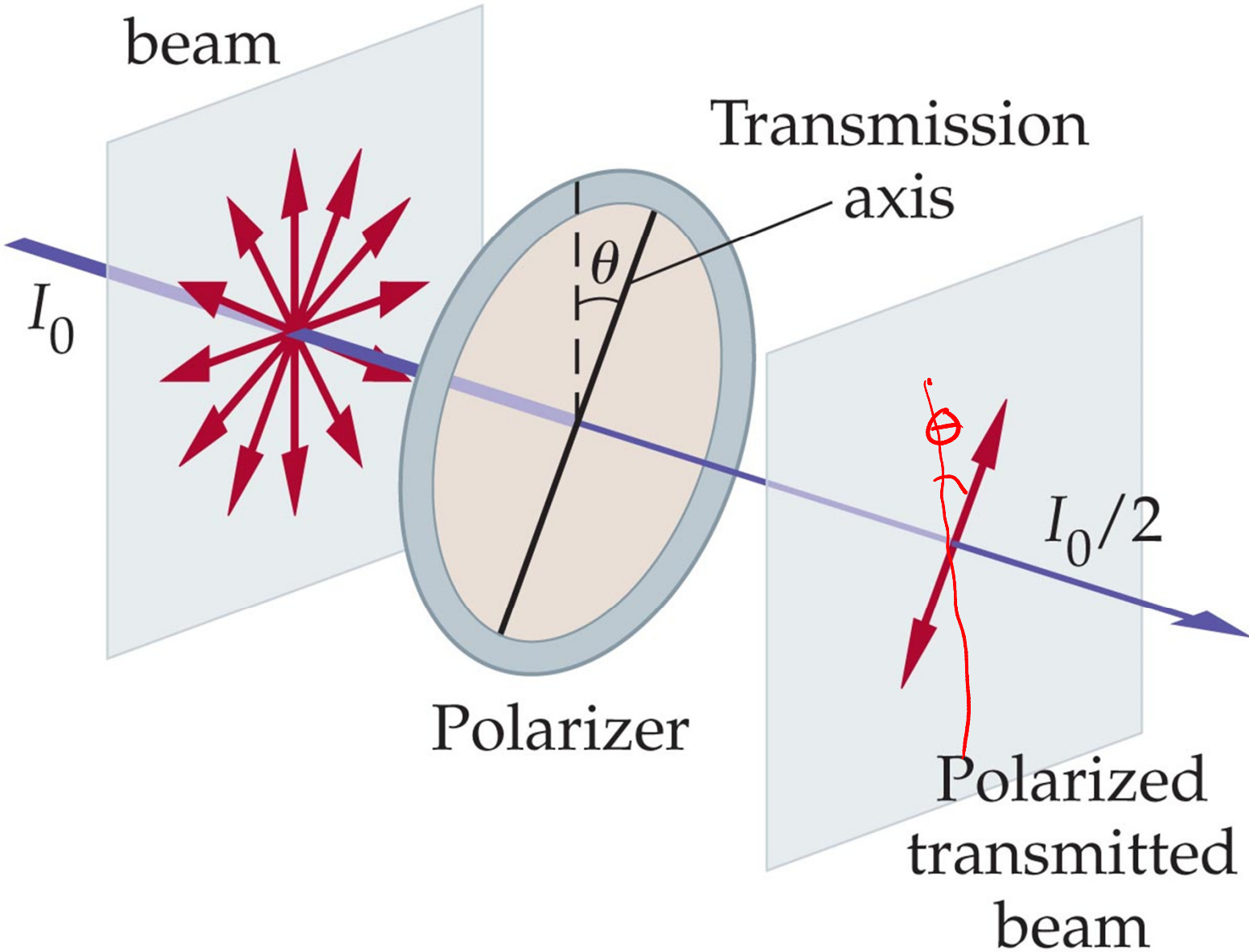
(a)



(b)



