Ratio of circumference and diameter of a circle

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Abstract

In this investigation, we examined the hypothesis that the circumference (C) and diameter (D) of a circle are directly proportional. We measured the circumference and diameter of five circular objects ranging from 2 cm to 7 cm in diameter. Vernier calipers were used to measure the diameter of each object, and a piece of paper was wrapped around each cylinder to determine its circumference. Numerical analysis of these circular objects yielded the unitless C/D ratio of 3.14 ± 0.03, which is essentially constant and equal to pi. Graphical analysis lead to a less precise but equivalent estimate of 3.15 ± 0.11 for this same ratio. These results support commonly accepted geometrical theory which states that $C = \pi D$ for all circles. However, only a narrow range of circle sizes were analyzed, so additional data should be taken to investigate whether the constant ratio hypothesis applies to very large and very small circles.

Introduction

Going back at least as far back as the ancient greeks, people remarked that circles appear to share common proportions. Measurements of two different features of a circle should yield a single ratio, regardless of the absolute size of the object. In the case of circles, the distance around the circle (its circumference) and the distance across the circle on a line through the center (the diameter) are in the ratio of roughly 3:1 . Measurements and geometrical calculations of this proportion are well known to be 3.1415926 to eight significant figures, and this value is given the special symbol π, the greek letter pi.

In this lab, we have performed measurements to test the validity that the ratio is common to all circles. Finding that it is, then we obtain a value for the ratio from the measurements. Because the measurements are made on five different sized circles, two different measurement techniques are used, each appropriate to the size of the sample.

Materials and Methods:

Five round objects: "D" cell battery, two short pieces of PVC pipe, tomato soup can, penny coin

Metric ruler with millimeter resolution

Vernier caliper with 0.05 mm resolution

Five objects were chosen such that measurements of their circumference and diameter could be obtained easily and would be reproducible. Therefore, we did not use irregularly shaped objects or ones that could be deformed when measured. The diameter of each of the five objects was measured with either the ruler or caliper. The circumference and diameter of each object was measured with the same measuring device in case the two instruments were not calibrated the same. The circumference measurement was obtained by tightly wrapping a small piece of paper around the object, marking the circumference on the paper with a pencil, and measuring this distance with the ruler or caliper. The uncertainty specified with each measurement is based on the precision of the measuring device and the experimenter's estimated ability to make a reliable measurement.

Data, Calculations, Results:

The raw measurements for the five objects are as follows.

Object Description	Diameter (cm)	Circumference. (cm)	Measuring Device
Penny coin	1.90 ± 0.01	5.93 ± 0.03	Vernier caliper, paper
"D" cell battery	3.30 ± 0.02	10.45 ± 0.05	Vernier caliper, paper
PVC cylinder A	4.23 ± 0.02	13.30 ± 0.03	Vernier caliper, paper
PVC cylinder B	6.04 ± 0.02	18.45 ± 0.05	Plastic ruler, paper
Tomato soup can	6.6 ± 0.1	21.2 ± 0.1	Plastic ruler, paper

Table 1: raw measurements for five objects, and the methods used to obtain the measurements.

The C/D value for the penny is (5.93 cm)/(1.90 cm) = 3.12 (no units). The precision of the ratio can be estimated using the error propagation formula:

[so give the formula appropriate to this particular calculation]

Results with the calculation and final uncertainty for all five objects are given in the table below.

Object Description	Diameter (cm)	Circumfer. (cm)	C/D calculated (no units)
Penny	1.90 ± 0.01	5.93 ± 0.03	3.12 ± 0.02
"D" cell battery	3.30 ± 0.02	10.45± 0.05	3.17 ± 0.02
PVC cylinder A	4.23 ± 0.02	13.30 ± 0.03	3.14 ± 0.02
PVC cylinder B	6.04 ± 0.02	18.45 ± 0.05	3.06 ± 0.01
Tomato soup can	6.6 ± 0.1	21.2 ± 0.1	3.21 ± 0.05

Table 2: the raw measurements, and the calculated ratio of circumference/diameter, with its measurement uncertainty.

Average C/D = 3.14 ± 0.03, where 0.03 is the standard error of the 5 values.

Analysis and discussion

From this empirical investigation, the average C/D ratio is 3.14 ± 0.03 (no units). This ratio agrees with the accepted value of pi (3.1415926…). The uncertainty associated with the average C/D ratio is the standard error of the five C/D values, which is equal to the standard deviation (0.06) divided by the square root of N, which in this case is 5 since there were five measurements.

While the five C/D values do not agree within their estimated uncertainties, the variation between these values is relatively small (only about 0.06/3.14 = 2%), which suggests that the C/D ratio is a constant value. The reason for the imperfect agreement may be that the individual uncertainties were underestimated or perhaps is a consequence of the "paper" method used for measuring the diameters of the object. The paper may have slipped while we made the mark, but this "slip effect" should only be a random error, which would not affect the average value of our measurements for C, since there is no reason to believe that the paper would have consistently slipped in the same direction (either too high or too low) every time.