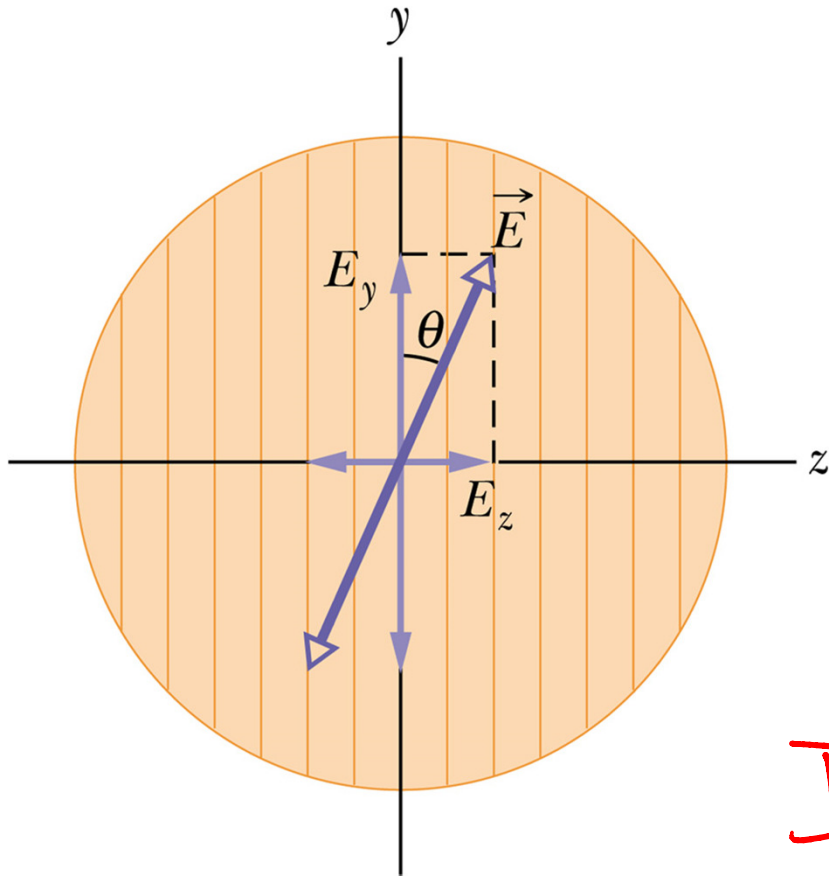
Unpolarized
incident
beam



Transmission
axis

Polarizer

Polarized
transmitted
beam



$$E_1 = |E| \cos \theta$$

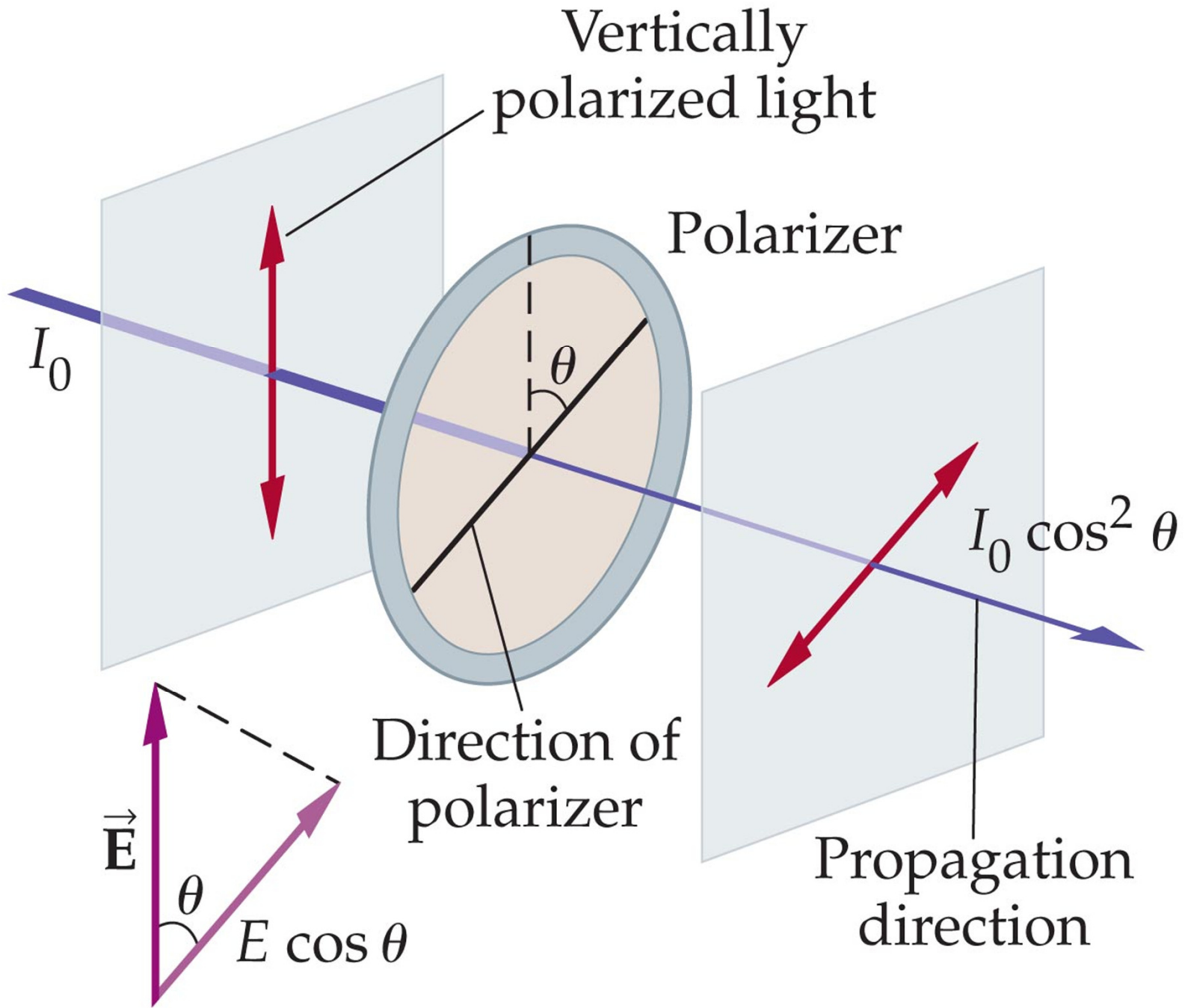
$$E_2 = |E| \sin \theta$$

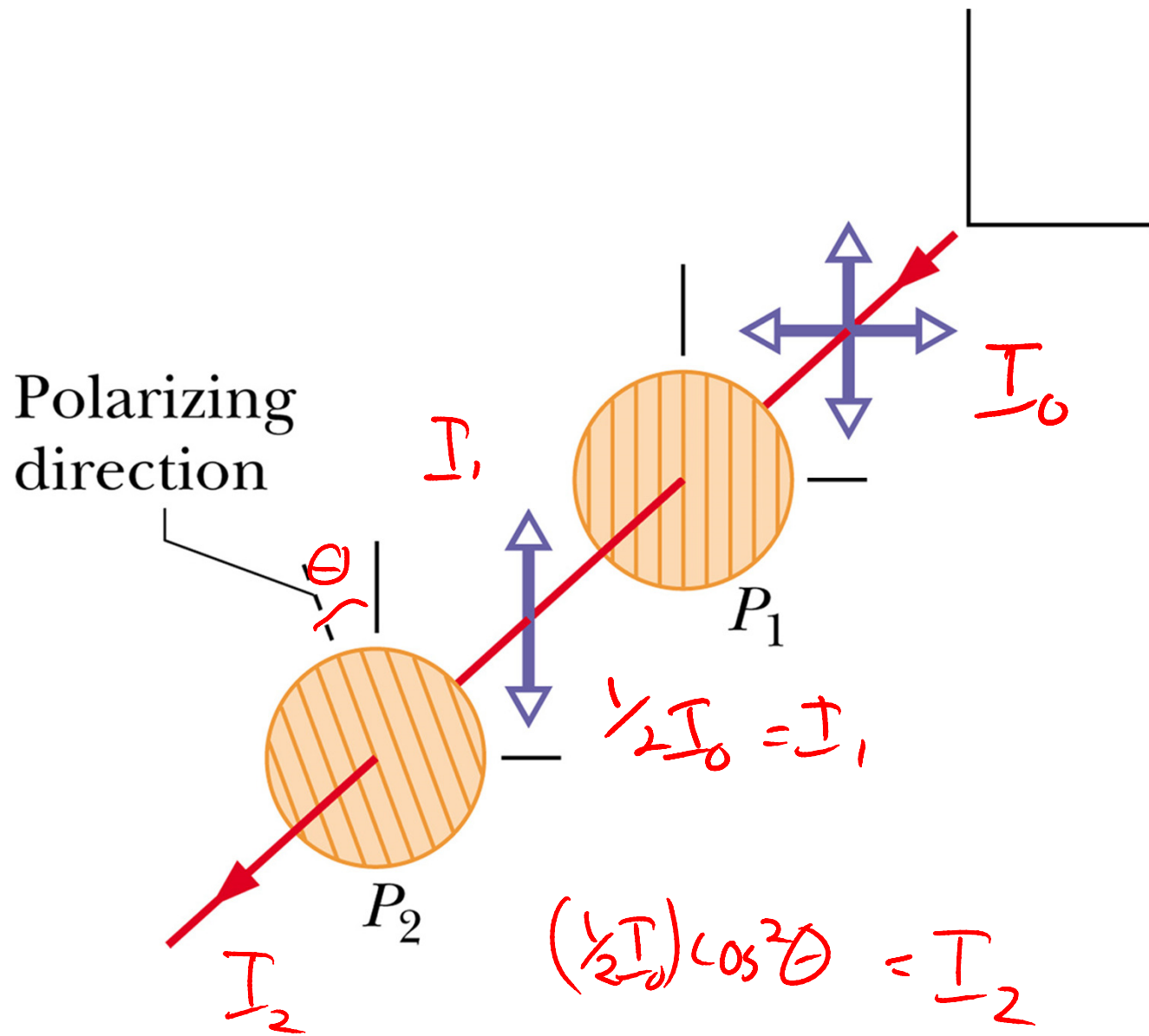
\downarrow
 This E gets
 through