Another way to visualize and calculate this constant circle ratio is by graphing the circumference versus diameter for each object. Graphs are especially useful for examining possible trends over the range of measurements. They are especially successful if the axis labels and titles are big and readable.

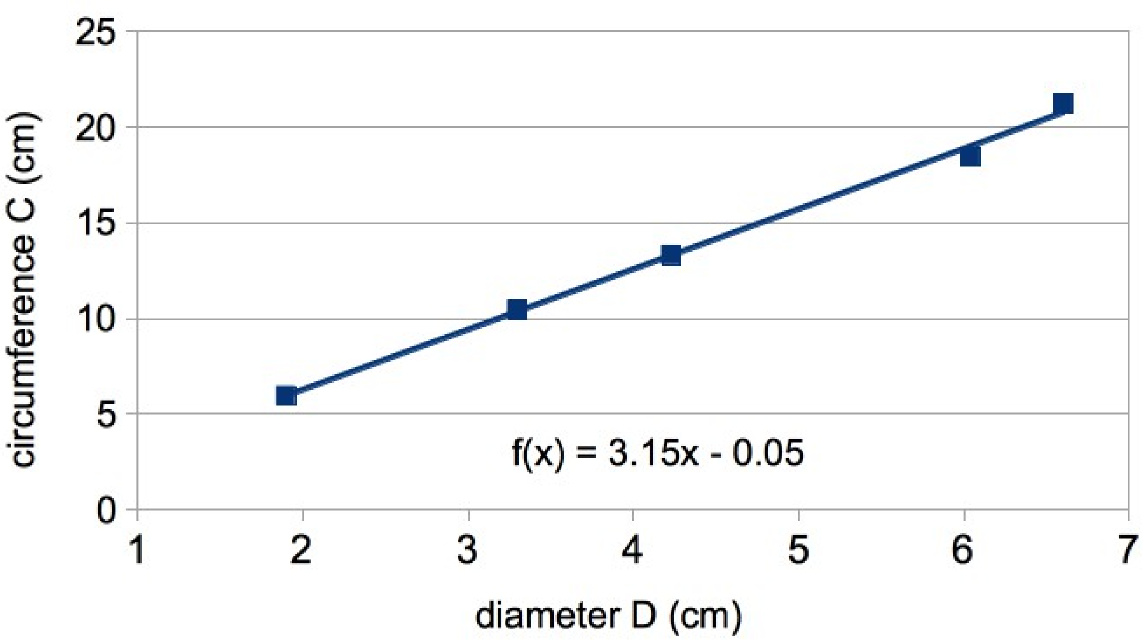

Figure 1. Graphical illustration of the correlation of circumference and diameter with a least-squares fit done by the spreadsheet program. The error bars are too small to see.

If C is proportional to D, we should get a straight line through the origin. From our numerical results, we would expect the slope of the C vs. D graph to be equal to pi. The slope of the best fit line is (3.15 ± 0.11), which is equal to pi within its uncertainty. The intercept is essentially zero: (-0.05 ± 0.5).

Further discussion

Our results support the original hypothesis for five circles ranging in size from 2 cm to 7 cm in diameter. The C/D ratio for our objects is essentially constant (3.14 ± 0.03) and equal to the known value of π . The specified uncertainty is the standard error of the C/D ratio for the five objects. Graphical analysis also supports the "directly proportional" hypothesis. The line has an intercept (-0.05 ± 0.5) that is equal to zero within the uncertainty and a slope (3.15 ± 0.11) equal to pi . The larger uncertainty from the graphical analysis suggests that the random measurement errors may be larger than estimated in the numerical analysis. A more extensive investigation of this C/D relationship over a wider range of circle sizes should be performed to verify that this ratio is indeed constant for all circles.

The uncertainty in the measurements could be due to the paper-wrapping method of measuring the circumference, circles that may not be perfect, and the limited precision of the measuring devices. The use of paper to measure the circumference was probably the most significant source of uncertainty. It is unlikely, however, that this measurement technique biased our results, since the technique probably gave measurements of C that were too high in some cases and too low in others.

The C/D ratio for a perfect circle was defined long ago by the Greek symbol: π = 3.14159… Our measured value appears to be consistent with the accepted value of π within the limits of our experimental uncertainty. This unique C/D ratio has many important applications wherever circles or spheres are encountered. More information about π can be found at: http://en.wikipedia.org/wiki/Pi

Citations

This lab report was modified from one at UNC Chapel Hill, with major sections taken verbatim. I dare you to compare this one to the original and figure out what I thought was an improvement!

physics.unc.edu/undergraduate-program/labs/sample-report

Some helpful guides to effective scientific lab-report style writing (halfway down this page) https://www.ncsu.edu/labwrite/res/res-homepage.html

And obviously the wikipedia page http://en.wikipedia.org/wiki/Pi