$$I_{\text{thru}} = I_0 \cos^2 \theta$$

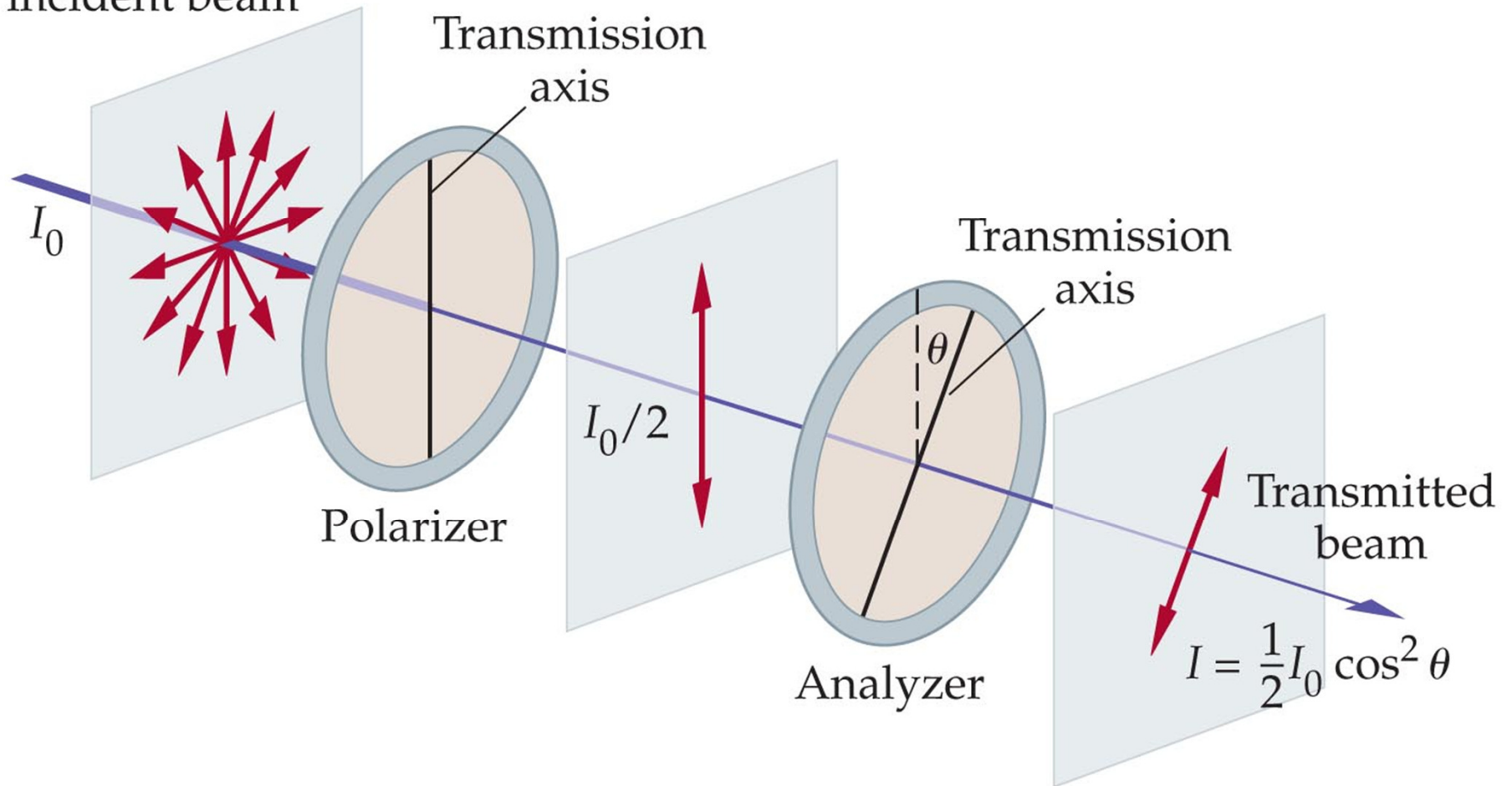
$$I \propto E_{\text{avg}}^2 = \frac{E_{\text{avg}}^2}{c \mu_0}$$

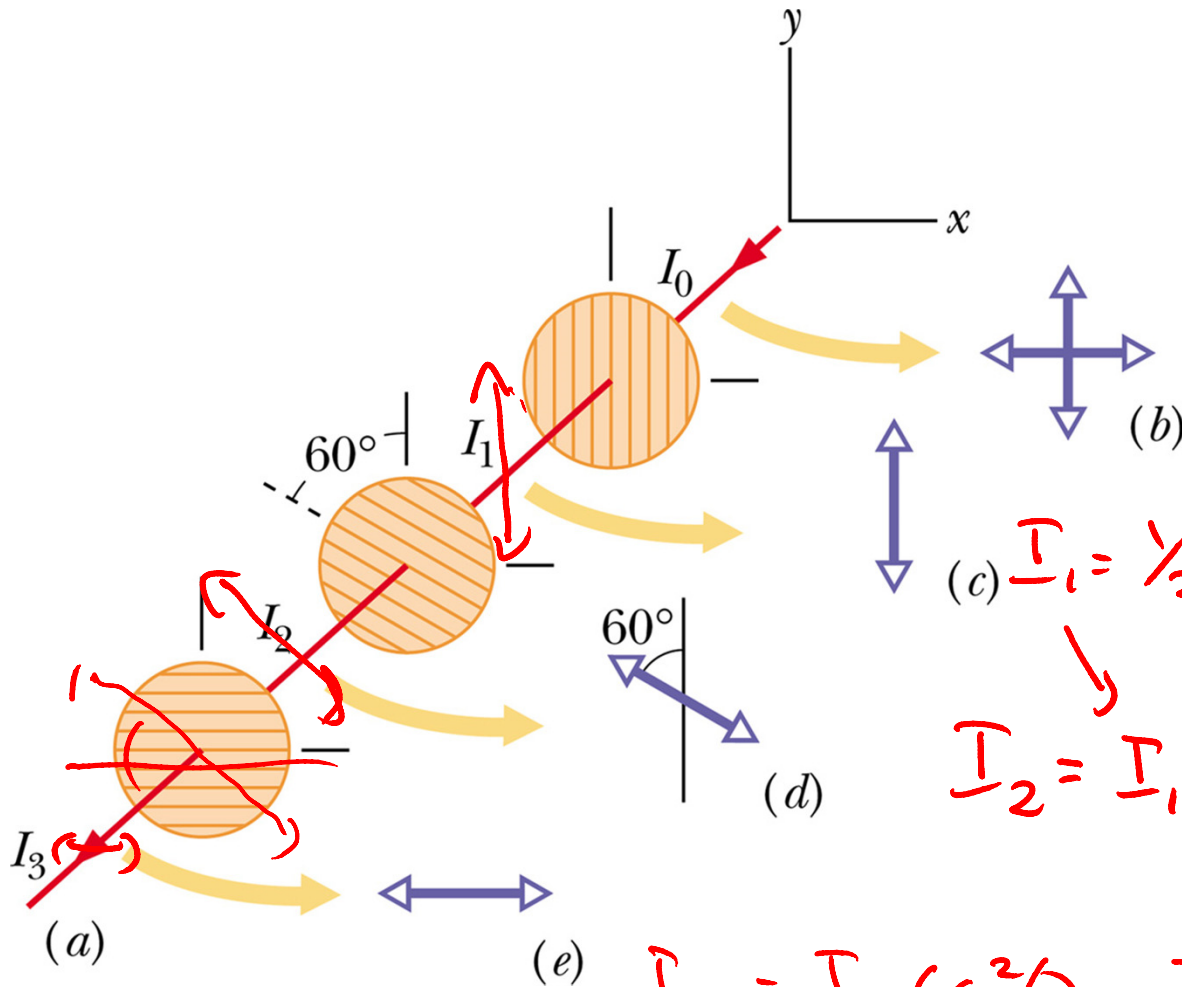
$$I \propto E^2$$





Unpolarized incident beam

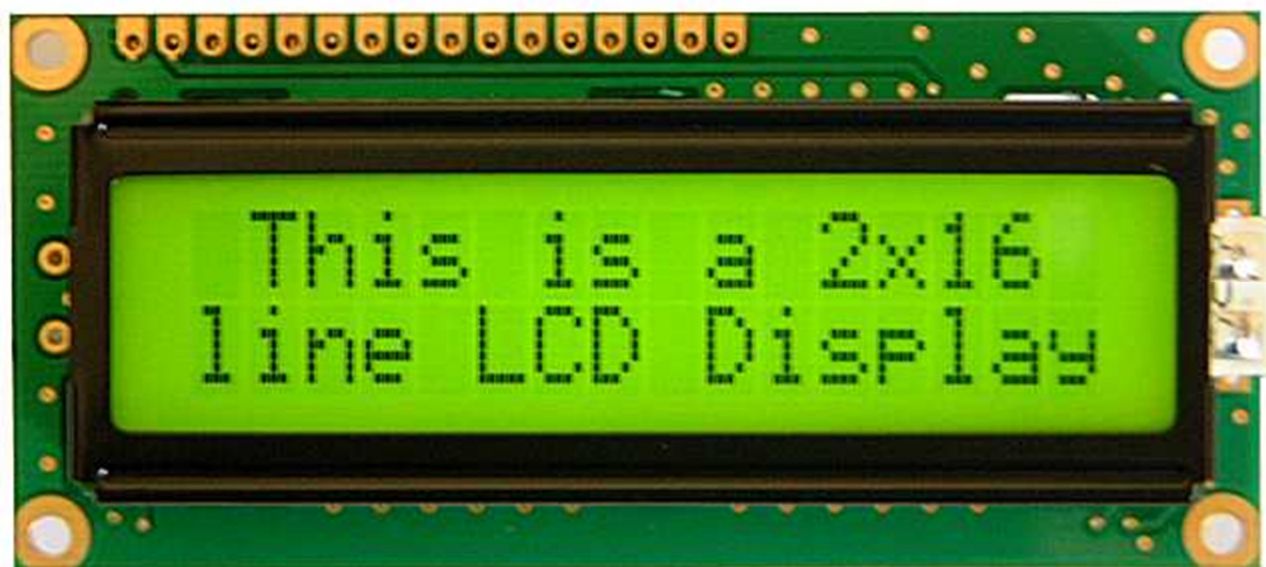


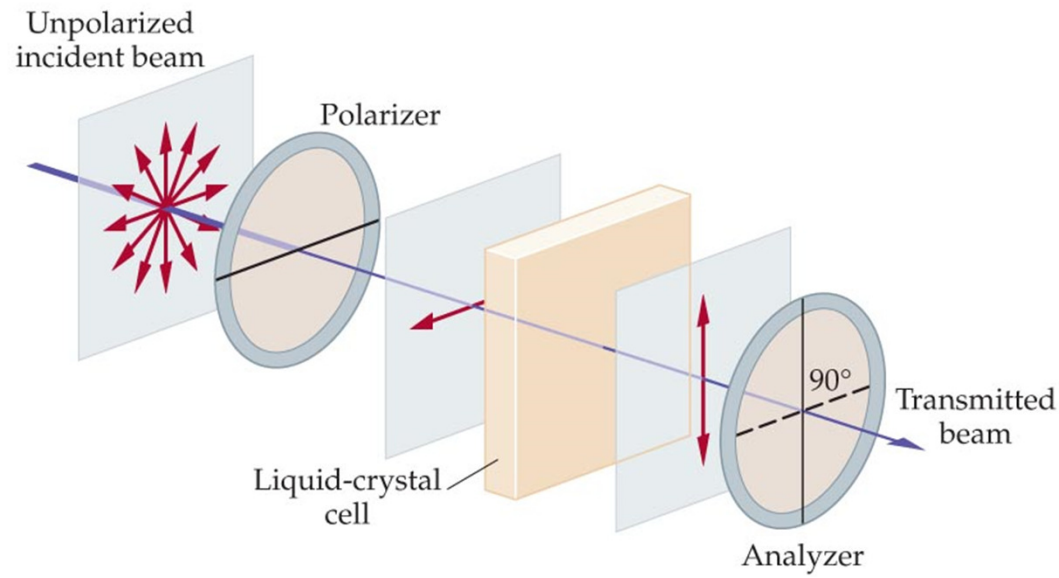


(c) $I_1 = \frac{1}{2} I_0$

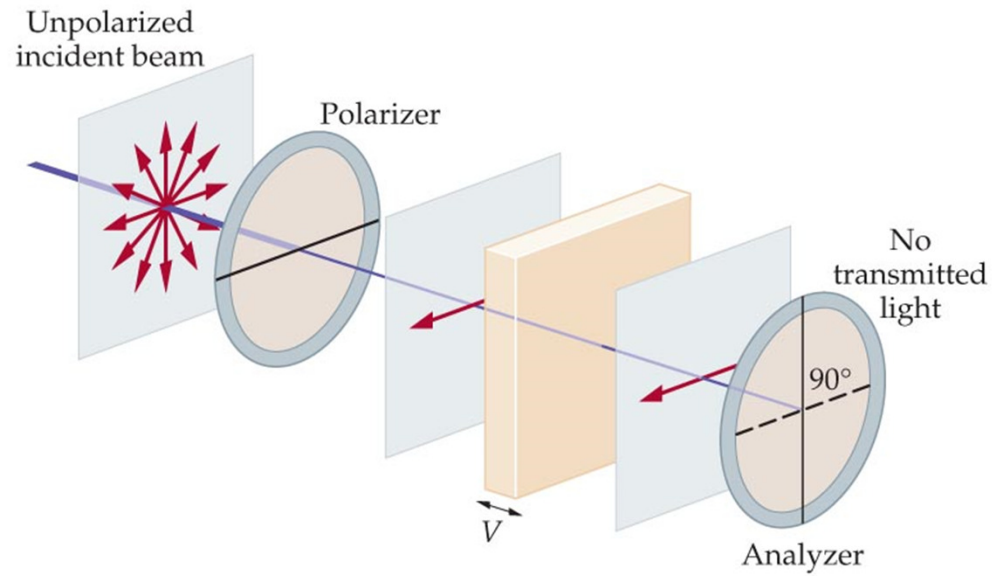
$I_2 = I_1 \cos^2(60^\circ) = \frac{1}{2} I_0 \cos^2(60^\circ)$

$I_3 = I_2 \cos^2(\theta) = I_2 \cos^2(90-60)$





(a) Off (transmitted light gives bright background)



(b) On (dark characters formed where no light is transmitted)

